NEW ENTROPIC UNCERTAINTY RELATIONS AND INEQUALITIES FOR TOMOGRAPHIC ENTROPIES

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Abstract

Entropic uncertainty relations are reviewed and new inequalities are presented for symplectic tomographic entropies. The tomographic entropies for qubit states of composite systems are discussed and strong subadditivity conditions are presented.

S. De Nicola, R. Fedele, M. A. Man'ko, and V. I. Man'ko, "New uncertainty relation for tomographic entropy: Application to squeezed states and solitons," Los Alamos ArXiv quant-ph/0607200 v1; *Eur. Phys. J.* B, **52**, 191–198 (2006).

M. A. Man'ko, V. I. Man'ko, and R. V. Mendes, "A probability operator symbol framework for quantum information," Los Alamos ArXiv quant-ph/0602189 v1; *J. Russ. Laser Res.*, **27**, 507–532 (2006).

Introduction

Our aim is to discuss some new entropic inequalities for quantum states of systems with continuous variables (position) and discrete variables (spin).

The inequalities can be obtained using probability representations of quantum states based on quantum tomography approach.

In the probability representation of quantum mechanics, the states are described by standard probability distributions (tomographic-probability distributions).

In view of this, all the results and notion from probability theory can be naturally associated with quantum states.

In the talk, we discuss the Shannon entropy and known entropic inequalities for the Shannon entropy. We adopt these inequalities and apply them for studying the quantum-system properties.

Entropy and Entropic Uncertainty Relations

Below we consider the notion of entropy for functions $\psi(x)$ (vectors $\mid\psi\rangle$).

In the information-theory context, entropy is related to an arbitrary probability-distribution function [C. E. Shannon, Bell. Tech. J., **27**, 379 (1948).]

For example, given the probability distribution $P(n)$, where n is a discrete random variable, i.e.,

$$P(n) \geq 0, \tag{1}$$

and the normalization condition holds

$$\sum_n P(n) = 1, \tag{2}$$

one has, by definition, the entropy

$$S = -\sum_n P(n) \ln P(n) = -\langle \ln P(n) \rangle. \tag{3}$$

In quantum mechanics, for continuous variables wave function $\psi(x)$ provides the probability-distribution density of position

$$P(x) = |\psi(x)|^2. \tag{4}$$

The corresponding entropy S_x reads (see, for example, V. V. Dodonov and V. I. Man'ko, *Invariants and the Evolution of Nonstationary Quantum Systems*, Proceedings of Lebedev Physical Institute, Nauka, Moscow, 1987, Vol. 183 [translated by Nova Science, N.Y., 1989])

$$S_x = -\int |\psi(x)|^2 \ln |\psi(x)|^2 \, dx. \tag{5}$$

In the momentum representation, the wave function reads

$$\widetilde{\psi}(p) = \frac{1}{\sqrt{2\pi}} \int \psi(x) e^{-ipx}\, dx \qquad (\hbar = 1). \tag{6}$$

The corresponding entropy S_p related to the momentum-probability density $|\widetilde{\psi}(p)|^2$ reads

$$S_p = -\int |\widetilde{\psi}(p)|^2 \ln |\widetilde{\psi}(p)|^2\, dp. \tag{7}$$

It is worthy noting that one can construct entropies S_x and S_p not only in quantum mechanics. If the function $\psi(x)$ is replaced by a signal function $f(t)$ depending on time t, the function $\widetilde{\psi}(p)$ is replaced by the function $\widetilde{f}(\omega)$ describing the signal spectrum.

In this case, the entropy of the signal

$$S_t = -\int |f(t)|^2 \ln |f(t)|^2\, dt \tag{8}$$

and the entropy of its frequency spectrum

$$S_\omega = -\int |\widetilde{f}(\omega)|^2 \ln |\widetilde{f}(\omega)|^2\, d\omega \tag{9}$$

provide some information characteristics of the signal. Analogous approach can be applied also for soliton solutions of nonlinear equations.

From mathematical point of view, there exists correlation of entropies S_x and S_p (S_t and S_ω), since function $\psi(x)$ [$f(t)$] determines Fourier component $\widetilde{\psi}(p)$ [$\widetilde{f}(\omega)$]. This means that entropies S_x and S_p have to obey some constrains.

