



מכון ויצמן למדע

WEIZMANN INSTITUTE OF SCIENCE

Dynamical Protection of Multipartite Entanglement from Decoherence:

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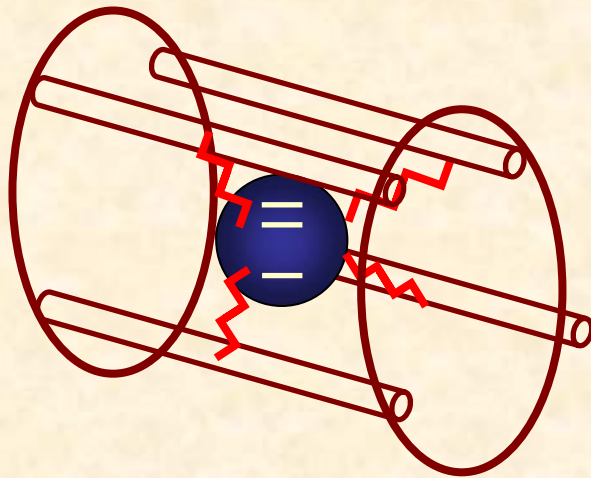
Sponsors: EU, ISF

Synopsis

- Multipartite decoherence causes disentanglement:
 - Error correction: high overhead & threshold
 - Dynamical decoupling [Zanardi, Lidar, Viola] : All particles must be subject to very strong and fast pulses, averaging out environment
 - Decoherence-free subspace (DFS): formation impeded by asymmetric coupling to a bath.
- We propose universal dynamical control for suppressing multipartite decoherence: relaxed requirements
- It may allow high-fidelity gate & storage operations in realistic scenarios without error correction

Single Particle Decoherence

1) Finite T + noisy fields
Ion trap

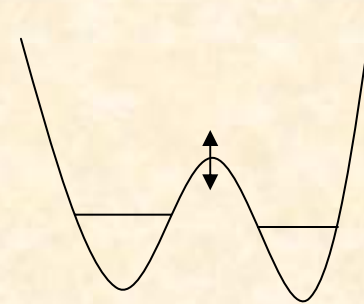


Keller et al.
Nature **431**,
1075 (2004)

Häffner et al.
Nature **438**
643 (2005)

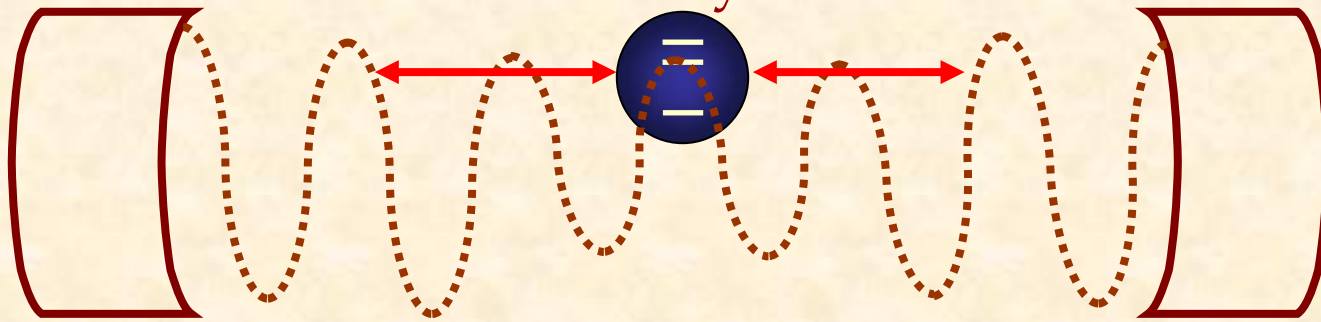
3) Josephson Junction analogs
Barone, Kofman, GK, PRL (2004)

4) BEC in double-well potential



Schumm et al.
Nature Physics
1, 57 (2005)

2) Finite T + radiative decay
Ion in cavity



Kreuter et al.
PRL **92**
203002 (2004)

Control of Single Particle Decoherence

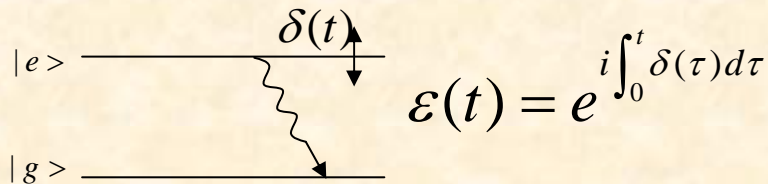
A. G. Kofman and G. Kurizki,
 Nature **405**, 546 (2000), PRL **87**, 270405 (2001)
 Facchi & Pascazio, Prog. In Opt. **42**, 147 (2001)

A. G. Kofman and G. Kurizki PRL **93**,130406(2004)

Amplitude Noise (AN): population change

Off-diagonal $H_{\text{int}} = \hat{B}\hat{\sigma}_x$

Off-resonant modulation (AC Stark shifts)



Decay rate: (T=0)

$$R(t) = 2\pi \int_{-\infty}^{\infty} d\omega \mathbf{G}_0(\omega) \mathbf{F}_t(\omega - \omega_a)$$

