Quantum Decoherence and Gravitational Waves

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Informations available on
http://www.spectro.jussieu.fr/Vacuum

→ **Quantum fluctuations and relativity**

PhD thesis (in French)

http://tel.ccsd.cnrs.fr

Gravitation and Decoherence

classicality ⟷ large systems (macroscopic) ⟷ gravity is the dominant force for large systems

✖ virus, human, planet, universe…

FEYNMAN :
appearence of classicality due to gravitation

R.P. Feynman, *Feynman Lectures on Gravitation (1963)*

- Qualitative argument of Feynman based on Planck scales :

$$t_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^5}} \simeq 5 \times 10^{-44}\,\mathrm{s} \qquad l_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^3}} \simeq 10^{-35}\,\mathrm{m} \qquad m_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}} \simeq 22\,\mu\mathrm{g}$$

✖ The Planck mass is accessible !

- Gravitation leads to a decoherence effect for $m \geq m_{\mathrm{P}} \iff \ell_C = \dfrac{\hbar}{mc} \leq \ell_{\mathrm{P}}$

Gravitational decoherence scenarios

- Penrose mechanism : quantum state reduction.
  - A quantum superposition inherently possess a time uncertainty due spacetime curvature which in return leads to an energy uncertainty ΔE.
  - ΔE reduces to the Newtonian self energy of the superposition.
  - Decoherence time $\tau = \hbar/\Delta E$.

R. Penrose, Gen. Rel. Grav. **28** (1996), 581

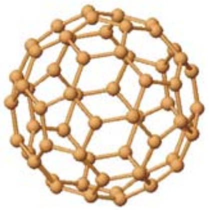
- GEID (Gravitational Environnement Induced Decoherence)
  - A quantum superposition interacts with a fluctuating gravitational environnement.
  - Many gravitational fluctuations have been studied → I focus on gravitational waves.
  - Use of Feynman-Vernon functional influence.

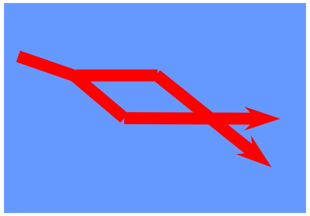
Gravity induced quantum/classical transition

Quantum Domain

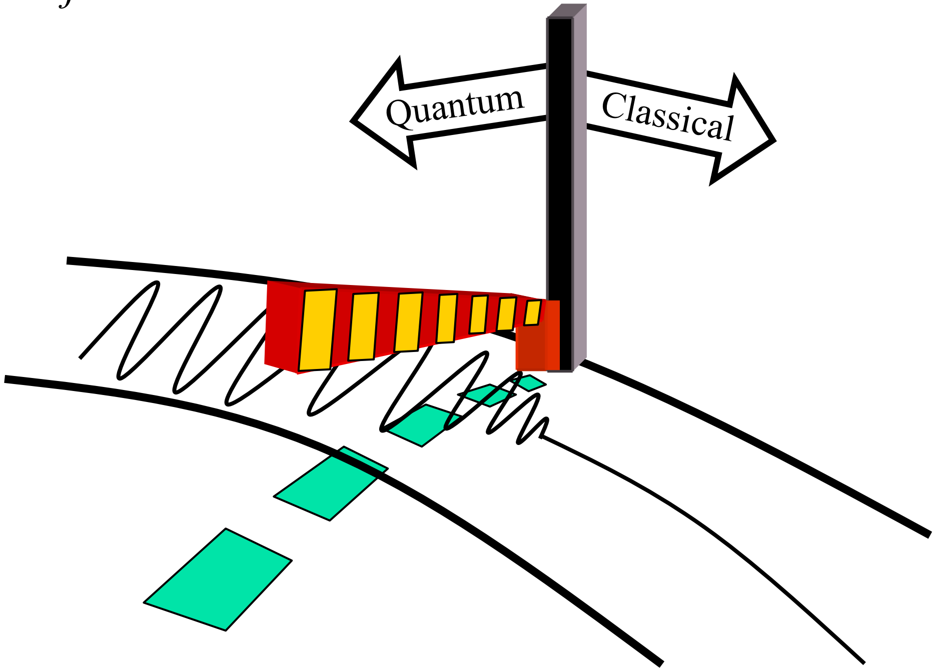
Microscopic objects

C_{60}





interferences



Classical Domain

Macroscopic objets

No interferences

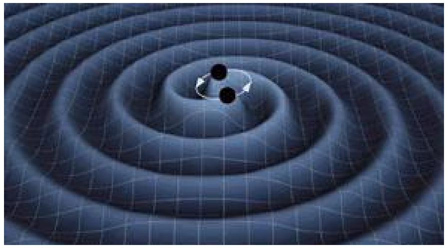
- Where is the borderline ?
- Is it observable ?

The gravitational environment

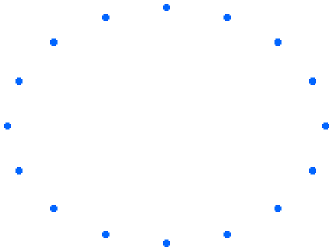
- Gravitational waves (GW) are freely propagating solutions of linearized Einstein equations (Einstein 1918).
  - Analog to electromagnetic radiation.

