Finite-size effects in solid-state quantum information processing

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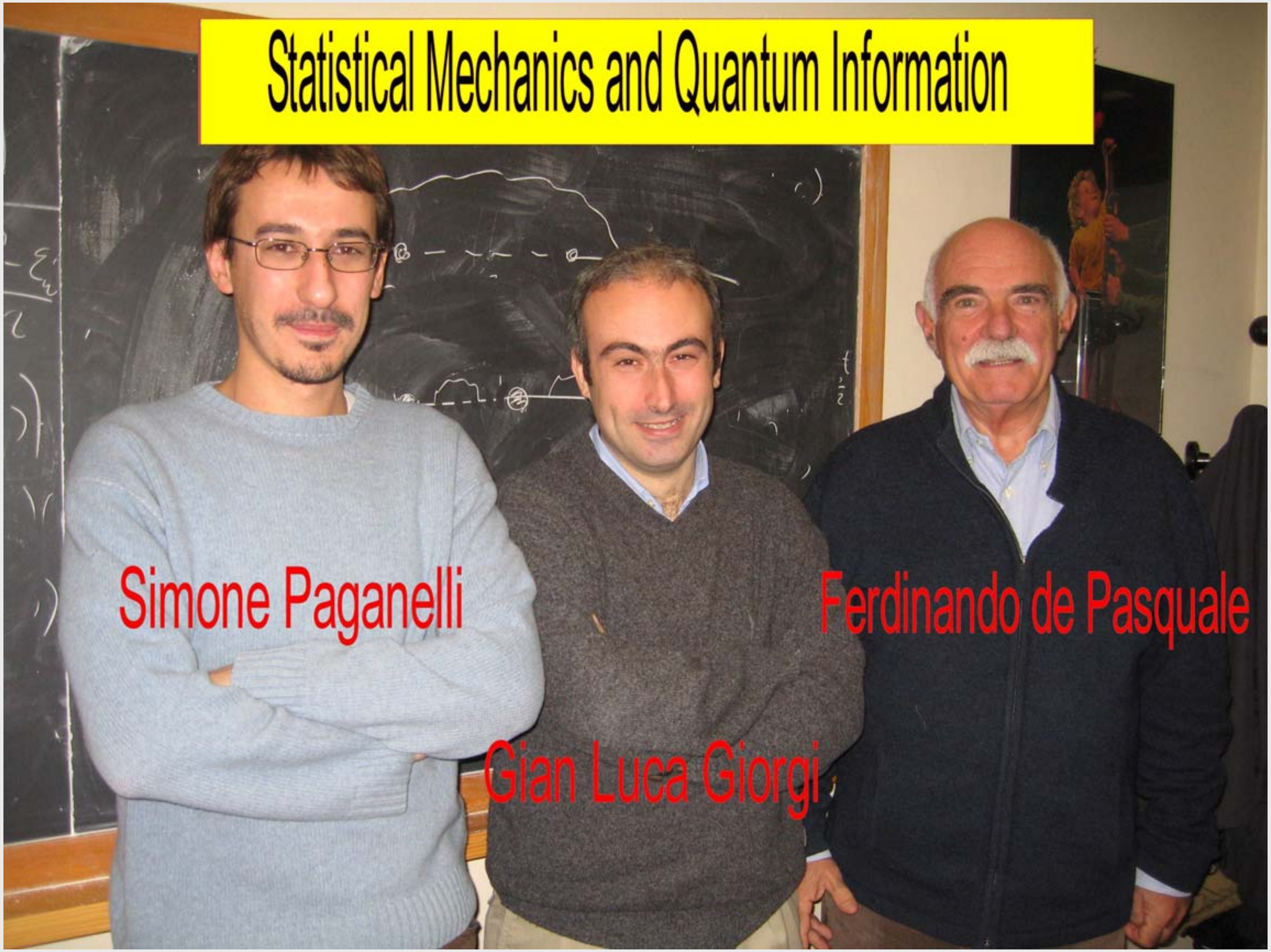
Collaboration