These constrains are well-known entropic uncertainty relations (some inequalities). For one-mode system, the inequalities read (see review Lebedev Proceedings, Vol. 183)

$$S_x + S_p \geq \ln(\pi e), \qquad S_t + S_\omega \geq \ln(\pi e). \tag{10}$$

For Gaussian wave functions (Gaussian signals) describing the states without correlations of position and momentum, e.g., the ground state of the harmonic oscillator

$$\psi(x) = \pi^{-1/4} e^{-x^2/2}, \qquad \widetilde{\psi}(p) = \pi^{-1/4} e^{-p^2/2}, \tag{11}$$

one has $S_x^{(0)} = S_p^{(0)} = 2^{-1} \ln(\pi e)$.

Consequently, $S_x^{(0)} + S_p^{(0)} = \ln(\pi e)$.

For squeezed and correlated states, wave functions have the Gaussian form, i.e., $\psi(x) = \mathcal{N} \exp(-ax^2 + bx)$, $a = a_1 + ia_2$, $a_1 > 0$, $b = b_1 + ib_2$, the normalization constant $\mathcal{N} = (2a_1)^{-1/4} \pi^{-1/4} e^{-b_1^2/4a_1}$, and the product of the position and momentum uncertainties reads

$$\sigma_x \sigma_p = [4(1 - R^2)]^{-1}. \tag{12}$$

Here R is the correlation coefficient of the position and momentum, i.e.,

$$R = \frac{\frac{1}{2}\langle \hat{q}\hat{p} + \hat{p}\hat{q} \rangle - \langle \hat{q} \rangle \langle \hat{p} \rangle}{\sqrt{\sigma_x \sigma_p}}, \qquad |R| < 1, \tag{13}$$

and for squeezed but not correlated states $R = 0$.

The sum of entropies for squeezed and correlated states reads

$$S_x + S_p = \ln(\pi e) + \ln\frac{1}{\sqrt{1-R^2}} \geq \ln(\pi e). \qquad (14)$$

For squeezed states, the entropy S_x differs from S_p.

For multimode quantum systems (or multicomponent signals), the entropy uncertainty relation reads

$$S_{\vec{x}} + S_{\vec{p}} \geq N\ln(\pi e), \qquad (15)$$

where N is the number of degrees of freedom of the system and

$$\begin{aligned} S_{\vec{x}} &= -\int |\psi(\vec{x})|^2 \ln|\psi(\vec{x})|^2\, d\vec{x}, \\ S_{\vec{p}} &= -\int |\widetilde{\psi}(\vec{p})|^2 \ln|\widetilde{\psi}(\vec{p})|^2\, d\vec{p}. \end{aligned} \qquad (16)$$

The functions $\psi(\vec{x})$ and $\widetilde{\psi}(\vec{p})$ are connected by the Fourier transform

$$\widetilde{\psi}(\vec{p}) = (2\pi)^{-N/2}\int \psi(\vec{x})e^{-i\vec{p}\vec{x}}\, d\vec{x}. \qquad (17)$$

For the Gaussian wave function corresponding to factorized squeezed state of several modes,

$$S_{\vec{x}} + S_{\vec{p}} = N\ln(\pi e). \qquad (18)$$

Symplectic Tomography

There exists invertable map, which is tomographic map of function ψ. In terms of the density operator (matrix) $\hat{\rho}$, the symplectic tomogram, which is the probability-density of random position X, can be presented in the form

$$w(X,\mu,\nu) = \mathrm{Tr}\,\hat{\rho}\ \delta(X-\mu\hat{q}-\nu\hat{p}), \tag{19}$$

where $\hat{\rho}$ is the density operator, μ and ν are real parameters, $\hat{q}$ and $\hat{p}$ are the position and momentum operators.

For pure state $\hat{\rho}_\psi = \mid \psi\rangle\langle\psi \mid$, transform (19) yields

$$w(X,\mu,\nu) = \frac{1}{2\pi|\nu|}\left|\int \psi(y)\exp\left(\frac{i\mu}{2\nu}y^2 - \frac{iX}{\nu}y\right)dy\right|^2. \tag{20}$$

The function $w(X,\mu,\nu)$ (the state tomogram) is the position-probability density of the position X, i.e.,
$w(X,\mu,\nu) \geq 0$ and $w(X,\mu,\nu)\,dX = 1$.