Accelerated dipole of charge ← detection — emission → Electromagnetic radiation

Accelerated quadrupole of mass ← detection — emission → Gravitational radiation



- Effect on freely falling particles



- GW propagate at the speed of light.
- GW have 2 transverses polarizations.
- Perturbation of the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad h_{\mu\nu} \ll 1$$

Gravitational waves background

- GW backgrounds ! collection of harmonic oscillators

$$h_{ij}(t) = \sum_\alpha \lambda_{\alpha,ij}\, h_\alpha(t)$$

$$S_{\mathcal{E}} = \frac{c^2}{32\pi G} \sum_\alpha \int_{t_i}^{t_f} \frac{1}{2}\left(\dot{h}_\alpha^2(t) + \omega^2 h_\alpha^2(t)\right) \mathrm{d}t$$

- h_α are normal variables of the field and can be quantized ! $\hat{h}_{ij}$

$$\sum_\alpha \equiv \frac{2}{\pi c^3} \sum_{\gamma=+,\times} \int \frac{\mathrm{d}\hat{\mathbf{k}}}{4\pi} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, \omega^2$$

polarizations *directions* *frequencies*

- Classical metric perturbations are characterized by a noise spectrum $S_h[\omega]$:

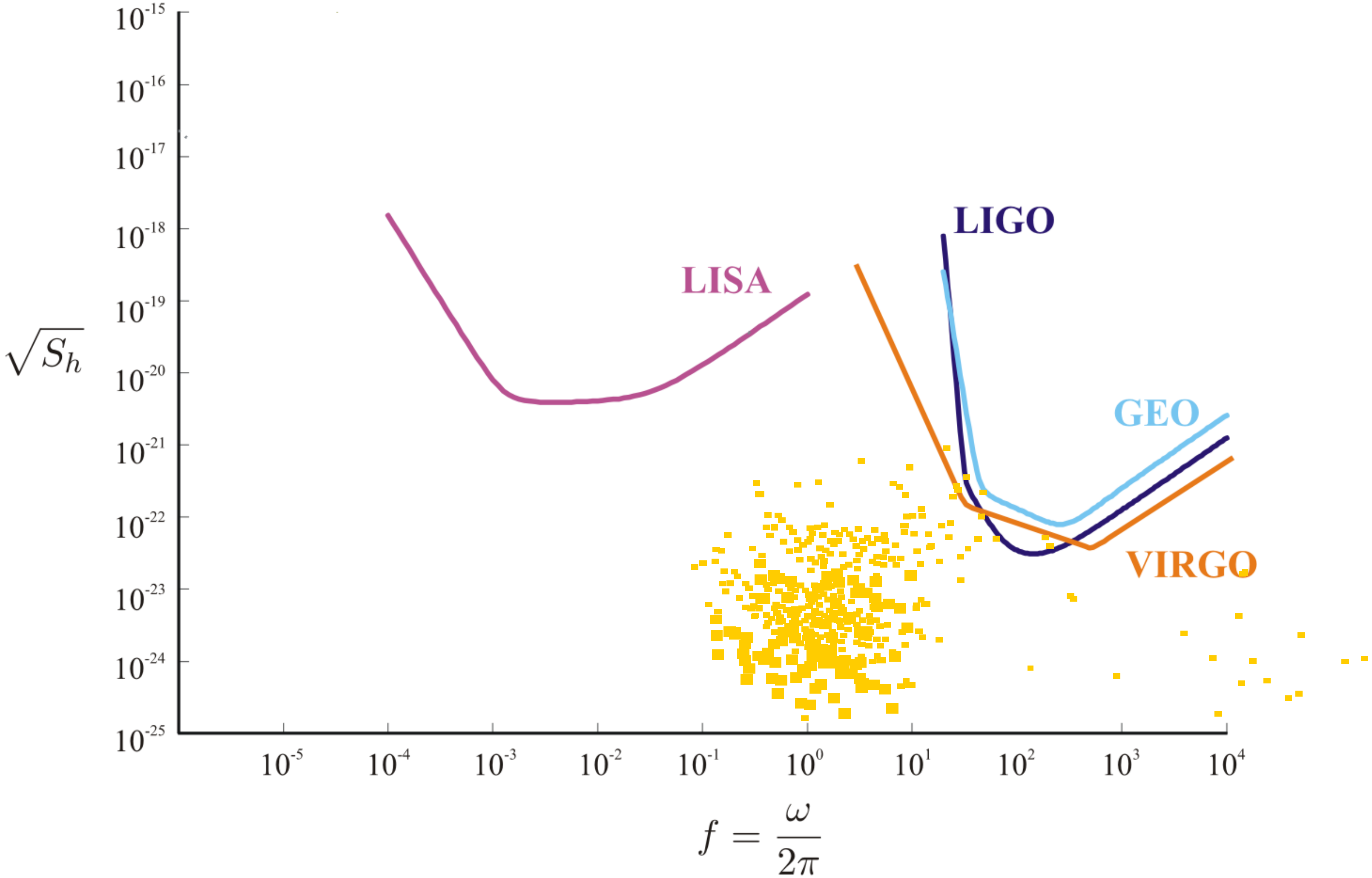
$$\langle h_{ij}(t) h_{kl}(0)\rangle = \delta_{ijkl} \int \frac{\mathrm{d}\omega}{2\pi} S_h[\omega] e^{-i\omega t}$$

$$S_h[\omega] = \frac{16G}{5c^5} k_{\mathrm{B}} T_{\mathrm{gw}}[\omega] = \frac{16G}{5c^5} n_{\mathrm{gw}}[\omega] \hbar\omega$$

- no blackbody for gravitational radiation) frequency dependent noise temperature
- GW detectors need $n_{gw} \sim 10^{31}$ gravitons at 1kHz