Reservoir coupling spectrum

$$\mathbf{G}_0(\omega) \rightarrow \rho(\omega) |\mu(\omega)|^2$$

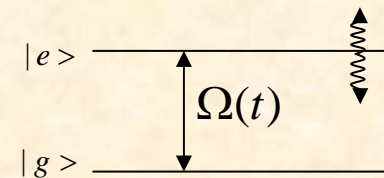
Spectral intensity of modulation

$$\mathbf{F}_t(\omega) = |\epsilon_t(\omega)|^2$$

Phase noise (PN): dephasing

Diagonal $H_{\text{int}} = \hat{B}\hat{\sigma}_z$

Resonant modulation (flipping)



Dephasing rate:

$$R(t) = 2\pi \int_{-\infty}^{\infty} d\omega \mathbf{G}_r(\omega) \mathbf{F}_t(\omega - \omega_a)$$

Spectral density of dephasing

$$\mathbf{G}_r(\omega) = (2\pi)^{-1} \int d\omega \underbrace{\overline{\delta_r(t)\delta_r(0)}}_{\Phi(t)} e^{i\omega t}$$

Spectral intensity of flipping

$$\mathbf{F}_t(\omega) = |\epsilon_t(\omega)|^2$$

Control of Single Particle Decoherence

A. G. Kofman and G. Kurizki, Nature **405**, 546 (2000), PRL **87**, 270405 (2001), PRL **93**,130406(2004)

AN + Off-resonant modulation

Bath coupling spectrum

e.g. Cutoff

$$G(\omega) = \frac{\sqrt{\omega}}{\omega + \Gamma} \theta(\omega - \omega_0)$$

Impulsive phase modulation

Repetitive Weak Pulses

$$\varepsilon(t) = e^{i[t/\tau]\phi}, \quad \phi \ll \pi$$

single shifted peak

Repetitive π Pulses

$$\varepsilon(t) = e^{i[t/\tau]\pi}$$

two symmetric peaks

PN + Resonant modulation

Dephasing spectrum

e.g. Lorentzian

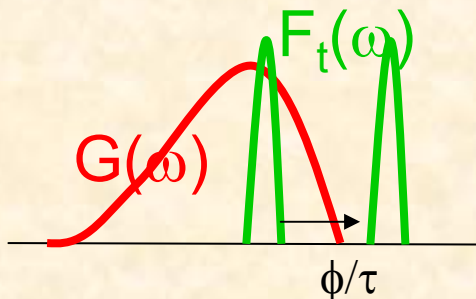
$$\Phi(\tau) = \frac{\gamma}{t_c} \exp(-\tau/t_c)$$

$$G(\omega) = \gamma \frac{1/t_c^2}{1/t_c^2 + \omega^2}$$

Modulation

$$\varepsilon(t) = e^{2i \int_0^t d\tau \Omega(\tau)}$$

CW: $F_t(\omega) = \delta(\omega - 2\Omega)$



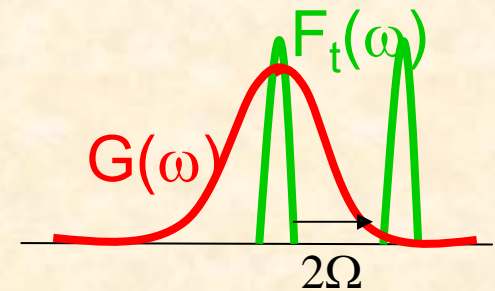
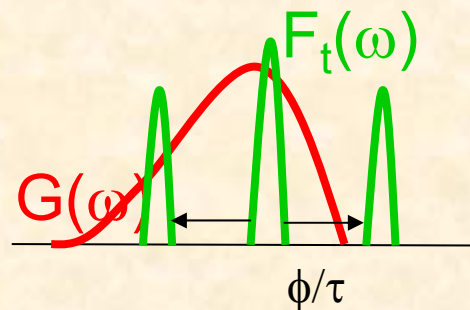
Dynamic decoupling.

Viola & Lloyd PRA **58** 2733 (1998)

Shiokawa & Lidar PRA **69** 030302(R) (2004)

Vitali & Tombesi PRA **65** 012305 (2001)

Agarwal, Scully, Walther PRA **63**, 044101 (2001)



