In-medium chiral dynamics and hypernuclear structure

(...on the microscopic origin of the Spin-Orbit interaction)

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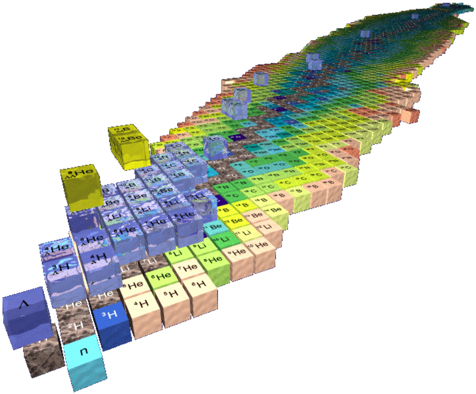
Les Houches Summer School

Extended nuclear chart table

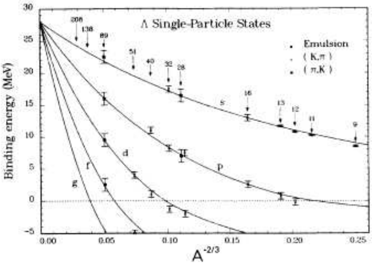


[From HYP 2006 web site]

- a microscopic description of the spin-orbit interaction.
- a common mechanism both for nuclei and hypernuclei.

About hypernuclei

- Normal nucleus: composed of nucleons (proton, neutron). At the quark level: p=(uud), n=(udd)
- Hypernucleus: not only nucleons but hyperons (quarks other than u and d)
- We will consider only Λ-hypernuclei. At the quark level Λ = (usd)
- Notation: ${}^{A}_{\Lambda}Z$
  - A: Total baryons
  - Z: Total charge
  - Λ: hyperon
- No Pauli effect, weak coupling



[D. J. Millener et al., PRC38 (1988) 2700]

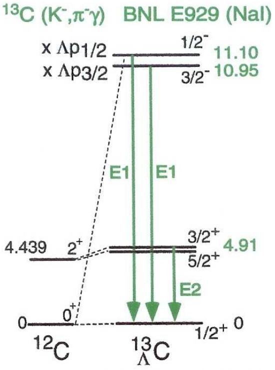
About hypernuclei

Concerning the spin-orbit interaction, hypernuclei are interesting systems because:

Recent spectroscopic investigations using (K^-, π^-) reactions suggested an extraordinary weakness of the Λ-nucleus spin-orbit interaction

$$\delta_{(p_{3/2}-p_{1/2})} = (152 \pm 54 \pm 36)\ keV$$

for ${}^{13}_{\Lambda}$C (20 times smaller than the $p_{3/2} - p_{1/2}$ spin-orbit splitting of 5 MeV for nucleons).



PRL 86 (2001) 4255

Theoretical interpretations

- Considering a simple relativistic mean field approach, the spin-orbit potential arise from a coherent superposition of scalar $V_S(r)$ and vector fields $V_V(r)$

$$V_{LS}(r) \sim -\frac{\frac{d}{dr}(V_S(r) - V_V(r))}{4M^2} \frac{\boldsymbol{l} \cdot \boldsymbol{\sigma}}{r}$$

with

$$V_S \simeq -500\ MeV \quad V_V \simeq 400\ MeV$$

- Arguments from quark models (the strange quark inside the Λ does not interact with the up and down quarks of the nucleons) suggest a 2/3 reduction

$$V_S^\Lambda = \frac{2}{3} V_S \quad V_V^\Lambda = \frac{2}{3} V_V$$

➡ impossible to obtain a small spin-orbit potential

Theoretical interpretations: solutions

- Brockmann and Weise: weak Λ-meson couplings ($1/3$ and even smaller).
- Pirner *et al.* kept the factor $2/3$ but introduced a strong tensor coupling

$$\mathcal{L} = -g_\omega \bar{\psi}\gamma^\mu \psi \omega_\mu + \frac{f_\omega}{2M_\Lambda}\bar{\psi}\sigma^{\mu\nu}\psi\partial_\nu\omega_\mu \quad \textit{with} \quad f_\omega = -g_\omega$$

➡ successful reproduction of experimental data, but correct?

- DDRH (Lenske *et al.*), Quark Meson Coupling model (Thomas *et al.*), many others...

[Phys. Lett. B **69** (1977) 167; Phys. Lett. B **246** (1990) 325; Phys. Rev. C **61** (2000) 064309;...]