Gravitational waves background

- Resolved binaries pulsars



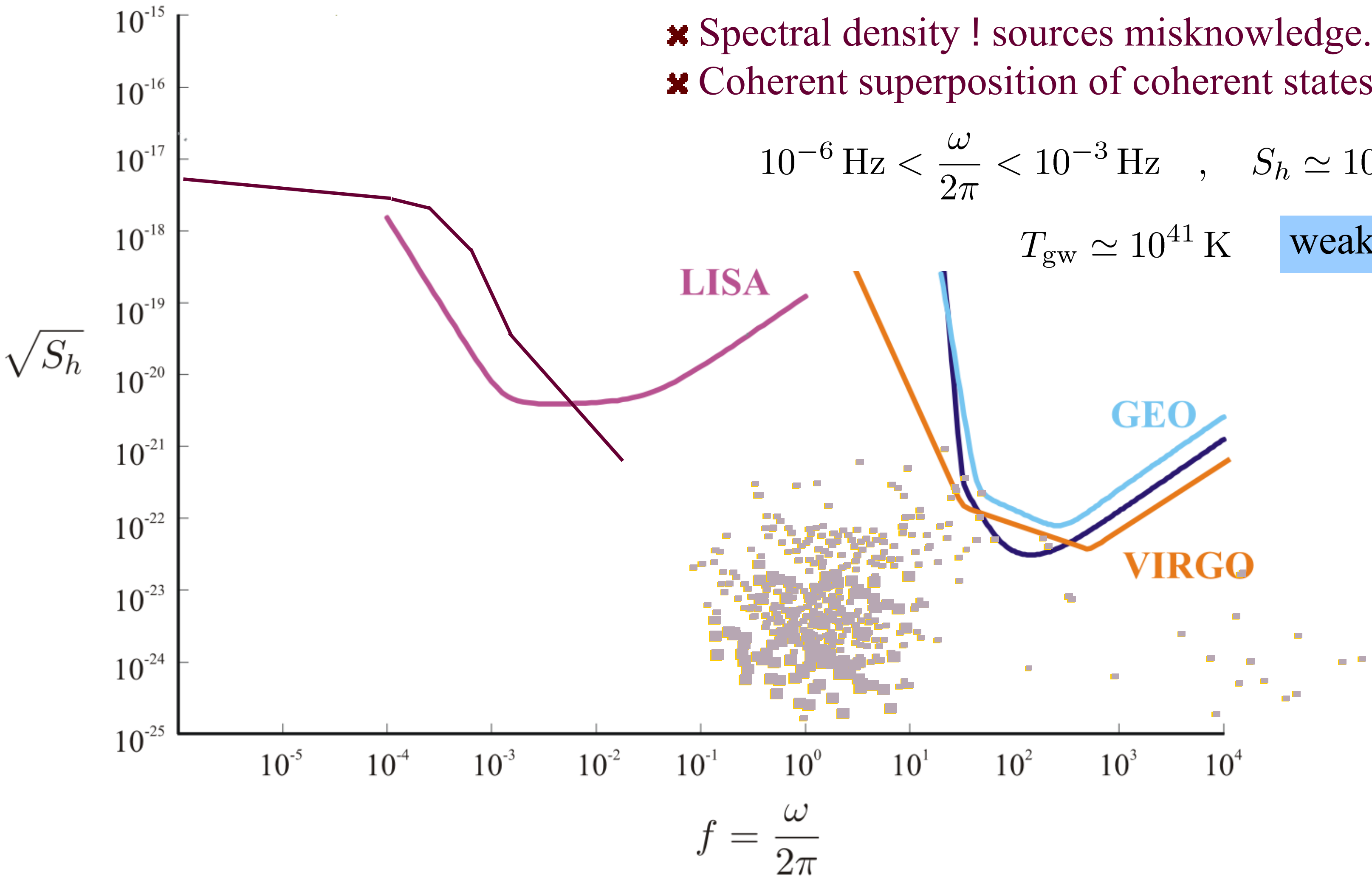
Gravitational waves background

- Binary confusion background
  - Spectral density ! sources misknowledge.
  - Coherent superposition of coherent states.

$$10^{-6}\,\mathrm{Hz} < \frac{\omega}{2\pi} < 10^{-3}\,\mathrm{Hz} \quad , \quad S_h \simeq 10^{-34}\,\mathrm{s}^{-1}$$