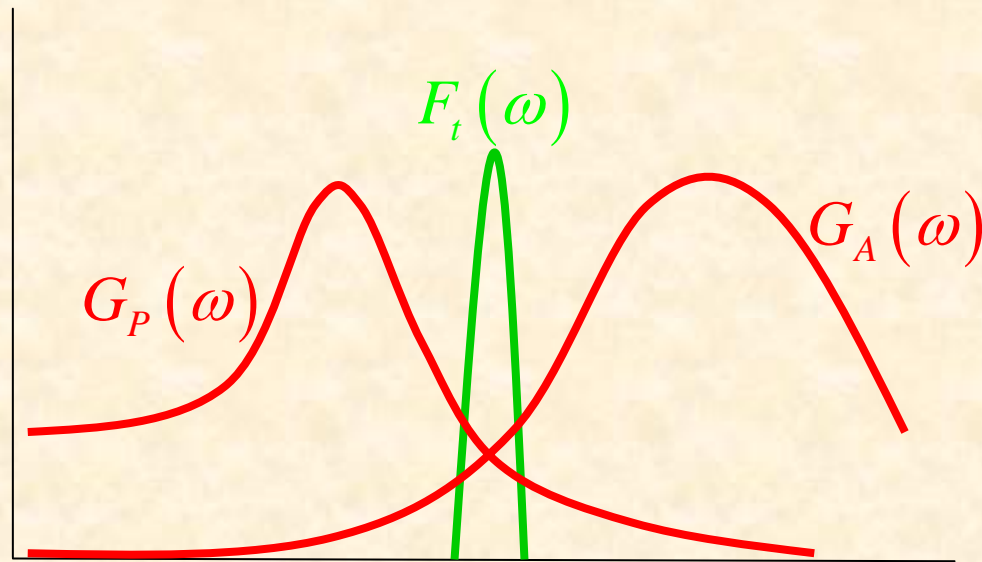
Combined Amplitude & Phase Noise

Gordon, Kofman & GK

Exact PN, 2nd order AN

Double convolution,
Off-resonant modulation:

$$R(t) = 2\pi \int d\omega \int d\omega' G_A(\omega) G_P(\omega') F_t(\omega - \omega')$$



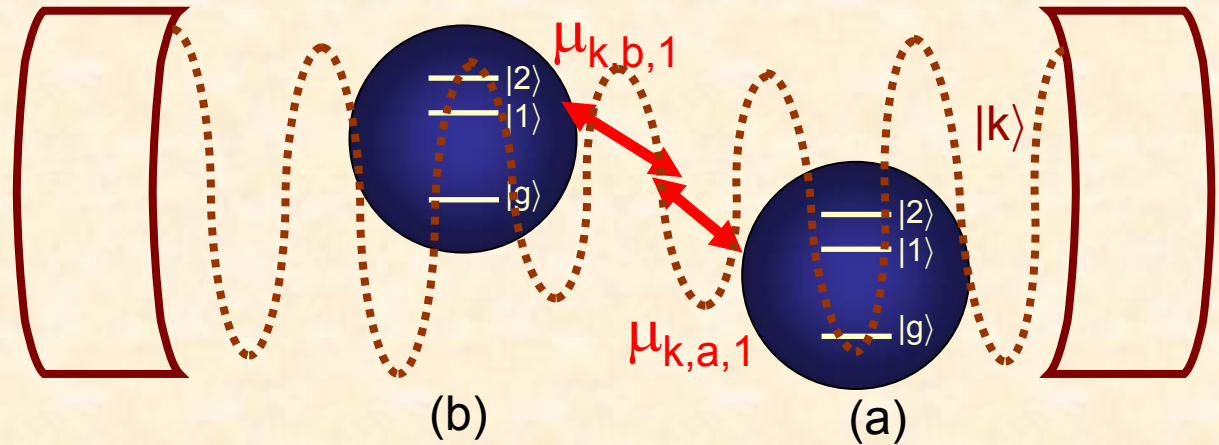
No decoupling for finite shifts!

Realistic Scenario:

Asymmetric Particle-Bath coupling

Gordon, Kofman GK Opt. Comm. 264, 398 (2006); quant-ph/0608175

Gordon & Kurizki, PRL 97, 110503 (2006), , quant-ph/0608174



- Particles:
 - Ions
 - Cold atoms
- Bath:
 - Cavity modes
 - Vibrational modes
- Bath-particle coupling: Position dependent. (& cross-decoherence)
- Can we restore symmetric coupling
 - Decoherence free subspace?

Viola et al. PRL **85**, 3520 (2000);

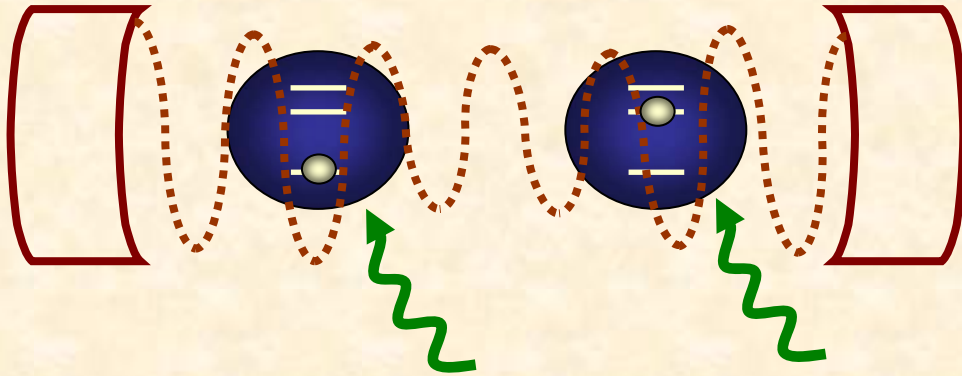
Wu & Lidar, PRL **88**, 207902 (2002)

