Aharonov-Bohm oscillations in a mesoscopic ring with two entangled magnetic impurities

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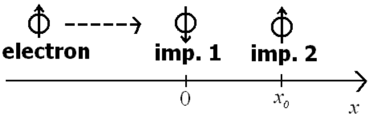
cond-mat/0612057 (2006)

Outline

1D Wire with two spin-1/2 impurities

F. Ciccarello *et al.*, New. J. Phys. **8**, 214 (2006)
F. Ciccarello *et al.*, sottomesso (2006), ArXiv:quant-ph/0611025



$$H = \frac{p^2}{2m^*} + J\,\boldsymbol{\sigma}\cdot\mathbf{S}_1\,\delta(x) + J\,\boldsymbol{\sigma}\cdot\mathbf{S}_2\,\delta(x - x_0)$$

- Multiple scattering between 1 and 2 with spin-flip
- Electron analogue of a Fabry-Perot with two "quantum" mirrors
- What happens when the impurities are in an entangled state?

Magnetic impurities
Sources of electron decoherence

- Static impurities $\Rightarrow$ Well fixed phase shifts $\Rightarrow$ Coherence preserved
- Magnetic impurities $\Rightarrow$ Undeterminate phase shifts $\Rightarrow$ Loss of coherence

S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge Univ. Press, Cambridge, 1995)

Stern A, Aharonov Y and Imry Y 1990 *Phys. Rev. A* **41** 3436

Imry Y 1997 *Introduction to Mesoscopic Physics* (New york: Oxford Univ. Press

Approach

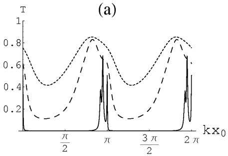
- Derivation of exact stationary states
- Technique: adapted *quantum waveguide theory*
- The distance between the impurities is a parameter of the model

Outline

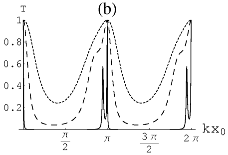
Perfect transparency

$J/k = 1(\cdots), 2(--), 5\ (—)$

- Typical case: $|\uparrow\downarrow\rangle$ (product state)



- Perfect transparency: $|\Psi^-\rangle = 2^{-1/2}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ (particular maximally entangled state)

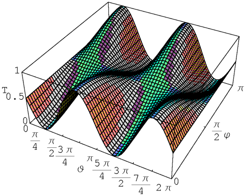


One spin-up family of states at the resonance condition

One spin-up family of impurity spin states

$$|\Psi(\vartheta,\varphi)\rangle = \cos\vartheta\,|\uparrow\downarrow\rangle + e^{i\varphi}\sin\vartheta\,|\downarrow\uparrow\rangle$$

Transmittivity vs. $\vartheta \in [0, 2\pi]$ and $\varphi \in [0, \pi]$ at $kx_0 = n\pi$



- maxima of T for $|\Psi^-\rangle = 2^{-1/2}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$
- minima of T for $|\Psi^+\rangle = 2^{-1/2}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$
  F. Ciccarello *et al.*, New J. Phys. **8**, 214 (2006)

Physical explanation

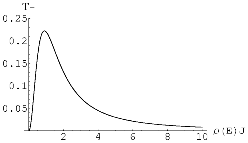
- For $kx_0 = n\pi$ the electron is found at $x = 0$ and $x = x_0$ with equal probability
  $\Rightarrow$ $\mathbf{S}_{12}^2$ additional constant of motion
- $\delta_k(x) = \delta_k(x - x_0) \Rightarrow$ Effective el.-imp. coupling V:
  $V = \frac{J}{2}\left(\mathbf{S}^2 - \sigma^2 - \mathbf{S}_{12}^2\right)\,\delta_k(x)$

...when $|\uparrow\rangle\,|\Psi^-\rangle$ is prepared:

- $\mathbf{S}_{12}^2\,|\uparrow\rangle\,|\Psi^-\rangle = 0$ $S_z\,|\uparrow\rangle\,|\Psi^-\rangle = 1/2$ (no degeneracy!)
  $\Rightarrow |\uparrow\rangle\,|\Psi^-\rangle \rightarrow |\uparrow\rangle\,|\Psi^-\rangle$: spin-flip is effectively quenched!
  $\Rightarrow V = 0$ and $H = \frac{p^2}{2m^*}$: perfect transparency!