$$T_{\mathrm{gw}} \simeq 10^{41}\,\mathrm{K}$$

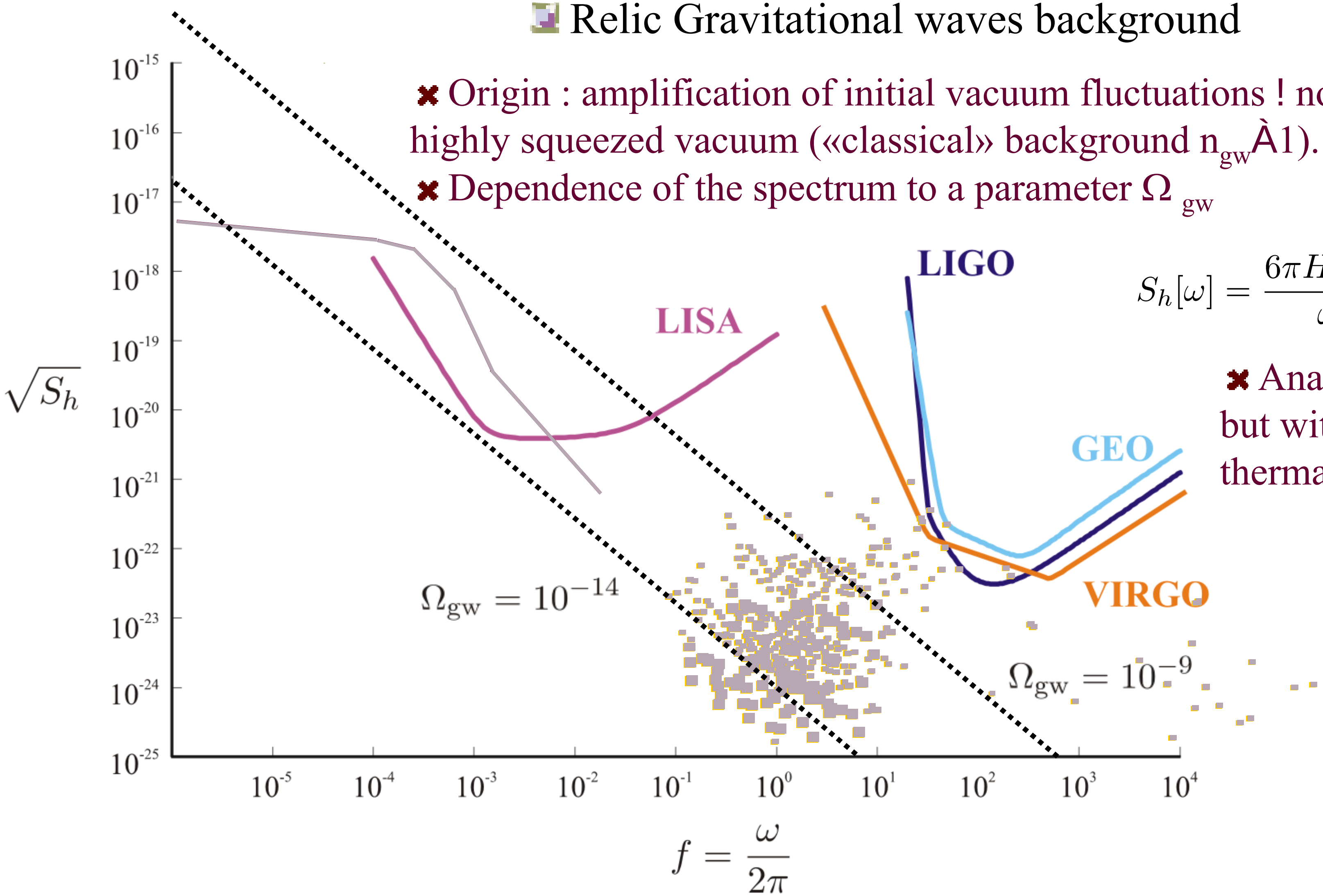
weak coupling



Gravitational waves background

Relic Gravitational waves background

- Origin : amplification of initial vacuum fluctuations ! nowadays a highly squeezed vacuum («classical» background n_{gw}À1).
- Dependence of the spectrum to a parameter Ω_{gw}



$$S_h[\omega] = \frac{6\pi H_0^2 \Omega_{gw}}{\omega^3}$$

- Analog to CMB but with a non thermal spectrum.

Coupling to gravitational waves

- Linear coupling of matter to gravitation : $\mathcal{L}_{\mathcal{S}/\mathcal{E}} = \frac{1}{2} h_{\mu\nu} T^{\mu\nu}$
  - $T^{\mu\nu}$ energy momentum tensor

- Quadrupole coupling in the long wavelength approximation : $\lambda_{\rm gw} \geq 3\,000\,\mathrm{km}$

$$S_{\mathcal{S}/\mathcal{E}} = \frac{1}{4} \int \mathrm{d}t \; h_{ij}(t) \ddot{Q}^{ij}(t) = \frac{1}{4} \sum_{\alpha} \lambda_{\alpha,ij} \int \mathrm{d}t \; h_{\alpha}(t) \ddot{Q}^{ij}(t)$$

$$Q^{ij}(t) = \frac{1}{c^2} \int \mathrm{d}^3\vec{x} \left(x^i x^j - \frac{1}{3} \delta^{ij} x^k x_k \right) T^{00}(t, \vec{x})$$

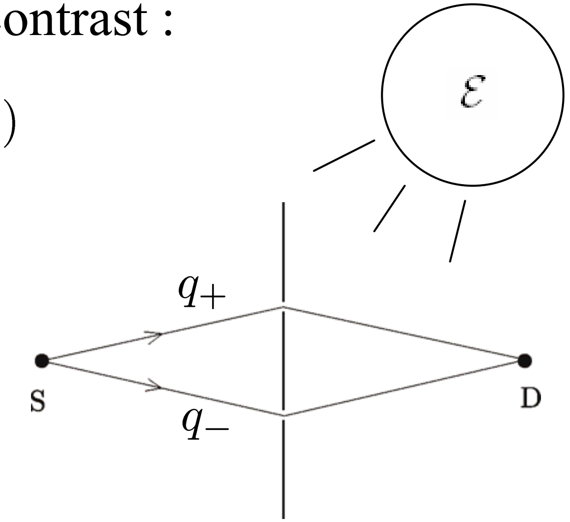
  - equivalent to the dipole approximation of electromagnetism
  - expression in the TT gauge

Decoherence in interferometers

B. Lamine, *PhD* Thesis (2004)

- Coherence of an interferometer characterized by its contrast :

$$\mathcal{I} = |\mathcal{A}_+ + \mathcal{A}_-|^2 = |\mathcal{A}_+|^2 + |\mathcal{A}_-|^2 + 2\mathrm{Re}(\mathcal{A}_+\mathcal{A}_-^*)$$



- In presence of an environnement :

$$\mathcal{I} = |\mathcal{A}_+|^2 + |\mathcal{A}_-|^2 + 2\mathrm{Re}(\mathcal{A}_+\mathcal{A}_-^*\mathcal{F}[q_+, q_-])$$

$$\langle\mathcal{E}_+|\mathcal{E}_+\rangle = \langle\mathcal{E}_-|\mathcal{E}_-\rangle = 1 \qquad \mathcal{F}[q_+, q_-] = \langle\mathcal{E}_+|\mathcal{E}_-\rangle$$

- $F[q_+(t), q_-(t)]$: Feynman-Vernon influence functional ! coherence between two paths $q_+(t)$ and $q_-(t)$.