DD assumptions: fast, strong, frequent

Dynamically Modified Decoherence Matrix Elements

Gordon, Kofman GK Opt. Comm. 264, 398 (2006); quant-ph/0608175

Gordon & Kurizki, PRL 97, 110503 (2006), , quant-ph/0608174

$$\begin{pmatrix} \alpha_{a,1}(t) \\ \alpha_{a,2}(t) \\ \alpha_{b,1}(t) \\ \alpha_{b,2}(t) \end{pmatrix} = T_+ e^{-\begin{pmatrix} J_{aa,11} & J_{aa,12} & J_{ab,11} & J_{ab,12} \\ J_{aa,21} & J_{aa,22} & J_{ab,21} & J_{ab,22} \\ J_{ba,11} & J_{ba,12} & J_{bb,11} & J_{bb,12} \\ J_{ba,21} & J_{ba,22} & J_{bb,21} & J_{bb,22} \end{pmatrix} t} \begin{pmatrix} \alpha_{a,1}(0) \\ \alpha_{a,2}(0) \\ \alpha_{b,1}(0) \\ \alpha_{b,2}(0) \end{pmatrix}$$


Diagonal elements: Individual particle decoherence

$$J_{jj,nn}(t) = 2\pi \int_{-\infty}^{\infty} d\omega G_{jj,nn}(\omega) |\varepsilon_{t,j,n}(\omega - \omega_{j,n})|^2$$

Dynamically Modified Decoherence Matrix Elements

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Off-diagonal elements: Cross-decoherence

$$J_{jj',nn'}(t) = 2\pi \int_{-\infty}^{\infty} d\omega G_{jj',nn'}(\omega) \varepsilon_{t,j,n}^*(\omega - \omega_{j,n}) \varepsilon_{t,j',n'}(\omega - \omega_{j',n'})$$

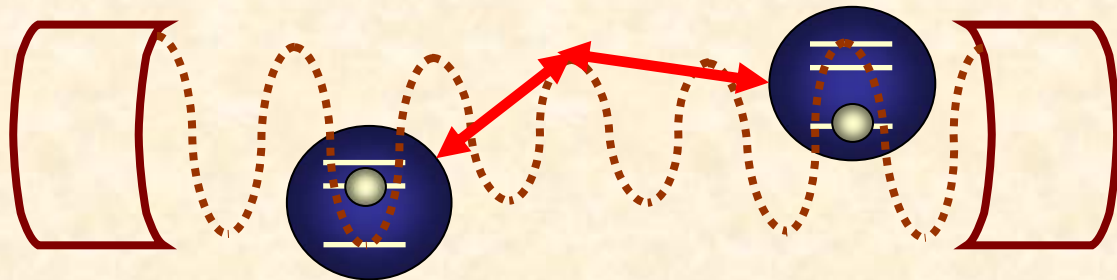
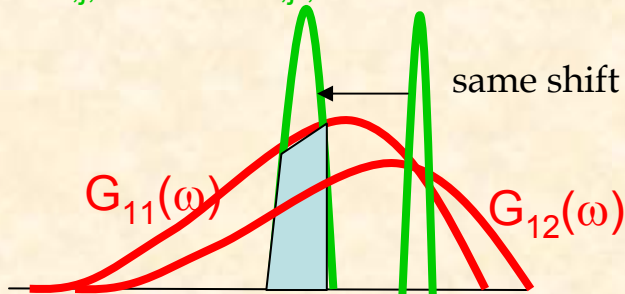
Distance dependent: how to control?

Global Modulation

$$\varepsilon_{j,n}(t) = \varepsilon(t) \quad \forall j,n$$

$$J_{jj',nn'}(t) = 2\pi \int_{-\infty}^{\infty} d\omega \mathbf{G}_{jj',nn'}(\omega) \varepsilon_t^*(\omega - \omega_{j,n}) \varepsilon_t(\omega - \omega_{j',n'})$$