The tomogram has homogeneity property which reads

$$w(\lambda X,\lambda\mu,\lambda\nu) = |\lambda|^{-1}w(X,\mu,\nu). \tag{21}$$

Also for the pure state, one has

$$w(X,1,0) = |\psi(X)|^2, \qquad w(X,0,1) = |\widetilde{\psi}(X)|^2, \tag{22}$$

where $\psi(X)$ $[\widetilde{\psi}(X)]$ is the wave function in the position (momentum) representation.

Tomogram (20) can be rewritten in the form

$$w(X,\mu,\nu) = \frac{1}{2\pi|\nu|}\left|\int \psi(y)\exp\left[\frac{i}{2}\left(\frac{\mu}{\nu}y^2 - \frac{2X}{\nu}y + \frac{\mu}{\nu}X^2\right)\right]dy\right|^2. \tag{23}$$

For $\mu = \cos t$ and $\nu = \sin t$, one has the optical tomogram

$$w(X,t) = \left|\int \psi(y)\exp\left[\frac{i}{2}\left(\cot t\ (y^2+X^2) - \frac{2X}{\sin t}\,y\right)\right]\frac{dy}{\sqrt{2\pi i\sin t}}\right|^2. \tag{24}$$

The optical tomogram of the quantum state given in terms of Wigner function was introduced in [J. Bertrand and P. Bertrand, Found. Phys., **17**, 397 (1987); K. Vogel and H. Risken, Phys. Rev. A, **40**, 2847 (1989)].

If we denote $\psi(X,t) =$

$$= \frac{1}{\sqrt{2\pi i\sin t}}\int \exp\left[\frac{i}{2}\left(\cot t\ (y^2+X^2) - \frac{2X}{\sin t}\,y\right)\right]\psi(y)\,dy, \tag{25}$$

then (24) means that $w(X,t) = |\psi(X,t)|^2$.

Comparing (25) with the known relation for the harmonic-oscillator wave function at time moment t

$$\psi(X,t) = \sqrt{\frac{m\omega}{2\pi i\hbar\sin\omega t}}\int dy\,\psi(y,0)\exp\left\{\frac{im\omega}{2\hbar}\left[(X^2+y^2)\cot\omega t - \frac{2Xy}{\sin\omega t}\right]\right\},$$

one can see that the wave function $\psi(X,t)$ corresponds to the wave function of a harmonic oscillator with $\hbar = m = \omega = 1$ taken at the time moment t provided the wave function $\psi(y,0)$ at the initial time equals to $\psi(y)$. In fact, we see the identity of (25) to the last formula for the wave function of harmonic oscillator with $\hbar = m = \omega = 1$ at the time moment t expressed in terms of the Green function of the Schrödinger evolution equation and the initial wave function of harmonic oscillator.

In optical tomography, one denotes

$$\mu = \cos\theta, \qquad \nu = \sin\theta.$$

Thus the parameter of time t for oscillator with unit frequency $\omega = 1$, i.e., $\omega t = t = \theta$, is equal to the parameter of angle θ.

Below we denote in optical tomograms the angle as t.

Thus, we pointed out that symplectic tomogram of a pure quantum state can be interpreted as modulus squared of the harmonic-oscillator's wave function for the pure state. This observation provides the possibility to apply entropic uncertainty relations known for wave functions and reviewed in previous sections to symplectic tomograms.

Tomographic Entropies

Since the symplectic tomogram is the standard probability distribution, one can introduce entropy associated with tomogram of quantum state [O. V. Man'ko and V. I. Man'ko, J. Russ. Laser Res., **18**, 407 (1997)] or with tomogram of analytic signal [M. A. Man'ko, J. Russ. Laser Res., **22**, 168 (2001)]. Thus one has entropy as the function of two real variables

$$S(\mu,\nu) = -\int w(X,\mu,\nu)\,\ln w(X,\mu,\nu)\,dX. \qquad (26)$$

We call this entropy symplectic entropy.