- After temporal averaging on the integration time :

$$\mathcal{V} = |\overline{\mathcal{F}[q_+, q_-]}| = |\overline{\langle\mathcal{E}_+|\mathcal{E}_-\rangle}|$$

✖ decoherence observable.

Influence functional

- The influence functional can be written as an exponential of linear and two quadratic kernels:

$$\mathcal{F}[q_+,q_-] = \exp\left(\frac{i}{4\hbar}\int_{t_i}^{t_f}(\ddot{Q}^{ij}[q_+(t)]-\ddot{Q}^{ij}[q_-(t)])\langle\hat{h}_{ij}(t)\rangle \mathrm{d}t\right)$$

$$\times\exp\left(-\frac{1}{4\hbar}\int_{t_i}^{t_f}\mathrm{d}t\int_{t_i}^{t_f}\mathrm{d}s\,(\ddot{Q}^{ij}[q_+(t)]-\ddot{Q}^{ij}[q_-(t)])\sigma_{ijkl}(t-s)(\ddot{Q}^{kl}[q_+(s)]-\ddot{Q}^{kl}[q_-(s)])\right)$$

$$\times\exp\left(-\frac{1}{4\hbar}\int_{t_i}^{t_f}\mathrm{d}t\int_{t_i}^{t_f}\mathrm{d}s\,(\ddot{Q}^{ij}[q_+(t)]+\ddot{Q}^{ij}[q_-(t)])\xi_{ijkl}(t-s)(\ddot{Q}^{kl}[q_+(s)]-\ddot{Q}^{kl}[q_-(s)])\right)$$

$i\Delta\varphi_{\mathrm{lin}}$

$-\frac{1}{2}\Delta\varphi^2_{\mathrm{noise}}$

$-\frac{i}{2}\Delta\varphi^2_{\mathrm{diss}}$

- With the following kernels :

$$\sigma_{ijkl}(\tau) = \frac{1}{4\hbar}\left(\langle\hat{h}_{ij}(\tau)\cdot\hat{h}_{kl}(0)\rangle - \langle\hat{h}_{ij}(\tau)\rangle\langle\hat{h}_{kl}(0)\rangle\right)$$

- Noise kernel.

$$\xi_{ijkl} = \frac{1}{4\hbar}\langle[\hat{h}_{ij}(\tau),\hat{h}_{kl}(0)]\rangle = i\delta_{ijkl}\int_0^\infty \mathrm{d}\omega\,\frac{2G\omega}{5\pi c^5}\sin(\omega\tau)$$

- Dissipation kernel (imaginary) (spontaneous emission of GW).

Gravitational decoherence

- Double limit of the gravitational waves environment :
  - low dissipation (weak coupling) : $\xi_{ijkl}(\tau) \simeq 0$
  - high noise (large effective temperature) : $\sigma_{ijkl}(\tau)$ is a classical correlation function.

- This limit is formally equivalent to G ! 0 and T_{gw} ! 1 with a finite product.

$$\mathcal{F}[q_+, q_-] = \exp\left(i\Delta\varphi_{\text{lin}} - \frac{1}{2}\Delta\varphi^2_{\text{noise}} \right)$$

- Relic gravitational waves : $\langle \hat{h}_{ij}(t) \rangle = 0$ ~~$\Delta\varphi_{\text{lin}}$~~
  - « Quantum » decoherence by a squeezed vacuum.

- Binary confusion background : $\langle \hat{h}_{ij}(\tau) \cdot \hat{h}_{kl}(0) \rangle - \langle \hat{h}_{ij}(\tau) \rangle \langle \hat{h}_{kl}(0) \rangle \simeq 0$ ~~$\Delta\varphi^2_{\text{noise}}$~~
  - « Classical » decoherence due to the fluctuations of the linear term when averaging over the integration time.

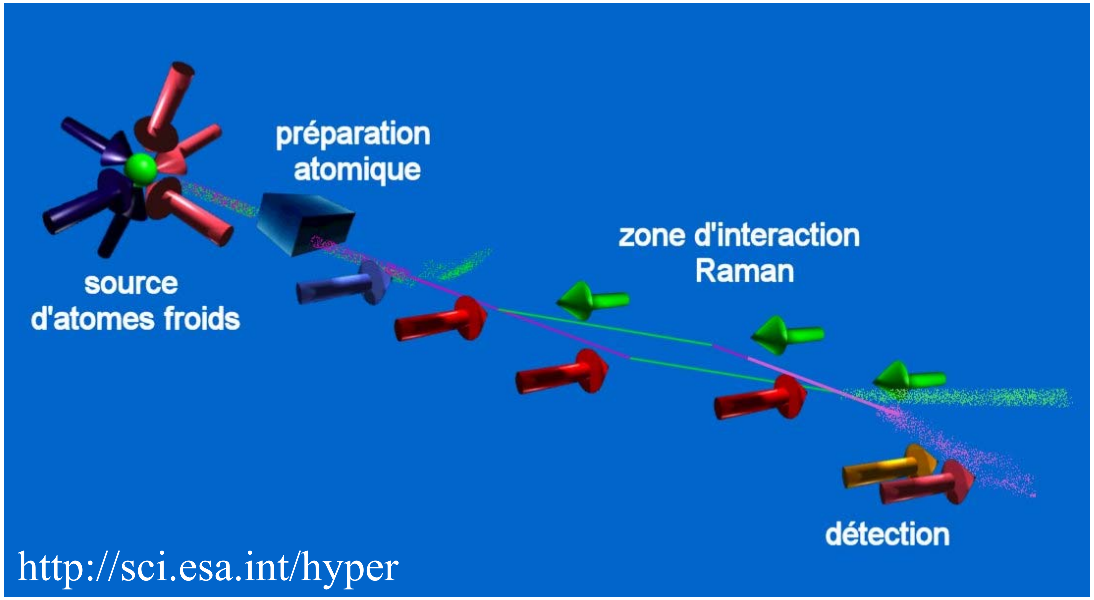
$$\mathcal{V} = \left| \overline{\mathcal{F}[q_+, q_-]} \right| = \exp\left(-\frac{1}{2}\Delta\varphi^2 \right)$$