$$|\varepsilon_{t,j,n}(\omega)|^2 = |\varepsilon_{t,j',n'}(\omega)|^2$$



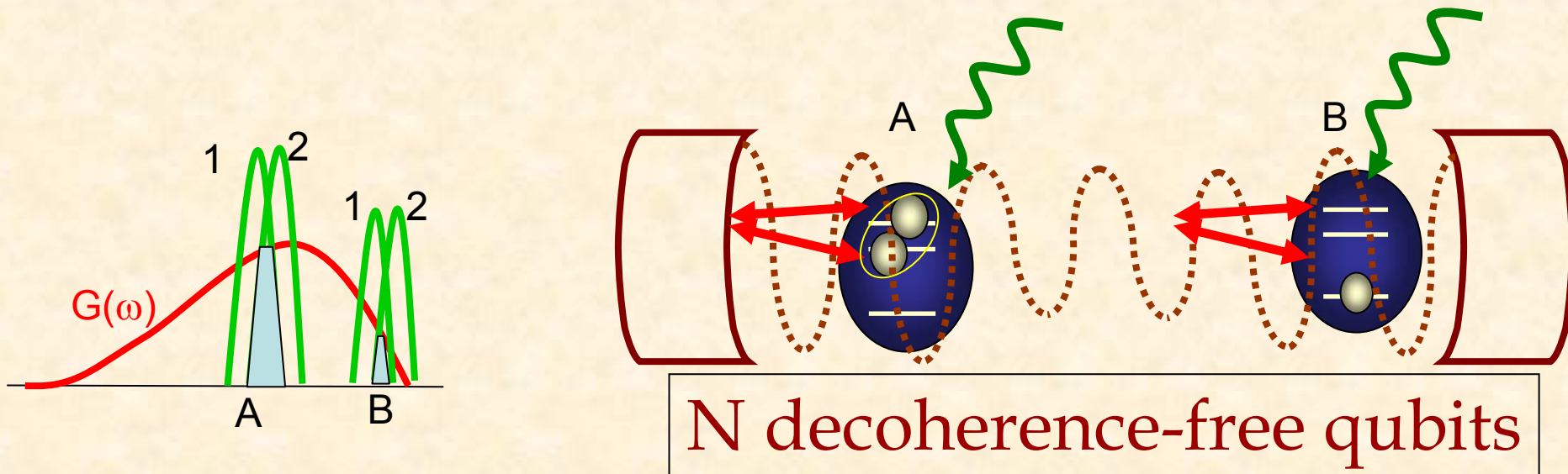
- $N(N+1)/2$ elements uncontrollable by single $\varepsilon_t(\omega)$!
- Cross-coupling, different couplings to bath: persist

Independent Trapping by Local Modulation

Gordon & Kurizki, PRL 97, 110503 (2006), , quant-ph/0608174

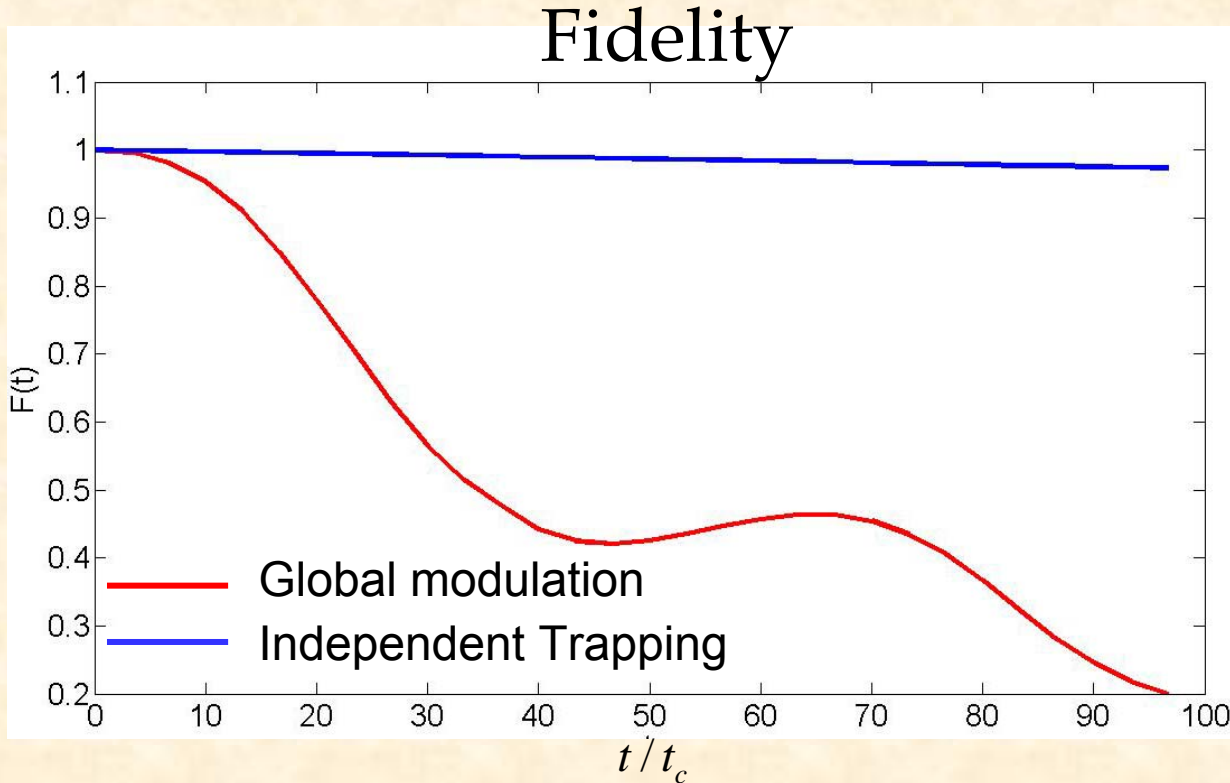
- For N three-level particles
- Anti-symmetric state of each particle $|-\rangle_j = 1/\sqrt{2}(|1\rangle_j - |2\rangle_j)$
= Intra-particle Decoherence Free Subspace
- Equating intraparticle decoherence rates (impose des. interf.)
- Eliminating interparticle cross-decoherence
modulation need not lead to decoupling

(works even for SE - broad G)