Generation of entangled states

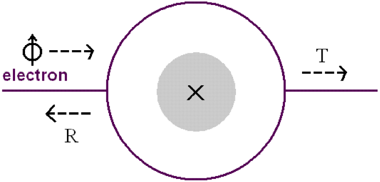
- for $kx_0 = n\pi \quad |\uparrow\rangle\,|\downarrow\downarrow\rangle \rightarrow t_\uparrow\,|\uparrow\rangle\,|\downarrow\downarrow\rangle + t_\downarrow\,|\downarrow\rangle\,|\Psi^+\rangle$
- $T_\downarrow = |t_\downarrow|^2$: probability that the electron is transmitted in $|\downarrow\rangle$ that is that the impurities are projected in $|\Psi^+\rangle$
- $|\Psi^+\rangle \rightarrow |\Psi^-\rangle$ through a local field acting on one of the two impurities

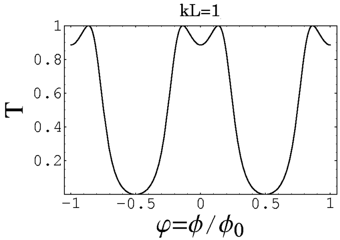


A. T. Costa, Jr., S. Bose, and Y. Omar, Phys. Rev. Lett. **96**, 230501 (2006)
F. Ciccarello *et al.*, New J. Phys. **8**, 214 (2006)
K. Yuasa, H. Nakazato, accepted for publication in J. Phys. A, ArXiv:quant-ph/0609217
G. L. Giorgi, and F. De Pasquale, Phys. Rev. B **74**, 153308 (2006)

Mesoscopic ring

Aharonov-Bohm (AB) effect (S. Washburn, and R. A. Webb, Adv. Phys. **35**, 375, 1986)

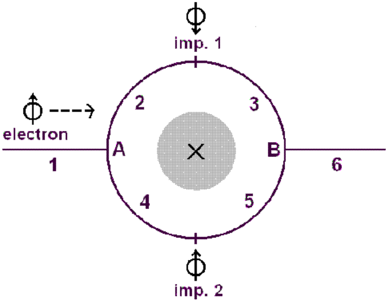




- k: incident electron wave vector
- L: circumference
- Magnetic flux: ϕ
- $\phi_0 = hc/e$: quantum flux

Mesoscopic ring with two magnetic impurities

F. Ciccarello, G. M. Palma, and M. Zarcone, submitted (2006) ArXiv: cond-mat/0612057 (2006)

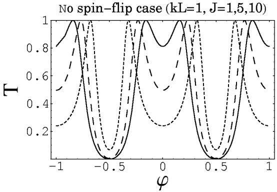


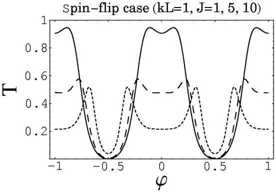
- Electron-impurity coupling: $-J\,\boldsymbol{\sigma}\cdot\mathbf{S}_i\,\delta(x_i - L/4)$
- $k_1 = k + (e\phi/\hbar cL)$, $k_2 = k - (e\phi/\hbar cL)$
- Wavefunction in the upper arm: lin. comb. of e^{ik_1x} and e^{-ik_2x}
- Wavefunction in the lower arm: lin. comb. of e^{ik_2x} and e^{-ik_1x}

AB effect with 1 magnetic impurity

S. K. Joshi *et al.*, Phys. Rev. B **64**, 075320 (2001)

- No spin-flip case: electron $|\uparrow\rangle$, impurity $|\uparrow\rangle$
- Spin-flip case: electron $|\uparrow\rangle$, impurity $|\downarrow\rangle$





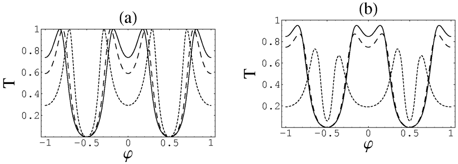
Scattering with spin-flip $\Rightarrow$ Loss of coherence
$\Rightarrow$ Reduction of the amplitude of AB oscillations

AB oscillations with 2 magnetic impurities

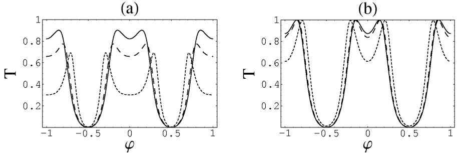
F. Ciccarello *et al.*, submitted (2006) ArXiv: cond-mat/0612057 (2006)

$kL = 1$, $J/k = 1$ (—), $2(--)$, $5(\cdots)$

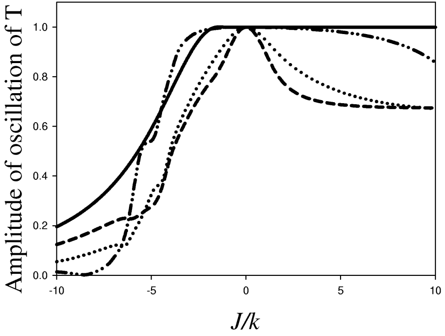
- Product states : $|\uparrow\rangle\,|\uparrow\uparrow\rangle$ e $|\uparrow\rangle\,|\uparrow\downarrow\rangle$