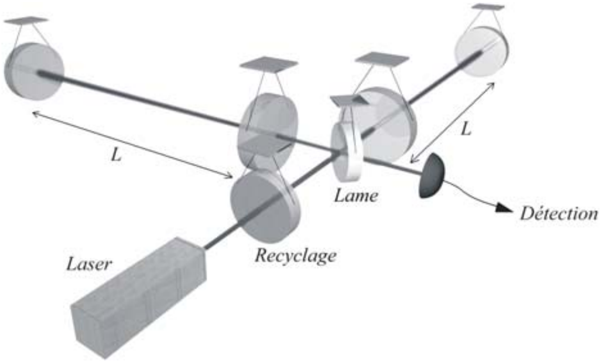
$$\Delta\varphi^2 = \overline{\Delta\varphi^2_{\text{noise}}} + \overline{\Delta\varphi^2_{\text{lin}}}$$

Application to atomic interferometers

S. Reynaud & *al.*, Gen. Rel. Grav. **36** (2004), 2271

- The lasers used for stimulated Raman transition provide nearly freely falling mirrors and beam splitters for atoms.

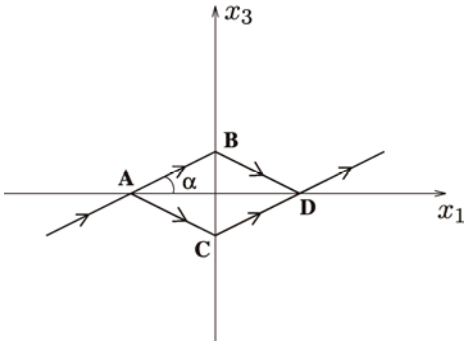




- It behaves as a GW detector.

- Dephasing in the eikonal approximation

$$\Delta\varphi_{\rm lin} = \frac{1}{2\hbar}\oint h_{\mu\nu}\frac{P^\mu P^\nu}{m}\,{\rm d}t$$



Discussion of a simplified model

B. Lamine & *al,* Eur. Phys. J. ***D20*** (2002), 165

- Assume a white noise model $S_h[\omega] = C^{te}$ for the gravitational background

$$\Delta\varphi^2 = (2\Omega_{\text{at}} \sin(2\alpha))^2 \, S_h \, 2\tau_{\text{at}} \qquad \Omega_{\text{at}} = \frac{m_{\text{at}} v_{\text{at}}^2}{2\hbar}$$

  - Brownian diffusion of the phase.
  - For photons, replace Ω_{at} by the frequency Ω_{phot} of photon.

- Relevant parameters for the estimation of decoherence :

  - kinetic energy (in comoving frame) of the probe (not its mass energy !),
  - geometry (aperture sin (2α)),
  - level of the noise spectrum (S_h),
  - exposition time of the probe to the GW environment

the comparison of mass with Planck mass enters here, but only as one of the parameters

Discussion

B. Lamine & *al*, Phys. Rev. Lett. **96** (2006), 050405

- All existing interferometers verify $\Delta\varphi^2 \ll 1$

- At present time, gravitational decoherence is unobservable :
  - too slow for microscopic probes,
  - too fast for macroscopic probes (example of the center of mass motion of the Moon).