Numerical Example: Independent Trapping

Godon & Kurizki, PRL 97, 110503 (2006), , quant-ph/0608174

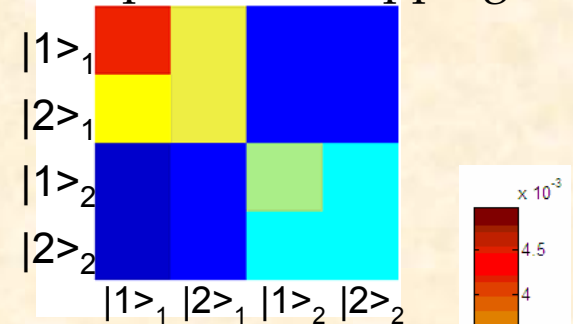


$$|\Psi(0)\rangle = 1/\sqrt{2}(|-\rangle_a |g\rangle_b \pm |g\rangle_a |-\rangle_b)$$

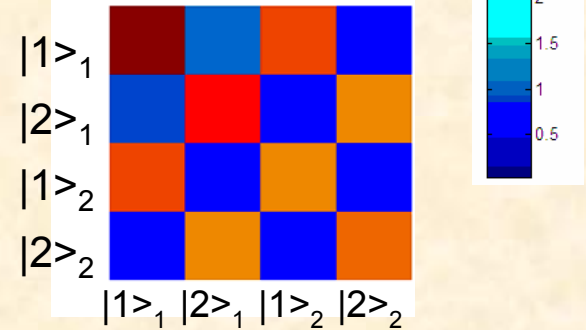
$$|-\rangle_j = 1/\sqrt{2}(|1\rangle - |2\rangle)_j$$

Decoherence Matrices

Independent Trapping



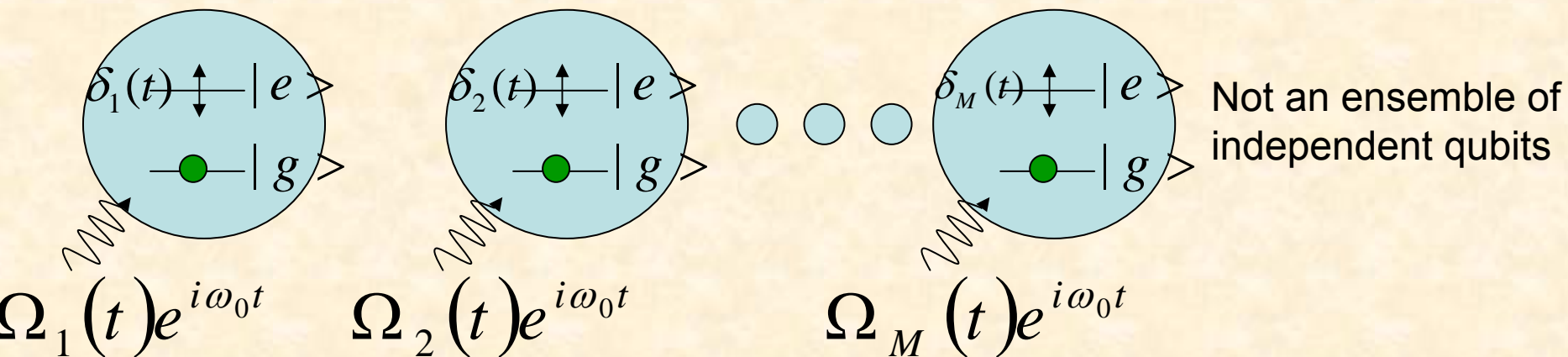
Global Modulation



Singly excited N-particle MES may be always protected by 2N local fields !

Multi-qubit dephasing control

Gordon et al. Opt. Comm. **264**, 398 (2006), quant-ph/0608175



Each qubit undergoes (different) proper dephasing

Each qubit independently driven (addressability)

Cross-correlation Function

$$\overline{\delta_i(t)\delta_j(t-\tau)} = \Phi_{ij}(\tau)$$

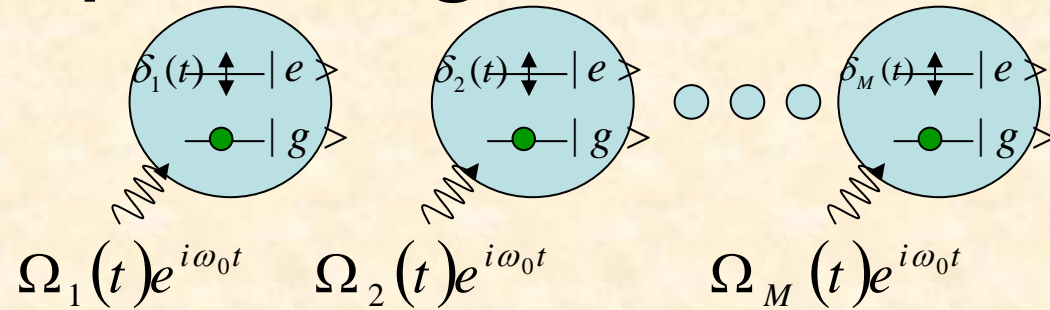
$$\varepsilon_i(t) = \exp\left(2i\int_0^t dt' \Omega_i(t')\right)$$

$$\Phi_{ij}(\tau) = \frac{2\gamma}{t_{c,i} + t_{c,j}} \frac{\exp\left(-\tau/2t_{c,i} - \tau/2t_{c,j}\right)}{1 + |r_i - r_j|/r_c}$$