... a microscopic explanation of the anomalously small Λ-nucleus spin-orbit force is still missing.

χPT as low-energy QCD

- Nuclei are aggregates of quarks and gluons in the hadronic phase of QCD.
- Confinement $\Rightarrow$ Eigenstates of H_{QCD} are colour-singlet hadrons.
- Spontaneously broken Chiral $SU(2)_R \times SU(2)_L$ Symmetry:
  - Non trivial vacuum: Chiral quark condensate

    $$|\langle \bar{q}q \rangle| \simeq 1.5\ fm^{-2}$$

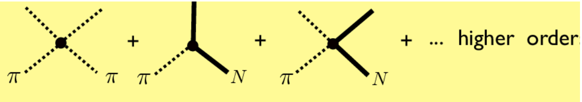
  - Low mass collective excitations: Pions are Goldstone bosons, they interact weakly at low energy/momentum.
  - Characteristic mass gap in the hadron spectrum:

    $$4\pi f_\pi \sim 1\ GeV$$

Low-energy χPT Lagrangian

- Low energy QCD: Effective Field theory for weakly interacting Goldstone bosons (Pions)
- Interacting systems of pions and nucleons

$$\mathcal{L}_{eff} = \mathcal{L}_{\pi}(U, \partial U) + \mathcal{L}_{\pi N}(\Psi_N, U, \ldots)$$
$$U(x) = exp(i\tau_a \pi_a(x)/f_\pi)$$



- Small scale expansion

$$\frac{Q}{4\pi f_\pi} \quad \frac{energy/momentum/pion\ mass}{mass\ gap\ of\ order\ 1\ GeV}$$

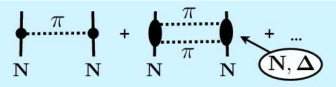
[Literature: Weinberg; Gasser and Leutwyler; Ecker; Scherer; Bernard, Kaiser and Meissner,...]

In-medium χPT extension

- Pions (π) and Delta isobars (Δ) as explicit degrees of freedom

  [N. Kaiser, S. Fritsch and W. Weise, Nucl. Phys. A **697** (2002) 255; **700** (2002) 343; **750** (2005) 259]

  - π exchange processes in the presence of a filled Fermi sea



  - short-distance dynamics: contact interactions

- In-medium propagator

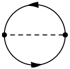
$$G_N = (\not p + M_N) \left[\frac{i}{p^2 - M_N^2 + i\epsilon} - 2\pi\delta(p^2 - M_N^2)\theta(k_f - |\vec{p}^2|)\theta(p_0) \right]$$

full — vacuum — medium

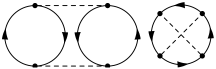
In-medium χPT extension

-

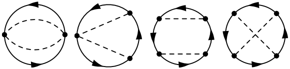
1π – exchange $\mathcal{O}(q^3)$



Iterated 1π – exchange $\mathcal{O}(q^4)$



Irreducible 2π – exchange $\mathcal{O}(q^5)$



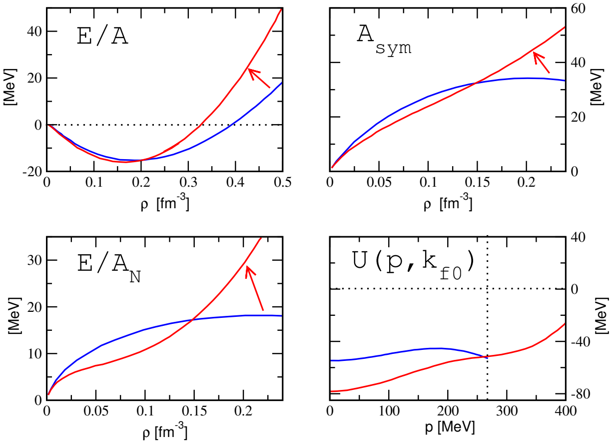
[Δ-intermediate states are not shown for simplicity]

- Expansion of $\mathcal{E}(k_f)$ in powers of the Fermi momentum k_f

$$\mathcal{E}(k_f) = E(k_f)/A = \sum_n \mathcal{F}_n(k_f/m_\pi, k_f/m_\Delta)$$

$$= \frac{3k_f^2}{10M_N} + \mathcal{C}_3\frac{k_f^3}{M_N^2} + \mathcal{C}_4\frac{k_f^4}{M_N^3} + \mathcal{C}_5\frac{k_f^5}{M_N^4} + \mathcal{C}_6\frac{k_f^6}{M_N^5} + \ldots$$