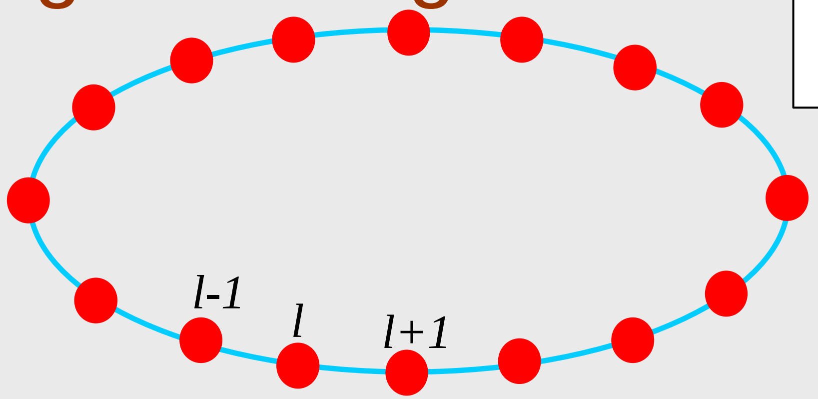
Outlook

- Energy spectrum of a quantum ring in the presence of a variety of impurities
- Finite-size effect
- Application to quantum information:
  - Quantum state transfer
  - Flux qubit in a normal loop
  - Two-spin entanglement generation

Quantum rings

Tight-binding model

$$H = -w \sum_k \left(c_l^\dagger c_{l+1} + c_l^\dagger c_{l-1} \right)$$



The model is easily solved in the Fourier space

$$c_k^\dagger = \frac{1}{\sqrt{N}} \sum_l e^{i\frac{2\pi kl}{N}} c_l^\dagger$$

$$H = \sum_k \varepsilon_k c_k^\dagger c_k \qquad \varepsilon_k = -2w\cos\frac{2\pi k}{N}$$

Quantum rings

The spectrum is represented by a set of discrete energy levels

If extra terms are added to the Hamiltonian, the spectrum modifies

In many practical situations, the thermodynamical limit can be performed

We will show that there are cases where the large N limit is unfit to fully take into account the model

Quantum rings modified by impurities

- Different impurity levels can be added
  - External sites coupled to the chain (Fano - Anderson)

$$H = \sum_k \varepsilon_k c_k^\dagger c_k + \Omega\left(c_A^\dagger c_A + c_B^\dagger c_B\right) - g\left(c_A^\dagger c_0 + c_B^\dagger c_L + h.c.\right)$$

  - Extra energy on a site (impurity scattering)

$$H = \sum_k \varepsilon_k c_k^\dagger c_k + g c_0^\dagger c_0$$

  - Two sites act as magnetic impurities (s-d)

$$H = \sum_{k,\sigma} \varepsilon_k c_{k,\sigma}^\dagger c_{k,\sigma} + g\left(\vec{S}_0 \cdot \vec{\sigma}_0 + \vec{S}_L \cdot \vec{\sigma}_L\right)$$

Quantum rings modified by impurities

- Depending on the model, the energy correction scales in a different way with N
  - Fano-Anderson model: $N^{-1/2}$
  - Impurity scattering model: N^{-1}
  - S-D model: N^{-1}
- Different behaviours are expected

Fano-Anderson Model

$$H = \sum_k \varepsilon_k c_k^\dagger c_k + \Omega\left(c_A^\dagger c_A + c_B^\dagger c_B\right) - \frac{g}{\sqrt{N}} \sum_k \left[c_k^\dagger \left(c_A + e^{ikL} c_B \right) + h.c. \right]$$

- Eigenvalues:

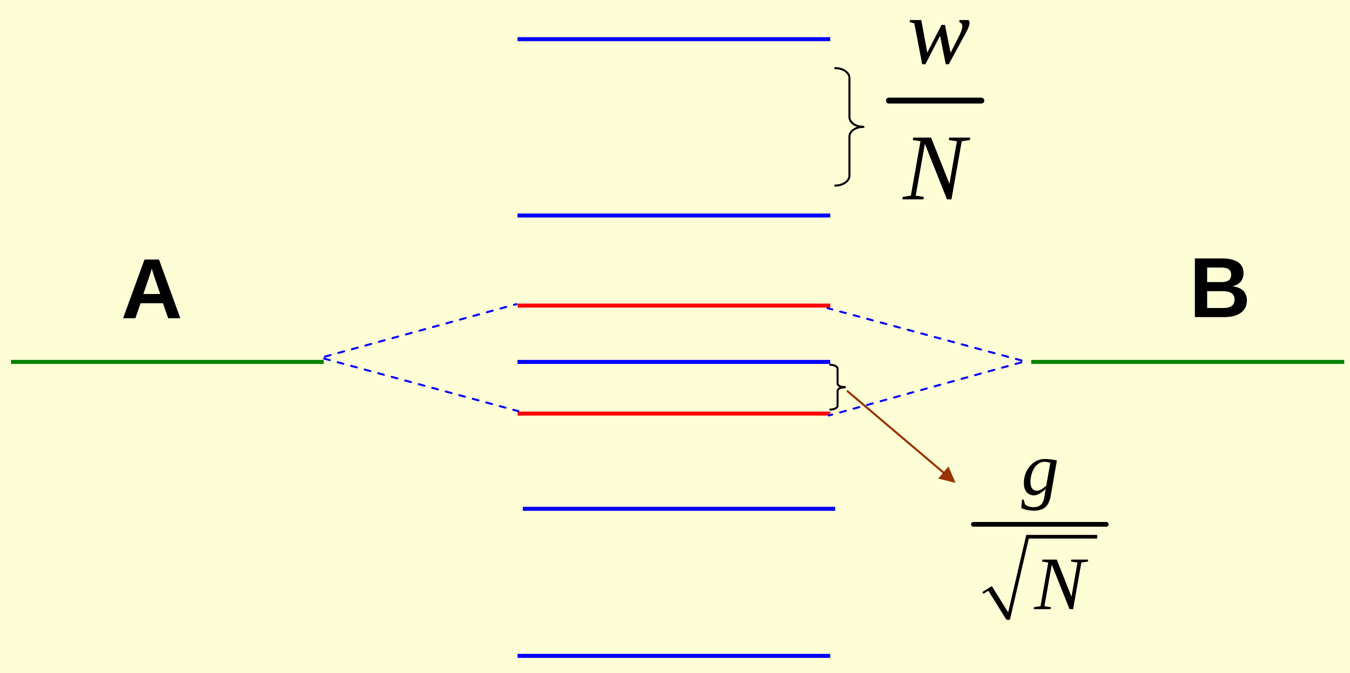
$$\left[\omega - \Omega - g^2 \Lambda_0(\omega)\right]^2 - g^4 \Lambda_L^2(\omega) = 0$$

$$\Lambda_d(\omega) = \frac{1}{N} \sum_k \frac{e^{ikd}}{\omega - \varepsilon_k}$$

- Weak-coupling limit *(g ⑥w)*
- Resonance ($\Omega = \varepsilon_k^* = 0$)