- Entangled states: $|\uparrow\rangle\,|\Psi^{+}\rangle$ e $|\uparrow\rangle\,|\Psi^{-}\rangle$



Amplitude of AB oscillations



$kL = 1$

$|\uparrow\rangle\,|\uparrow\uparrow\rangle$ (——) J. M. Mao *et al.*, J. Appl. Phys. **73**, 1853 (1993)

$|\uparrow\rangle\,|\uparrow\downarrow\rangle$ $(\cdot\cdot\cdot\cdot\cdot)$ Spin-flip and decoherence

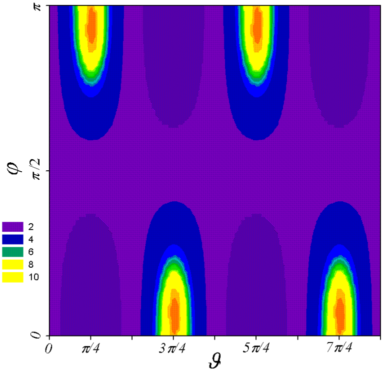
$|\uparrow\rangle\,|\Psi^+\rangle$ (— —) Spin-flip and decoherence

$|\uparrow\rangle\,|\Psi^-\rangle$ $(-\cdot\cdot\cdot-)$ **Nearly coherent behaviour**

Family of one spin-up impurity spins states

$$|\Psi(\vartheta, \varphi)\rangle = \cos\vartheta\,|\uparrow\downarrow\rangle + e^{i\varphi}\sin\vartheta\,|\downarrow\uparrow\rangle$$

Width of the range of J/k where the amplitude of AB oscillations is larger than 0.95 versus $\vartheta \in [0, 2\pi]$ and $\varphi \in [0, \pi]$



Physical explanation

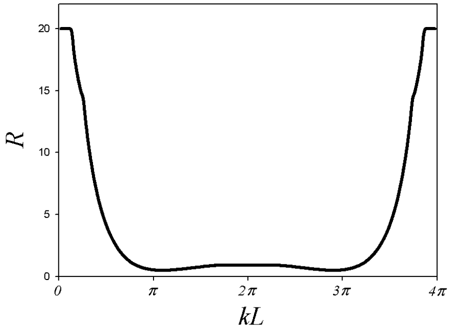
- For $\phi = 0$ $\forall kL$ the two arms of the ring are perfectly symmetric $\Rightarrow$ $\mathbf{S}_{12}^2$ is conserved
- $|\uparrow\rangle\,|\Psi^-\rangle \rightarrow |\uparrow\rangle\,|\Psi^-\rangle$: no spin-flip!
- When $\phi \neq 0$: $k_1 \neq k_2$ and symmetry is lost $\Rightarrow$ $\mathbf{S}_{12}^2$ is not conserved

...however

- For $\phi \neq 0$ the s. m. of the wavefunction is periodic in kL with period 4π
- For $kL = 4n\pi$ the system behaves as if $kL = 0$ that is as if $k_1 = -k_2 = e\phi/\hbar cL$ $\Rightarrow$ symmetry holds again $\Rightarrow$ $\mathbf{S}_{12}^2$ is conserved

Quasi conservation of $\mathbf{S}_{12}^2$

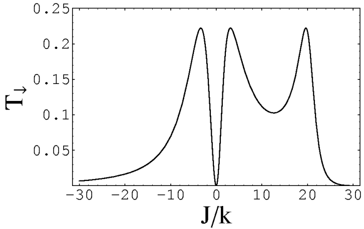
Width R of the range of J/k where the amplitude of AB oscillations is larger than 0.95 vs. kL for the initial state $|\uparrow\rangle\,|\Psi^-\rangle$



- R increases as $kL \to 0, 4\pi$
- Good robustness against deviations from $4n\pi$ ($R > 1$ for $kL < 0.8\pi$)
- Previous results shown in the case $kL = 1 \simeq 0.3\pi$

Scheme for generating $|\Psi^-\rangle$

At $\phi = 0$ strict conservation of $\mathbf{S}_{12}^2$ holds:
$|\uparrow\rangle\,|\downarrow\downarrow\rangle \rightarrow t_\uparrow\,|\uparrow\rangle\,|\downarrow\downarrow\rangle + t_\downarrow\,|\downarrow\rangle\,|\Psi^+\rangle$



- $kL = 1$
- One maximum for $J/k \simeq 3 \rightarrow$ within the nearly coherent range

Conclusions

- Magnetic impurities tipically induce loss of electron coherence in mesoscopic devices
- Coherence can however be preserved when the impurities are allowed to be in suitable entangled states (cooperative effects)
- Quantum entanglement as an inhibitor of electron decoherence