In-medium χPT results



[With Δ - Without Δ]

[At saturation: $\rho_{sat} = 0.157$ fm^{-3}, $\mathcal{E} = -16.0$ MeV, $K = 304$ MeV, $A_{sym} = 34.0$ MeV, $U(0, k_{f0}) = -78.2$ MeV]

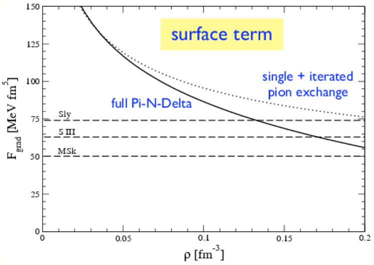
χ Spin-orbit for nucleons

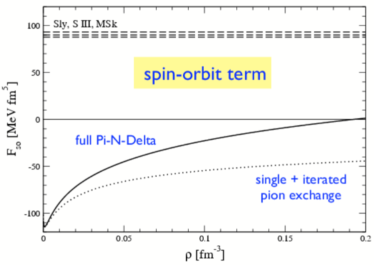
Density matrix expansion for inhomogeneous systems.

[S. Fritsch, N. Kaiser and W. Weise, Nucl. Phys. A **724** (2003) 47, Nucl. Phys. A **750** (2005) 259]

local density	kinetic energy density	spin-orbit density
$\rho(\mathbf{r}) = \frac{2\mathbf{k}_F^3(\mathbf{r})}{3\pi^2}$	$\tau(\mathbf{r})$	$\mathbf{J}(\mathbf{r})$

$$\mathcal{E}[\rho,\tau,\mathbf{J}] = \frac{\mathbf{E}(\mathbf{k_F})}{\mathbf{A}}\rho + \left(\tau - \frac{3}{5}\rho\,\mathbf{k_F^2}\right)\left[\frac{1}{2\mathbf{M_N}} - \mathbf{F}_\tau(\mathbf{k_F})\right] + (\nabla\rho)^2\,\mathbf{F_{grad}}(\mathbf{k_F}) + \nabla\rho\cdot\mathbf{J}\,\mathbf{F_{so}}(\mathbf{k_F}) + \ldots$$

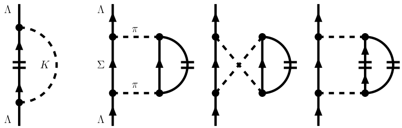


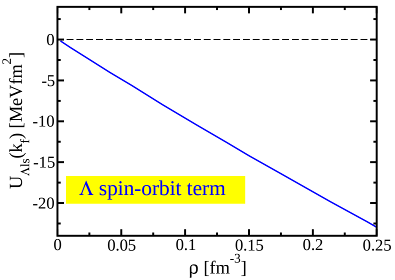


[Adapted from W. Weise talk at OCTS]

χ Spin-orbit for hyperons

- $SU(3)$ extension.
- Σ as intermediate states:





- Model-independent.
- Not a relativistic effect.
- Sizable and with a *wrong sign*.

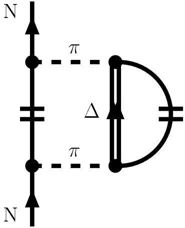
$$\mathcal{H}_{\Lambda ls} = U_{\Lambda ls}(k_f)\frac{1}{2r}\frac{d\ f(r)}{dr}\boldsymbol{\sigma}\cdot\boldsymbol{l}$$

[N. Kaiser and W. Weise, Phys. Rev. C **71** (2005) 015203]

Any solutions?

Summarizing

- χ spin-orbit term for nucleons: small, consistent with zero.
- χ spin-orbit term for hyperons: large with a wrong sign
- ➡ Additional contributions

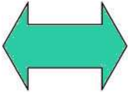


[Not present for hyperons; it cancels the negative contribution from 1π iterated exchange]

Possible solution: large scalar and vector fields (as suggested by RMF phenomenology) constrained by QCD Sum Rules.