Depends on distance between qubits

Multi-qubit dephasing control

Gordon et al. Opt. Comm. **264**, 398 (2006),
quant-ph/0608175



For each qubit: $|g\rangle, |e\rangle \Rightarrow |\pm\rangle$

Ensemble average,
density matrix:

$$\bar{\rho}(t) = \rho(0) - \frac{1}{4} \int_0^t dt' \int_0^{t'} dt'' \left[W_{off}(t'), [W_{off}(t''), \rho(0)] \right]$$

$$W_{off}(t)$$

$2^M \times 2^M$ matrix

$$n^{\text{th}} \text{ qubit-flip} \rightarrow \delta_n(t) \varepsilon_n(t)$$

Example: Two qubits

$$\begin{array}{l}
 |--\rangle \\
 |-\rangle \\
 |+\rangle \\
 |++\rangle
 \end{array}
 \rightarrow
 \begin{pmatrix}
 0 & \delta_2 \varepsilon_2 & \delta_1 \varepsilon_1 & 0 \\
 \delta_2 \varepsilon_2^* & 0 & 0 & \delta_1 \varepsilon_1 \\
 \delta_1 \varepsilon_1^* & 0 & 0 & \delta_2 \varepsilon_2 \\
 0 & \delta_1 \varepsilon_1^* & \delta_2 \varepsilon_2^* & 0
 \end{pmatrix}$$

Two-qubit dephasing control

Gordon et al. Opt. Comm. **264**, 398 (2006),
quant-ph/0608175

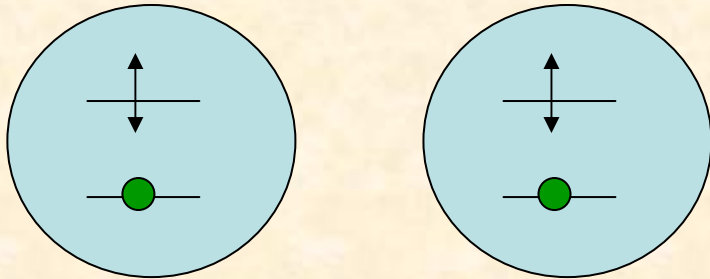
Pairwise DFS: M fields suffice

$$J_{jj'}(t) = \int d\omega G_{jj'}(\omega) \varepsilon_{t,j}(\omega) \varepsilon_{t,j'}(\omega)$$

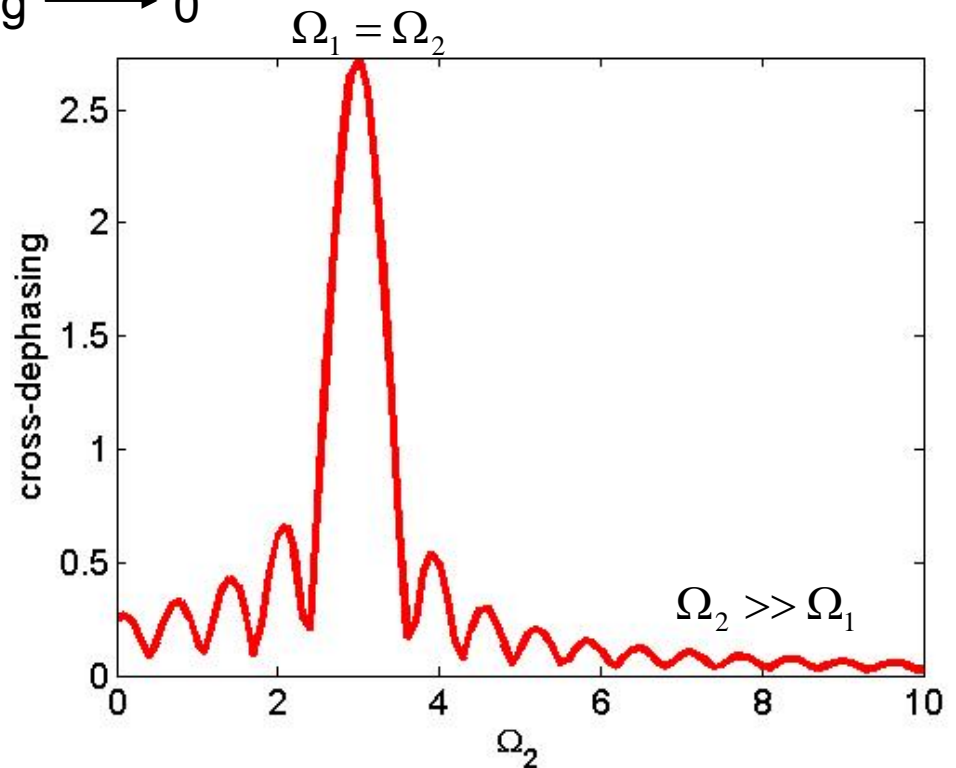
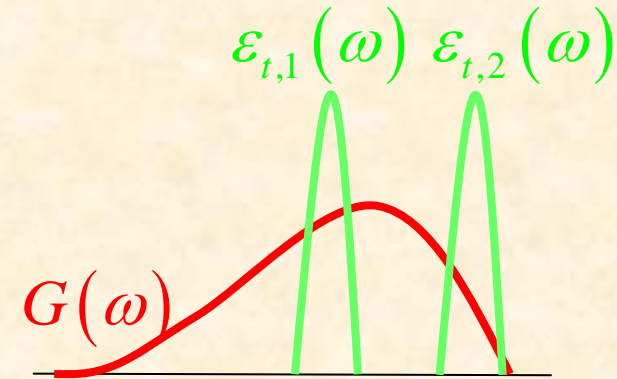
For different local modulations