$$\omega = 0, \pm \frac{g\sqrt{2}}{\sqrt{N}}$$

Fano-Anderson Model



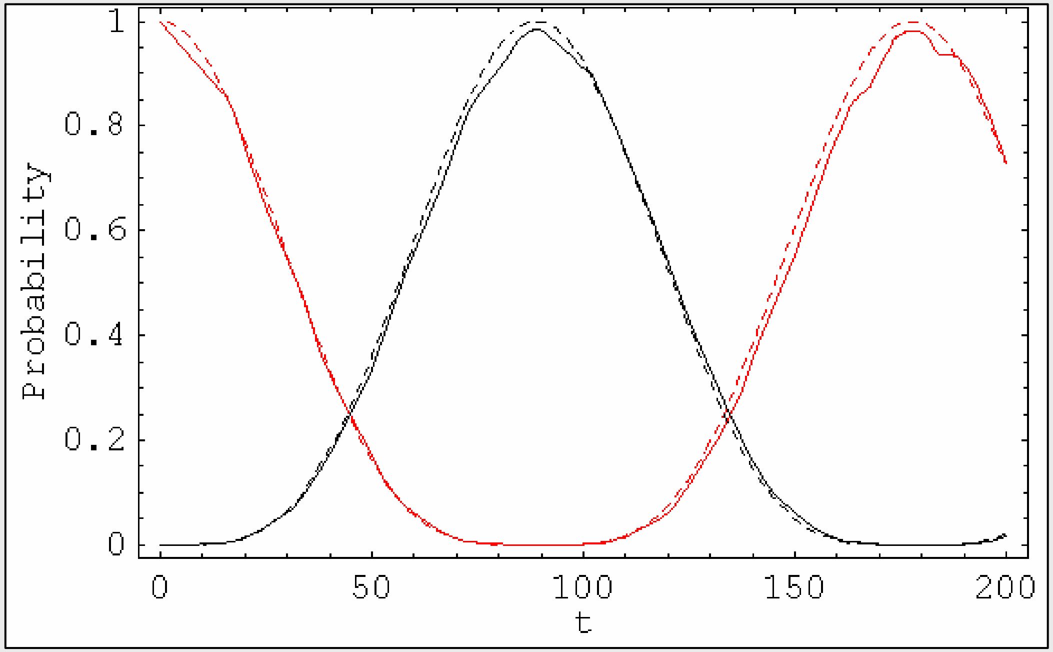
Energy band
(near the middle)

Resonance appears when $g/\sqrt{N} \gg w/N$

Quantum state transfer

- Alice encodes the information in her site
- Time evolution

$$\left|\Psi_A(t)\right\rangle = \cos^2\frac{gt}{\sqrt{N}}\left|\Psi_A\right\rangle + \sin^2\frac{gt}{\sqrt{N}}\left|\Psi_B\right\rangle + \frac{i}{2}\sin\frac{2gt}{\sqrt{N}}\left(\left|\Psi_{k*}\right\rangle + \left|\Psi_{-k*}\right\rangle\right)$$



P_A Exact
P_A Theory
P_B Exact
P_B Theory

$N=32$

$g=0.1\ w$

$\Omega=0 \qquad L=4$

S.Paganelli, F. de Pasquale, and G. Giorgi, Phys. Rev. A **74**, 012316 (2006)