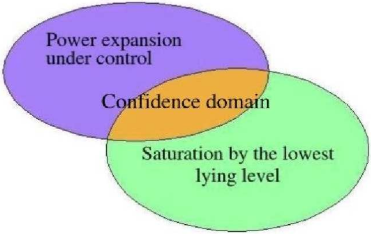
In-medium QCD sum rules

Description in terms of condensates: expectation values of local operators:$\langle \bar{q}q \rangle$, $\langle q^\dagger q \rangle$



(Phenomenological) Description in terms of physical intermediate states: nucleon pole + continuum

$$\Pi_N^{OPE}(q^2) = \sum_n C_n^i(q^2) \langle \hat{O}_n \rangle_{\rho_N} = \Pi_N^{phen}$$



- The success strongly depends on the existence of a domain, where the power expansion (in quark and gluon condensates) is under control and the sum over physical intermediate states is saturated by the lowest lying level.

QCD sum rules for nucleons and hyperons

Nucleons:

$$
\begin{aligned}
\Sigma_S^{(0)} &= M_N^*(\rho) - M_N = -\frac{\sigma_N M_N}{m_\pi^2 f_\pi^2}\rho_S = G_S^{(0)}\rho_S \\
\Sigma_V^{(0)} &= \frac{4(m_u + m_d)M_N}{m_\pi^2 f_\pi^2}\rho = G_S^{(0)}\rho \\
\frac{\Sigma_S^{(0)}}{\Sigma_V^{(0)}} &= \frac{G_S^{(0)}\rho_s}{G_V^{(0)}\rho} = -\frac{\sigma_N}{4(m_u + m_d)}\left(\frac{\rho_S}{\rho}\right) \simeq -1
\end{aligned}
$$

Strong scalar and vector mean fields ($300 - 400$ MeV) generated by in-medium changes of QCD condensates

[Cohen and Furnstahl, Phys. Rev. Lett. **67** (1991) 961]

Hyperons:

$$
\frac{(\Sigma_V)_\Lambda}{(\Sigma_V)_N} \sim \frac{(\Sigma_S)_\Lambda}{(\Sigma_S)_N} \sim 0.3 - 0.5 \quad \textit{with large uncertainties}
$$

[Jin and Furnstahl, Phys. Rev. C **49** (1994) 1190]

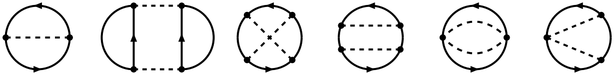
For finite nuclei:

- we suggest a Density Functional (DFT) strategy

[P. Finelli, N. Kaiser, D. Vretenar and W. Weise, Nucl. Phys. A **770** (2006) 1 and references therein]

$$E_0[j^\mu] = E_{free}[j^\mu] + \int d^3r \{\mathcal{E}^{(0)}[j^\mu] + \mathcal{E}_{exc}[j^\mu]\} + E_{coul}[j^\mu]$$

- $\mathcal{E}_{exc}$ from in-medium chiral pionic fluctuations



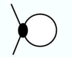
- $\mathcal{E}^{(0)}$ strong scalar and vector mean fields generated by in-medium changes of QCD condensates.

For finite nuclei:

construct an effective Lagrangian with density-dependent point coupling vertices

$$\mathcal{L}_{eff} = \bar{\Psi}(i\gamma_\mu\partial^\mu - M_N)\Psi - \frac{1}{2}\sum_{i=S,V,\ldots} G_i(\hat{\rho})(\bar{\Psi}\Gamma_i\Psi)^2 - \frac{1}{2}\sum_{i=S,V,\ldots} D_i(\hat{\rho})(\partial_\mu\bar{\Psi}\Gamma_i\Psi)^2 \quad \Gamma_i = \mathbb{1}, \gamma_\mu, \ldots$$

- matching at the level of nucleon self-energies

$$\Sigma_i(\rho) = G_i(\rho)\ \rho$$  $$+\ \Sigma_R$$

rearrangement terms arise for DD vertices

$$\Sigma_i^{(\pi)}(\rho) \Rightarrow G_i^{(\pi)}(\rho)\ \rho + \Sigma_R^{(\pi)}$$

$$\Sigma_i^{(0)} \Rightarrow G_i^{(0)}\ \rho$$

For finite nuclei:

Solve self-consistent Dirac equation

- for nucleons

$$h_N\psi_k = [-i\beta\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + M_N + \gamma_0(\Sigma_V^0 + \tau_3\Sigma_{TV}^0 + \Sigma_R^0) + \Sigma_S + \tau_3\Sigma_{TS}]\psi_k = \epsilon_k\psi_k$$

- and for the single hyperon

$$h_\Lambda\psi_\Lambda = [-i\beta\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + M_\Lambda + \gamma_0\Sigma_V^{\Lambda\,0} + \Sigma_S^\Lambda]\psi_\Lambda = \epsilon_\Lambda\psi_\Lambda$$

with rearrangement terms (Σ_R^0) only in the nucleon sector.