In view of the homogeneity and normalization conditions for tomogram, one has the additivity property

$$S(\lambda\mu,\lambda\nu) = S(\mu,\nu) + \ln|\lambda|. \qquad (27)$$

For pure state $\mid\psi\rangle$, in view of $w(X,1,0) = |\psi(X)|^2$ and $w(X,0,1) = |\widetilde{\psi}(X)|^2$, one obtains the entropies S_x and S_p putting in (26) $\mu = 1$ and $\nu = 0$

$$S(1,0) = S_x, \qquad (28)$$

or $\mu = 0$ and $\nu = 1$

$$S(0,1) = S_p. \qquad (29)$$

In view of inequality (10) $[S_x + S_p \geq \ln(\pi e)]$, one has the inequality for tomographic entropies

$$S(1,0) + S(0,1) \geq \ln(\pi e). \qquad (30)$$

For multimode system, the symplectic entropy reads

$$S(\vec{\mu},\vec{\nu}) = -\int w(\vec{X},\vec{\mu},\vec{\nu}) \, \ln w(\vec{X},\vec{\mu},\vec{\nu}) \, d\vec{X}. \qquad (31)$$

Since the symplectic entropy is related to entropies $S_{\vec{x}}$ and $S_{\vec{p}}$ of multimode-system state, one can use inequality $S_{\vec{x}} + S_{\vec{p}} \geq N \ln(\pi e)$ to obtain the entropic uncertainty relation in the form of inequality for symplectic entropies

$$S(\vec{1},\vec{0}) + S(\vec{0},\vec{1}) \geq N \ln(\pi e), \qquad (32)$$

where $\vec{\mu} = \vec{1} = (1,1,\ldots,1)$ and $\vec{\nu} = \vec{1} = (1,1,\ldots,1)$.

For the optical tomogram $w(X,t) =$

$$= \left| \int \psi(y) \exp\left[\frac{i}{2}\left(\cot t \, \left(y^2 + X^2\right) - \frac{2X}{\sin t} \, y\right)\right] \frac{dy}{\sqrt{2\pi i \sin t}} \right|^2,$$

entropy is defined by the formula

$$S(t) = -\int w(X,t) \, \ln w(X,t) \, dX. \qquad (33)$$

For pure state, $S(0) = S_x$ and $S(\pi/2) = S_p$.
In view of the expression of tomogram in terms of wave function $w(X,t) = |\psi(X,t)|^2$ and

$$\psi(X,t) = \frac{1}{\sqrt{2\pi i \sin t}} \int \exp\left[\frac{i}{2}\left(\cot t \, \left(y^2 + X^2\right) - \frac{2X}{\sin t} \, y\right)\right] \psi(y) \, dy,$$

one has the entropic uncertainty relation in the form

$$S(t) + S(t + \pi/2) \geq \ln \pi e. \qquad (34)$$

Thus we extended the entropic uncertainty relation to arbitrary value of parameter t using the interpretation of symplectic tomogram as modulus squared of the wave function of an "artificial harmonic oscillator."

Since symplectic and optical tomograms are connected

$$w(X, \mu = \cos t, \nu = \sin t) = w(X, t), \qquad (35)$$

the corresponding entropies are also connected

$$S(t) = S(\mu = \cos t, \nu = \sin t). \qquad (36)$$

For given symplectic entropy of any pure state $S(\mu, \nu)$, inequality $S(t) + S(t + \pi/2) \geq \ln \pi e$ reads

$$S(\cos t, \sin t) + S(-\sin t, \cos t) \geq \ln \pi e. \qquad (37)$$

In view of the homogeneity property

$$w(\lambda X, \lambda\mu, \lambda\nu) = |\lambda|^{-1} w(X, \mu, \nu),$$

optical $w(X, t)$ and symplectic $w(X, \mu, \nu)$ tomograms are also related as follows:

$$w(X, \mu, \nu) = \frac{1}{\sqrt{\mu^2 + \nu^2}}\, w\left(\frac{X}{\sqrt{\mu^2 + \nu^2}}\,, t\right). \qquad (38)$$

This means that for given optical tomogram $w(x, t)$ one can reconstruct symplectic tomogram $w(X, \mu, \nu)$.