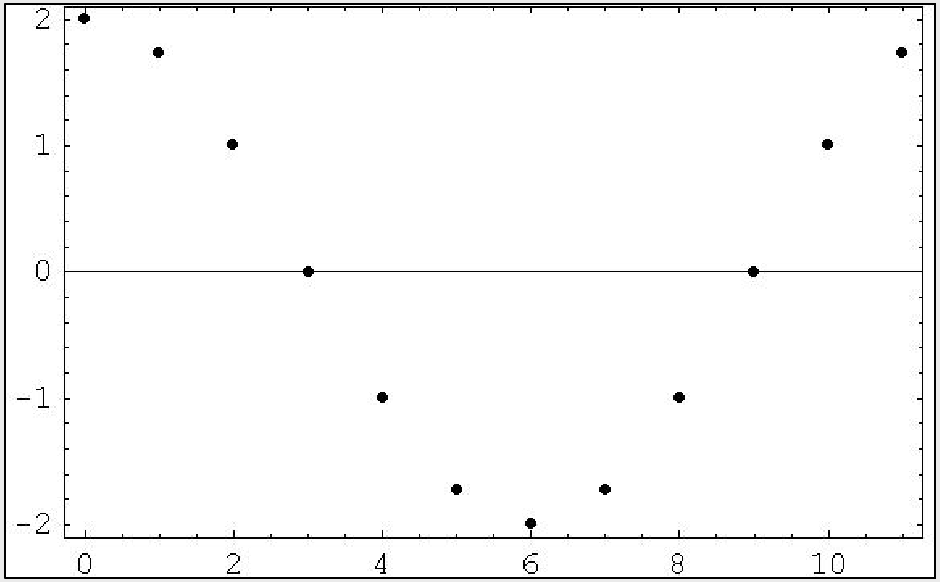
Impurity scattering model

$$H = \sum_k \varepsilon_k c_k^\dagger c_k + \frac{g}{N} \sum_{k,k'} c_k^\dagger c_{k'}$$

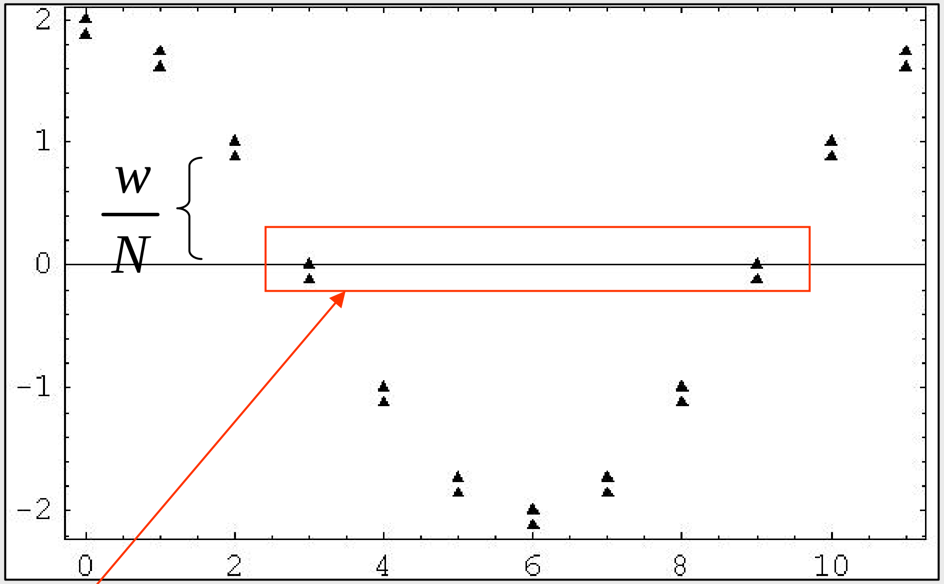
- Eigenvalues: $$\omega = \varepsilon_k ; 1 - \frac{g}{N} \sum_k \frac{1}{\omega - \varepsilon_k} = 0$$
- Weak-coupling limit *(g⑥w)* $$\omega = \varepsilon_k ; \omega = \varepsilon_k + \frac{2g}{N}$$
- Perturbation theory can be done for any N
- The degeneracy between k and $-k$ breaks down
- Eigenstates $$\left|\pm\right\rangle = \frac{1}{\sqrt{2}}\left(\left|k\right\rangle \pm \left|-k\right\rangle\right)$$