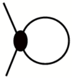
Density dependent couplings

PARAMETERS and their interpretation

- single particle potential:

$$U = S + V = G_S^{(0)} \rho_S + G_V^{(0)} \rho + \Delta U$$

$$\Delta U = g_3 \frac{k_F^3}{\Lambda^2} + g_4 \frac{k_F^4}{\Lambda^3} + g_5 \frac{k_F^5}{\Lambda^4} + g_6 \frac{k_F^6}{\Lambda^5}$$



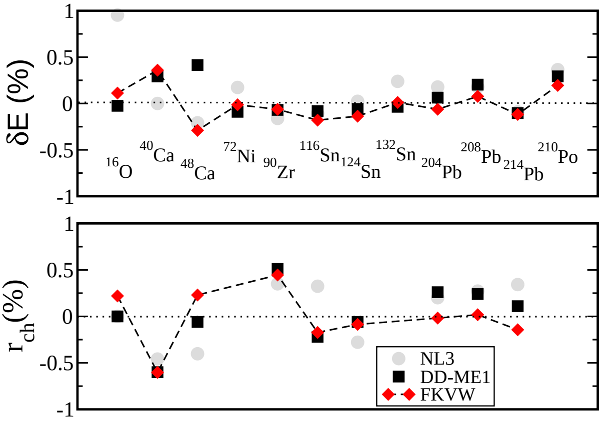
$(\Lambda = 0.6\,\mathrm{GeV})$

	coupling	fine-tuned	expected / predicted	
condensate fields	$G_S^{(0)}$	$-11.5\,\mathrm{fm}^2$	$-(11.0 \pm 1.5)\,\mathrm{fm}^2$	QCD sum rules
	$G_V^{(0)}$	$11.0\,\mathrm{fm}^2$	$(10.5 \pm 1.5)\,\mathrm{fm}^2$	
nuclear chiral (pion) dynamics:	g_3	-3.04	-3.31	in-medium ChPT
van der Waals + Pauli + short range (contact) terms	g_4	2.95	2.95	model-independent
	g_5	2.48	2.48	
	g_6	-4.00	$-$	
	D_S	-0.76	-0.7	surface (derivative) term

[Adapted from W. Weise talk at OCTS]

Density dependent couplings

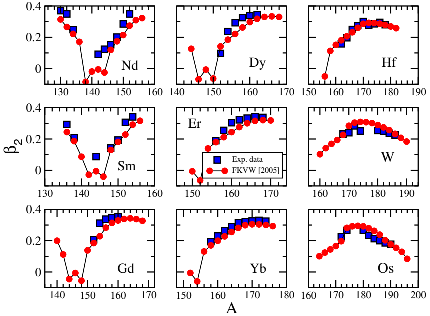
- 7 parameters for the nucleons ➡



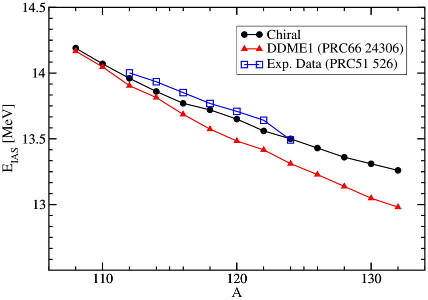
- 1 parameter for the hyperon (contact term) ➡ empirical Λ potential depth of about −28 MeV

Ground state and excited state properties

Deformations



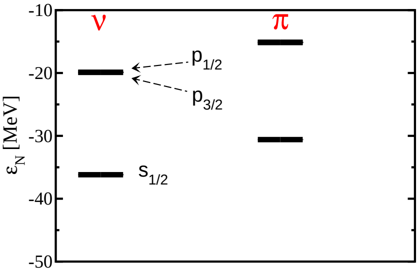
Isobaric Analogue States



Single particle levels for nuclei

^{16}O as a test case:

- Introducing chiral pionic fluctuations ($\mathcal{E}_{exc}$) ➡ binding.