Inserting

$$w(X,\mu,\nu)=\frac{1}{\sqrt{\mu^2+\nu^2}}\,w\left(\frac{X}{\sqrt{\mu^2+\nu^2}}\,,t\right)$$

into the basic equation defining the entropy

$$S(\mu,\nu)=-\int w(X,\mu,\nu)\ln w(X,\mu,\nu)dX,$$

in view of the additivity property of the tomographic entropy

$$S(\lambda\mu,\lambda\nu)=S(\mu,\nu)+\ln|\lambda|,$$

we obtain

$$S(t)=S(\sqrt{\mu^2+\nu^2}\cos t,\sqrt{\mu^2+\nu^2}\sin t)-\frac{1}{2}\,\ln(\mu^2+\nu^2). \tag{39}$$

And for symplectic entropy, the entropic uncertainty relation yields

$$\begin{aligned}&S\left(\sqrt{\mu^2+\nu^2}\cos t,\sqrt{\mu^2+\nu^2}\sin t\right)\\&+S\left(-\sqrt{\mu^2+\nu^2}\sin t,\sqrt{\mu^2+\nu^2}\cos t\right)-\ln(\mu^2+\nu^2)\geq\ln\pi e.\end{aligned} \tag{40}$$

This inequality is the main result which we want to get.

One can extend this inequality for multimode system

$$S\left(\sqrt{\mu_1^2+\nu_1^2}\cos t_1,\sqrt{\mu_2^2+\nu_2^2}\cos t_2,\ldots,\sqrt{\mu_N^2+\nu_N^2}\cos t_N,\right.$$
$$\left.\sqrt{\mu_1^2+\nu_1^2}\sin t_1,\sqrt{\mu_2^2+\nu_2^2}\sin t_2,\ldots,\sqrt{\mu_N^2+\nu_N^2}\sin t_N\right)$$
$$+S\left(-\sqrt{\mu_1^2+\nu_1^2}\sin t_1,-\sqrt{\mu_2^2+\nu_2^2}\sin t_2,\ldots,-\sqrt{\mu_N^2+\nu_N^2}\sin t_N,\right.$$
$$\left.\sqrt{\mu_1^2+\nu_1^2}\cos t_1,\sqrt{\mu_2^2+\nu_2^2}\cos t_2,\ldots,\sqrt{\mu_N^2+\nu_N^2}\cos t_N\right)$$
$$-\sum_{k=1}^{N}\ln\left(\mu_k^2+\nu_k^2\right)\geq N\ln(\pi e), \tag{41}$$

where entropy $S(\vec{\mu},\vec{\nu})$ is given by

$$S(\vec{\mu},\vec{\nu})=-\int w(\vec{X},\vec{\mu},\vec{\nu})\,\ln w(\vec{X},\vec{\mu},\vec{\nu})\,d\vec{X}.$$

Spin Entropic Inequalities

For any given quantum state with density operator ρ, the entropic characteristics is von Neuman entropy

$$S = -\mathrm{Tr}\,\hat{\rho}\ln\hat{\rho}.$$

For composite systems, there exist known inequalities for the von Neuman entropy.

For two-partite system with density operator $\hat{\rho}(1,2)$ depending on variables of the first subsystem 1 and the second subsystem 2, there exists the inequality called the subadditivity

$$S_{12} \leq S_1 + S_2,$$

where

$$\begin{aligned} S_{12} &= -\mathrm{Tr}\,\hat{\rho}(1,2)\ln\hat{\rho}(1,2), \\ S_1 &= -\mathrm{Tr}\,\hat{\rho}(1)\ln\hat{\rho}(1), \\ S_2 &= -\mathrm{Tr}\,\hat{\rho}(2)\ln\hat{\rho}(2). \end{aligned}$$

The operators $\hat{\rho}(1)$ and $\hat{\rho}(2)$ are density operators of two subsystems, respectively.

Now we present new tomographic entropic inequalities.

Consider a two-partite system with density matrix ρ_{12} and tomographic-probability distribution for two spin projections m_1 and m_2

$$w(m_1, m_2, u) = \langle m_1 m_2 \mid u^\dagger \rho_{12} u \mid m_1 m_2 \rangle,$$

u being a $(2j_1+1)(2j_2+1) \times (2j_1+1)(2j_2+1)$ unitary matrix, and Shannon entropy is defined by tomographic probability distributions

$$H_u\,(12) = -\sum_{m_1 m_2} w(m_1, m_2, u) \ln w(m_1, m_2, u).$$

From the tomographic probability for the first and second spin projections

$$w(m_1, u) = \sum_{m_2} w(m_1, m_2, u),$$

$$w(m_2, u) = \sum_{m_1} w(m_1, m_2, u),$$

one writes the tomographic entropies

$$H_u(1) = -\sum_{m_1} w(m_1, u) \ln\, w(m_1, u),$$

$$H_u(2) = -\sum_{m_2} w(m_2, u) \ln\, w(m_2, u).$$

For each fixed u, the tomographic probabilities are ordinary probability distributions.