Impurity scattering model

- Unperturbed spectrum



- Modified spectrum

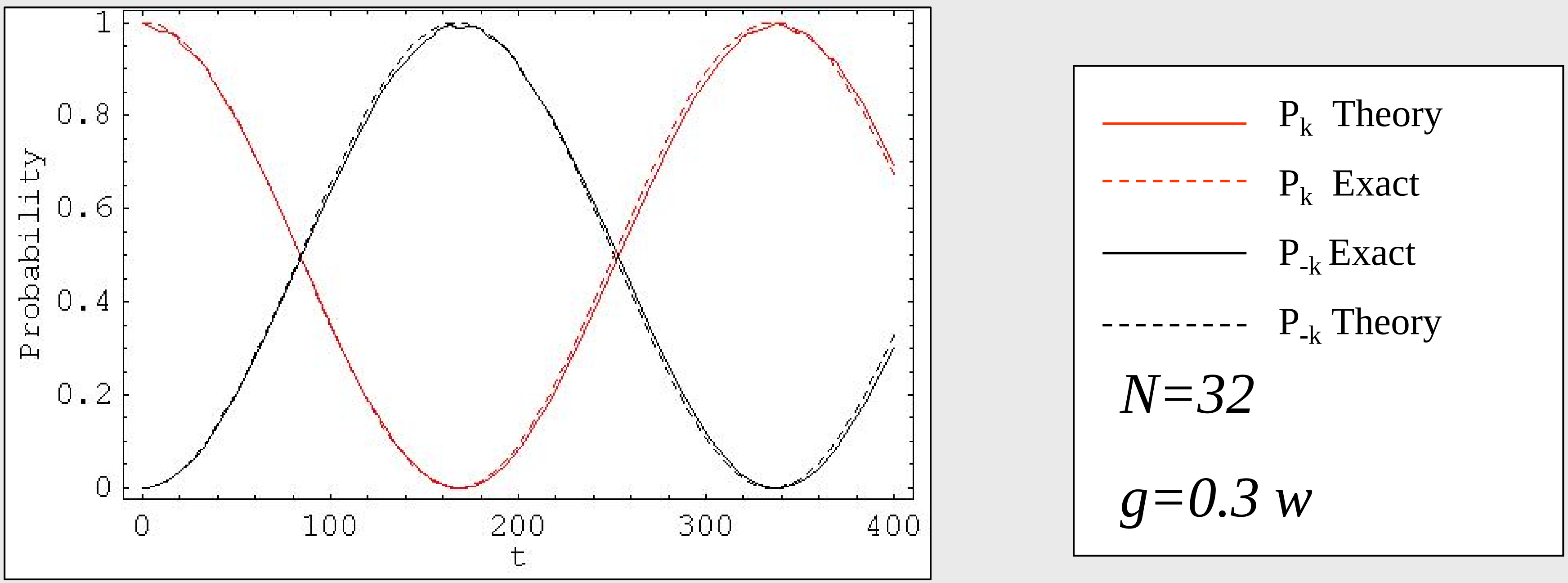


In the limit of g ⑥ w each doublet behaves as an isolated two-level system

Impurity scattering model

Two-level behaviour

$$\left|\Psi_k(t)\right\rangle = \cos\frac{gt}{N}\left|\Psi_k\right\rangle + i\sin\frac{gt}{N}\left|\Psi_{-k}\right\rangle$$



The two-level approximation holds also in the presence of extended contact

Current qubit in a normal metal

- One electron on the Fermi surface (N odd)
- A trapped magnetic flux ϕ shift the eigenenergies:

$$\varepsilon_{\pm} = -2w\cos\left(\frac{2\pi k_F}{N} \pm \frac{2\pi}{N}\frac{\phi}{\phi_0}\right)$$

- The old ground state $(|k_F> + \ |k_{-F}>)$ brings a current oscillating with the frequency $(\varepsilon_{+} - \varepsilon_{-})/2$
- This contribution sums to the constant persistent current derived from the electrons inside the Fermi surface

G. Giorgi and F. de Pasquale, submitted to PRB

S-D Model

$$H = \sum_{k,\sigma} \varepsilon_k c_{k,\sigma}^{\dagger} c_{k,\sigma} + g\left(\vec{S}_0 \cdot \vec{\sigma}_0 + \vec{S}_L \cdot \vec{\sigma}_L\right)$$

$$\sigma_l^x = \frac{1}{N} \sum_{q,q'} \left(a_{q,\uparrow}^{\dagger} a_{q',\downarrow} + a_{q,\downarrow}^{\dagger} a_{q',\uparrow}\right) e^{i(q-q')l}$$

$$\sigma_l^y = \frac{-i}{N} \sum_{q,q'} \left(a_{q,\uparrow}^{\dagger} a_{q',\downarrow} - a_{q,\downarrow}^{\dagger} a_{q',\uparrow}\right) e^{i(q-q')l}$$