S. Reynaud & *al*., Europhys. Lett. 53 (2001), 135

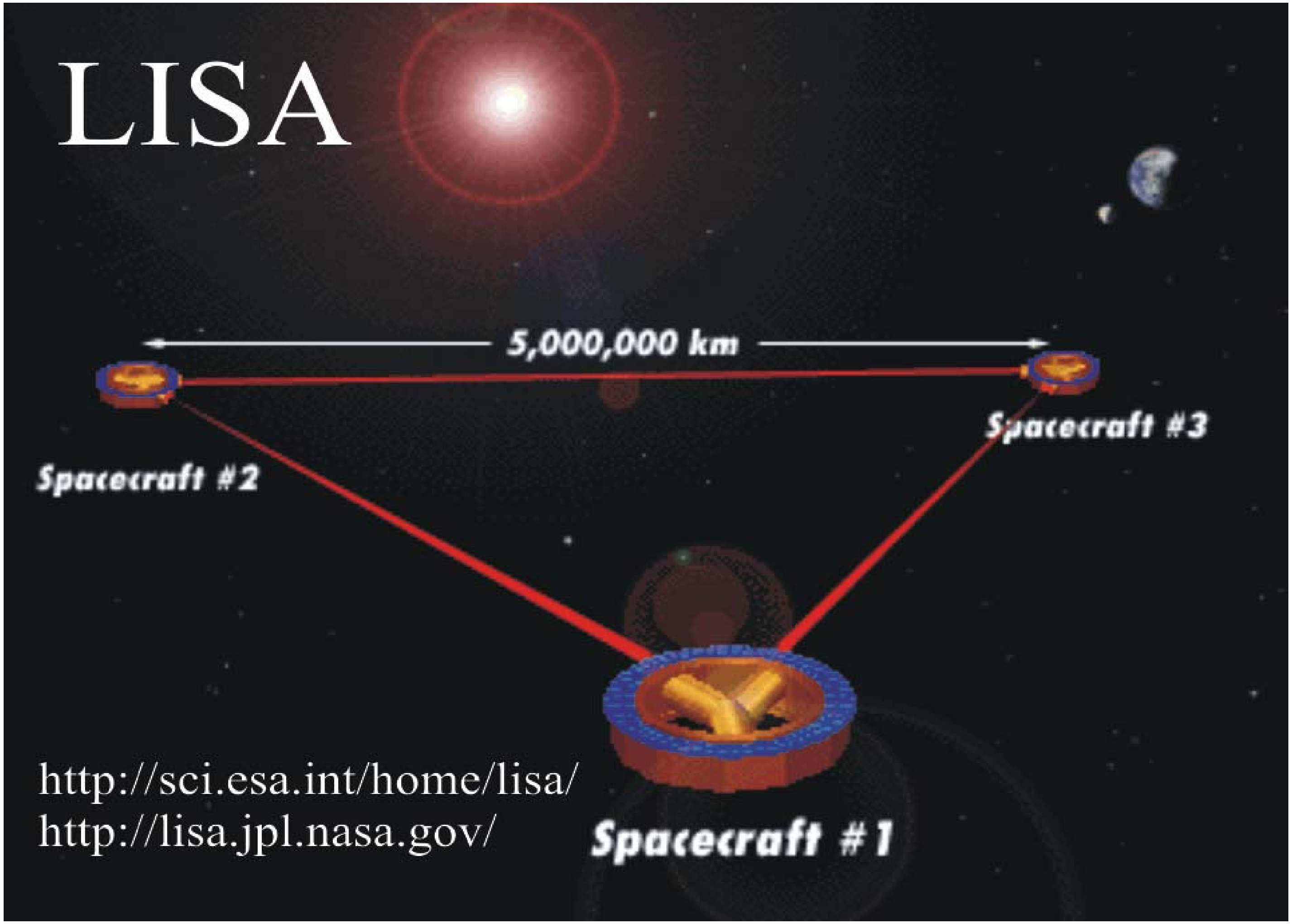
- Would it be possible to design experiments allowing one to explore the transition on « mesoscopic » objects ?

$$\Delta\varphi^2 \simeq (\Omega \sin\alpha)^2 S_h \tau \sim 1$$

$(\Omega \sin\alpha) \rightarrow \frac{\Delta E_k}{\hbar}$

- high energy splitting

Optical interferometer



$$\tau_{\text{phot}} \simeq 10\,\text{s}$$

$$\Omega_{\text{phot}} \simeq 10^{15}\,\text{rad.s}^{-1}$$

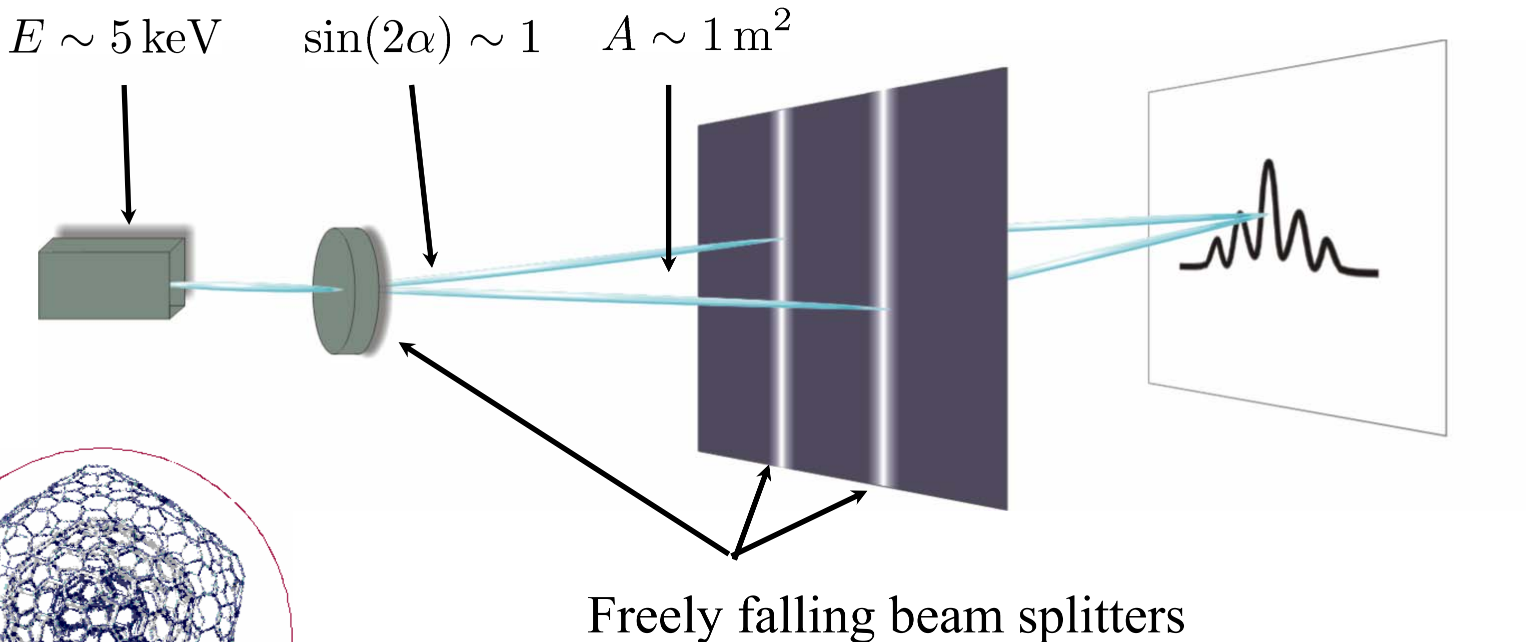
$$\Delta\varphi^2 \ll 1$$

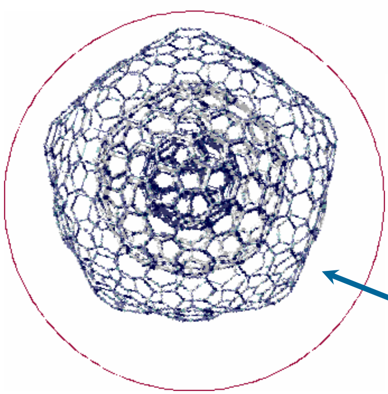
LISA is microscopic !