In classical probability theory, Shannon entropies associated with joint probability distributions of bipartite and tripartite systems satisfy known inequalities called subadditivity conditions and strong subadditivity conditions. We can adopt these inequalities and write them for quantum spin systems.

Therefore, by the subadditivity of classical entropy [see M. A. Man'ko, V. I. Man'ko, and R. V. Mendes, *J. Russ. Laser Res.*, **27**, 507–532 (2006); quant-ph/0602189 v1]

$$H_u(12) \leq H_u(1) + H_u(2).$$

This new inequality is a tomographic counterpart of the subadditivity condition for von Neuman entropy of bitartite quantum system.

Strong subadditivity existing in classical probability theory also holds for any u for tripartite system

$$H_u(123) + H_u(2) \leq H_u(12) + H_u(23). \tag{42}$$

This is a tomographic counterpart of the known strong subadditivity which exists for the von Neumann entropy $S = -\mathrm{Tr}\,\hat{\rho}\ln\hat{\rho}$ of tripartite system, i.e.,

$$S\,(123) + S\,(2) \leq S\,(12) + S\,(23)\,. \tag{43}$$

Therefore, strong subadditivity for the von Neumann entropy (43) and the new strong subadditivity for the tomographic entropies (42) are the properties of all quantum tripartite states.

Conclusions

To conclude, we point out our main results.

Inequalities for optical tomographic entropy

$$S(t) + S(t + \pi/2) \geq \ln(\pi e)$$

and symplectic tomographic entropy

$$S\left(\sqrt{\mu^2+\nu^2}\cos t, \sqrt{\mu^2+\nu^2}\sin t\right)$$
$$+S\left(-\sqrt{\mu^2+\nu^2}\sin t, \sqrt{\mu^2+\nu^2}\cos t\right) - \ln(\mu^2+\nu^2) \geq \ln(\pi e),$$

being the generalizations of known in quantum mechanics entropic inequalities for probability distributions of conjugate position and momentum, are obtained for entropies associated with symplectic tomograms.

The new uncertainty relations obtained characterize the behavior of quantum state in quantum mechanics as well as the behavior of analytic signal in signal analysis or behavior of solitons in the theory of nonlinear equations.

The entropic uncertainty relation for symplectic tomographic entropy is obtained also for functions of several variables. The uncertainty relation is given by formula

$$S\left(\sqrt{\mu_1^2+\nu_1^2}\cos t_1, \sqrt{\mu_2^2+\nu_2^2}\cos t_2, \ldots, \sqrt{\mu_N^2+\nu_N^2}\cos t_N,\right.$$
$$\left.\sqrt{\mu_1^2+\nu_1^2}\sin t_1, \sqrt{\mu_2^2+\nu_2^2}\sin t_2, \ldots, \sqrt{\mu_N^2+\nu_N^2}\sin t_N\right)$$
$$+S\left(-\sqrt{\mu_1^2+\nu_1^2}\sin t_1, -\sqrt{\mu_2^2+\nu_2^2}\sin t_2, \ldots, -\sqrt{\mu_N^2+\nu_N^2}\sin t_N,\right.$$
$$\left.\sqrt{\mu_1^2+\nu_1^2}\cos t_1, \sqrt{\mu_2^2+\nu_2^2}\cos t_2, \ldots, \sqrt{\mu_N^2+\nu_N^2}\cos t_N\right)$$
$$-\sum_{k=1}^{N}\ln\left(\mu_k^2+\nu_k^2\right) \geq N\ln(\pi e).$$

The entropy under study, as any Shannon entropy, provides the informational characteristics of the signal or soliton.

Also the spin tomogram entropic inequalities are obtained which correspond to standard classical subadditivity condition and strong subadditivity condition.

The uncertainty relations for tomographic entropies are new additional properties of the quantum state, analytic signal, and nonlinear systems. The physical meaning of tomographic entropic uncertainty relations has to be understood better.