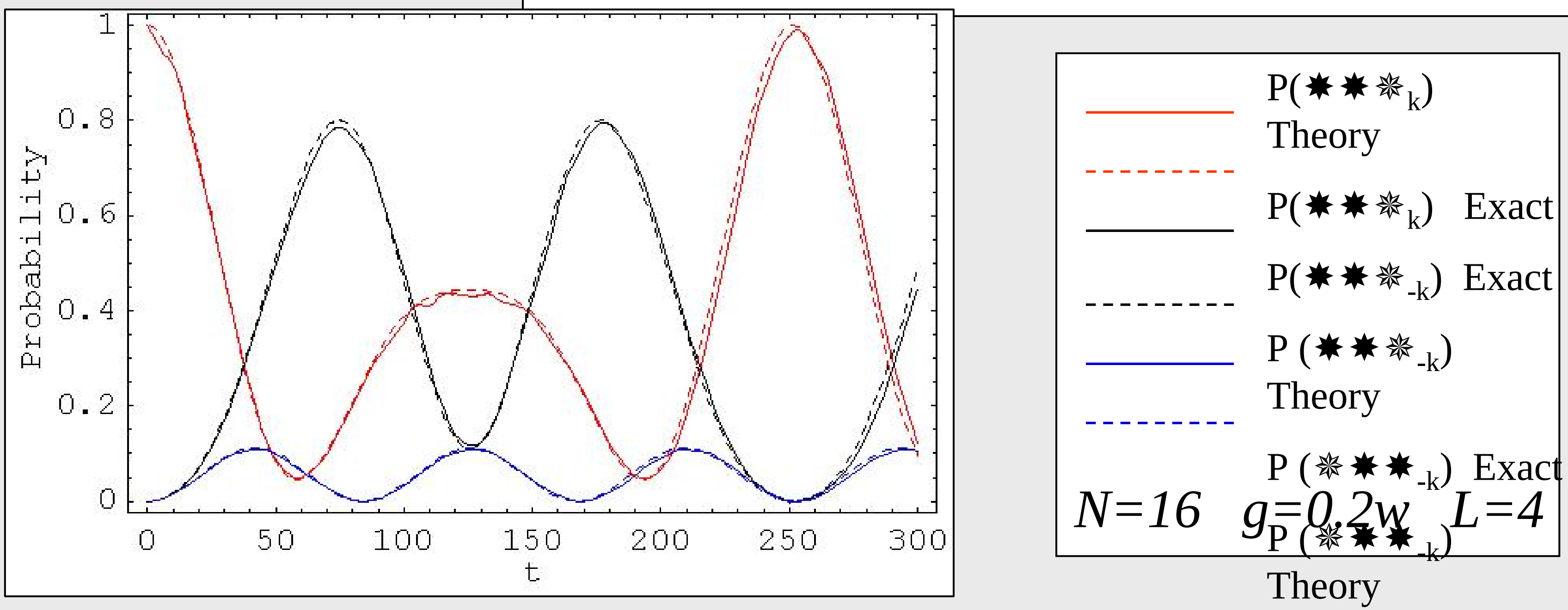
$$\sigma_l^z = \frac{1}{N} \sum_{q,q'} \left(a_{q,\uparrow}^{\dagger} a_{q',\uparrow} - a_{q,\downarrow}^{\dagger} a_{q',\downarrow}\right) e^{i(q-q')l}$$

- One excess electron is injected with spin down on the mode k, while both the impurities have spin up $\left|\uparrow\uparrow\right\rangle \left|\downarrow_k\right\rangle$