Good news for the fringes…

Though experiment

- $\Delta\varphi^2 \sim 1$ would need :

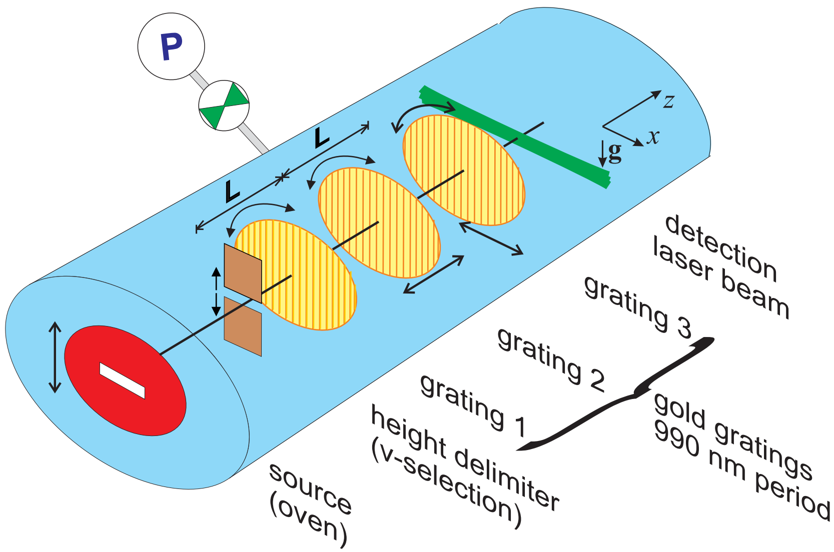




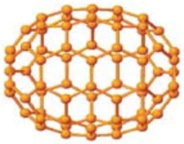
- Multi shell fullerene in supersonic beam (~ 3000 carbon atoms).

State of the art

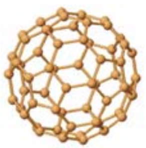
Zeilinger & *al.*, *Phys. Rev. Lett.* **88** (2002), 100404



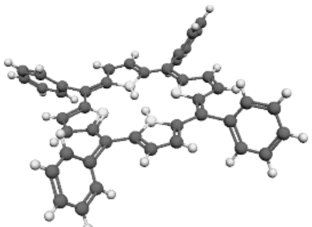
- Interferences with large molecules.



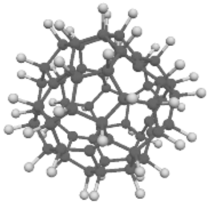
C_{70}



C_{60}



$C_{44}H_{30}N_4$

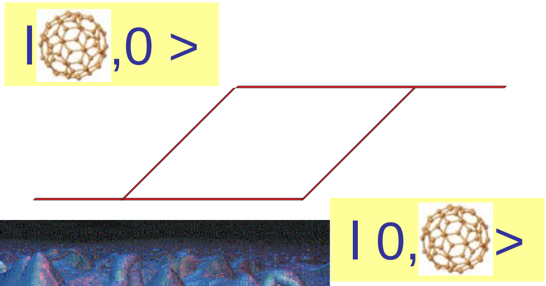


$C_{60}F_{48}$

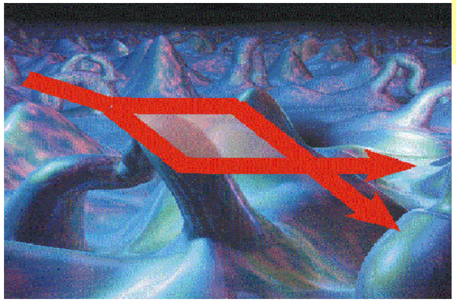
- Nevertheless, small separation angle, small kinetic energy and not freely falling beam splitters.

$E \sim 100\,\mathrm{meV}$ $\qquad \Delta\varphi^2 \ll 1$

Future ?



- Improving state of the art experiment à la Zeilinger with larger and hotter molecules ?



HYPER
(ESA)

- Going to space : increase of time of flight.

- Using Bose-Einstein condensate ? (N^2 amplification effect of decoherence)

- Looking for decoherence effects in condensed matter physics ?

- Schrödinger cats using micro-mechanical systems ?

S. Bose & *al.*, *Phys. Rev. A* **59** (1999), 3204

- Any idea is welcome !

Conclusions

- Gravitational waves set an ultimate frontier to quantum coherence

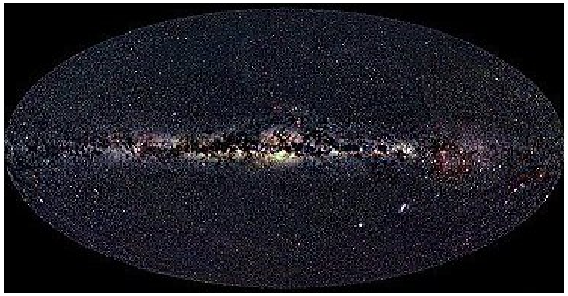
  ! Feynman intuition

- GW can be revealed by means of decoherence, in the same way than Brownian motion revealed the atomic nature of matter before we could observe atoms.

I.C. Percival, *Phys. World* **10** (1997) 48

- Gravitational waves astronomy : the next revolution in astrophysic ?

visible



Infra-red