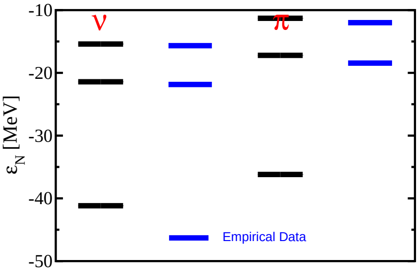


[*n* and *p* single particle energy levels from P. Finelli, N. Kaiser, D. Vretenar and W. Weise, Nucl. Phys. A **735** (2004) 449]

Single particle levels for nuclei

^{16}O as a test case:

- Introducing chiral pionic fluctuations ($\mathcal{E}_{exc}$) ➡ binding.
- Introducing background fields ($\mathcal{E}^{(0)}$) ➡ spin-orbit splitting correctly reproduced.

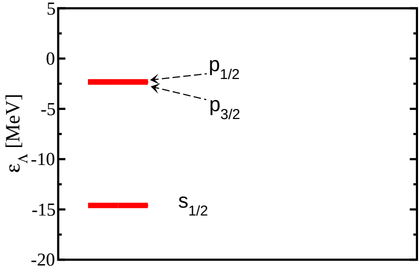


[*n* and *p* single particle energy levels from P. Finelli, N. Kaiser, D. Vretenar and W. Weise, Nucl. Phys. A **735** (2004) 449]

Single particle levels for hypernuclei

$^{17}_{\Lambda}O$ as a test case:

- Introducing chiral pionic fluctuations ($\mathcal{E}_{exc}$) ➡ binding



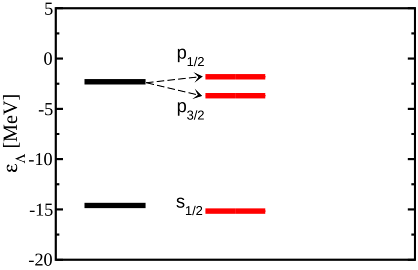
[Λ single particle energy levels]

using a factor 2/3 (corresponding to the quark model prediction)

Single particle levels for hypernuclei

$^{17}_{\Lambda}$O as a test case:

- Introducing chiral pionic fluctuations ($\mathcal{E}_{exc}$) ➡ binding
- Introducing background fields ($\mathcal{E}^{(0)}$) ➡ spin-orbit splitting



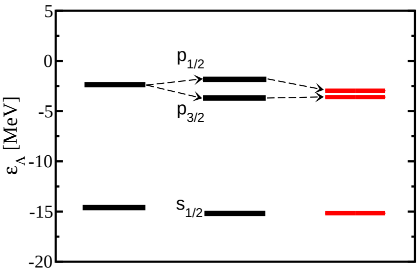
[Λ single particle energy levels]

using a factor $2/3$ (corresponding to the quark model prediction)

Single particle levels for hypernuclei

$^{17}_{\Lambda}$O as a test case:

- Introducing chiral pionic fluctuations ($\mathcal{E}_{exc}$) ➡ binding
- Introducing background fields ($\mathcal{E}^{(0)}$) ➡ spin-orbit splitting
- Introducing the chiral spin-orbit term ➡ spin-orbit reduction



[Λ single particle energy levels]

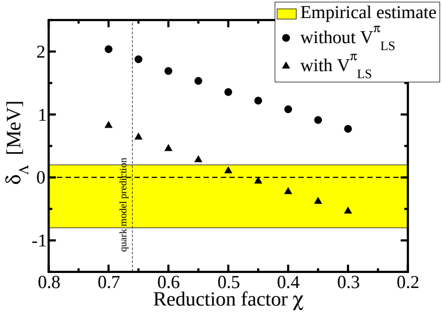
using a factor $2/3$ (corresponding to the quark model prediction)

General trend

Question: is it possible to have simultaneously

- a very small spin-orbit term in spite of a large and negative chiral contribution
- quark condensates in agreement with QCD sum rules predictions for the Λ hyperon

?



$$\chi = \frac{(\Sigma_V)_\Lambda}{(\Sigma_V)_N} = \frac{(\Sigma_S)_\Lambda}{(\Sigma_S)_N}$$

Answer: yes

Summary

- Exploration of the $SU(3)$ dynamics both for nuclei and hypernuclei
- Common mechanism for the generation of the spin-orbit interaction
- No *ad-hoc* solutions like the ω tensor term

Collaborators: Dario Vretenar, Norbert Kaiser, Wolfram Weise, Tamara Nikšić, Nils Paar.