S-D Model

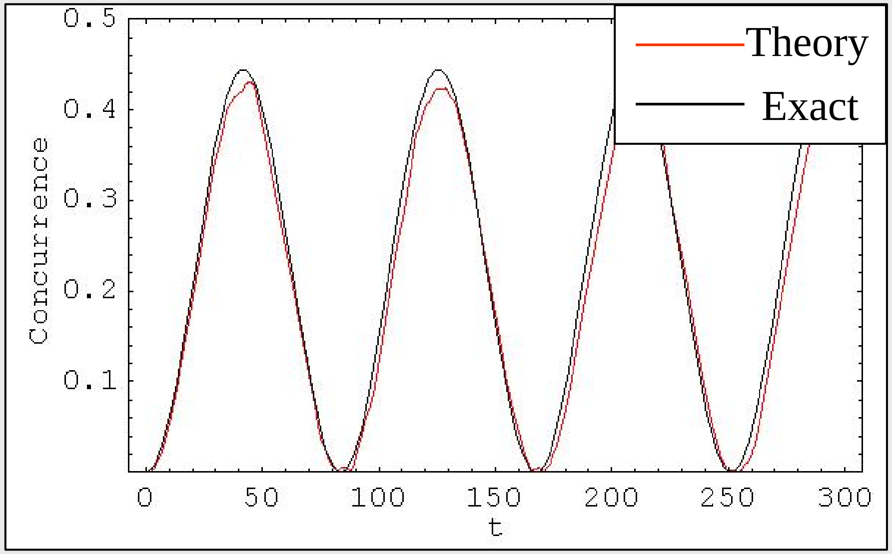
- Weak-coupling limit *(g ⑥w)*
- *k=π/2;*
- *L* even

$$\left|\uparrow\uparrow\right\rangle\left|\downarrow_k\right\rangle_t = \frac{1}{6}\left(3+e^{-2i(g/N)t}+2e^{4i(g/N)t}\right)\left|\uparrow\uparrow\right\rangle\left|\downarrow_k\right\rangle$$
$$+\frac{1}{6}\left(-3+e^{-2i(g/N)t}+2e^{4i(g/N)t}\right)\left|\uparrow\uparrow\right\rangle\left|\downarrow_{-k}\right\rangle$$
$$+\frac{1}{6}e^{-2i(g/N)t}\left(1-e^{6i(g/N)t}\right)\left(\left|\uparrow\downarrow\right\rangle+\left|\downarrow\uparrow\right\rangle\right)\left(\left|\uparrow_k\right\rangle+\left|\uparrow_{-k}\right\rangle\right)$$



Entanglement generation

- **Concurrence**



Deterministic generation of a finite amount of Entanglement

- **Entanglement of formation**

Projecting onto $\left(\left|\uparrow_k\right\rangle + \left|\uparrow_{-k}\right\rangle\right)/\sqrt{2}$ a maximally entangled state is created in a probabilistic way

$$P = \frac{4}{9}\sin^2\frac{3J}{N}t$$

G. Giorgi and F. de Pasquale, Phys. Rev. B **74**, 153308 (2006)

Disorder effects

Diagonal disorder

$$H_{disorder} = \sum_l \sigma_l c_l^\dagger c_l$$

In the k-space

$$H_{disorder} = \frac{1}{\sqrt{N}} \sum_l \sigma_{k-k'} c_k^\dagger c_k$$

Eigenvalues renormalize

$$\omega^* \rightarrow \bar{\omega}^* = \omega^* - \frac{\sigma^2}{N} \sum_k \frac{1}{\omega^* - \varepsilon_k}$$

Localization effects become important

When $\sigma / \sqrt{N} \;\square\; w / N$

$$\sigma^2 = \frac{1}{N} \sum_l \sigma_l^2$$

Disorder represents an upper bound: for N large enough the continuum limit becomes effective

Conclusions

- The finite-size effects in modified quantum rings allow a series of useful applications:
  - Quantum state transfer
  - Current (flux) qubit
  - Spin entanglement