$$\varepsilon_1(t) \neq \varepsilon_2(t)$$

Cross-dephasing $\rightarrow 0$



Individual local modulations
decouple the qubits
(eliminate cross dephasing)



Two-qubit dephasing control

Gordon et al. Opt. Comm. **264**, 398 (2006),
quant-ph/0608175

Initial entangled state:

$$|\psi_{\pm}(0)\rangle = \frac{1}{\sqrt{2}} (|ge\rangle \pm |eg\rangle) \quad \text{triplet/singlet}$$

Fidelity

$$F_{\pm}(t) = 1 - \frac{1}{2} \left[\underbrace{(J_{11}(t) + J_{22}(t))}_{\text{Dephasing of Individual qubits}} \pm \underbrace{(J_{12}(t) + J_{21}(t))}_{\text{Cross dephasing}} \right]$$

Modulation Scheme 1:

Equate dephasing and cross-dephasing rates $J_{jj'}(t) = J(t)$

$F_{-}(t) = 1 \rightarrow$ **Singlet**
Decoherence free subspace (DFS)

Viola et al. PRL **85**, 3520 (2000);
Wu & Lidar, PRL **88**, 207902 (2002)

Modulation Scheme 2:

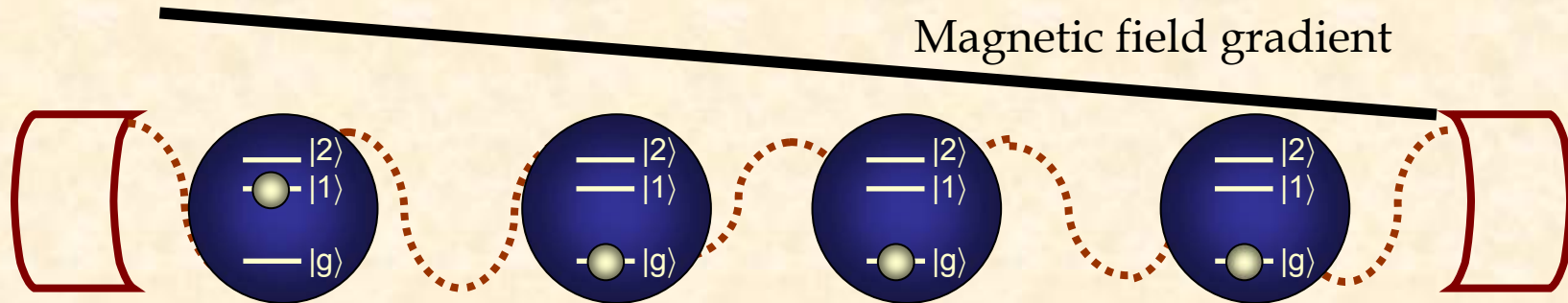
Eliminate cross-dephasing $J_{jj'}(t) = 0 \quad j \neq j'$

$F_{\pm}(t) = F(t) \rightarrow$ Equal fidelity for all Bell-states

Advantageous in quantum communications

Suggested Experimental Setup: Multiple $^{40}\text{Ca}^+$ Ions in Cavity

Kreuter et al. PRL **92** 203002 (2004); Barton et al. PRA **62** 032503 (2000)
D. Schrader et al., Phys. Rev. Lett. **93**, 150501 (2004).



Spatial addressability \rightarrow Spectral addressability
 $N=10$ ions \rightarrow 10 control fields
 50 gate fields

Three level system:

$$|g\rangle = 4^2S_{1/2}$$

$$|1\rangle = 3^2D_{3/2}$$

$$|2\rangle = 3^2D_{5/2}$$

$$\tau_1/\tau_2 = 1.026$$

Multiple ions in cavity:

Position in cavity: $\tau_a - \tau_b \approx 15\%$
 Two ions in each cavity
 + Local modulations
 $1/\sqrt{2}(|g\rangle_a|1\rangle_b - |1\rangle_a|g\rangle_b) = \text{DFS}$

Conclusions

- Universal approach to dynamical decoherence control allows to guess the optimal control fields for different (amplitude and phase) noises, in multipartite systems
- Local modulations (spectral addressability) can
 - Create a multipartite decoherence-free system despite asymmetric and/or cross-couplings to a bath
 - Enhance gate fidelity under local dephasing by specific control fields for each qubit pair
- Tradeoff: ancilla \rightarrow $2N$ or $N^2/2$ control fields

Know thine enemy