



Quantum manipulation of information by hyper-entangled two photon states

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- Entangled photon pairs generated by Spontaneous Parametric Down Conversion (SPDC). Qubits generally entangled in a single degree of freedom (limited to a 2×2 Hilbert space)

Increase the information content by enlarging the Hilbert space

Possible approaches:

- 1) Multiphoton entanglement (4, 5 photons, *)
- 2) d-level ($d > 2$) quantum systems (*qu-dits*)
- 3) **Two photon hyper-entanglement**

* Hefei group: 6 photons, quant-ph/0609130

Outline

- Polarization-momentum hyperentangled photon states
- Nonlocality tests, AVN test of quantum mechanics
- Growing with size quantum nonlocality
- Bell state analysis for quantum communication protocols
- Two photon linear cluster states
- Perspectives

Two photon hyperentanglement

$$|\Xi_{ab}\rangle = |Bell_1\rangle \otimes |Bell_2\rangle \dots \otimes |Bell_N\rangle$$

N different degrees of freedom, with $|Bell_i\rangle$ corresponding to one of the Bell states:

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}} [|00\rangle \pm |11\rangle]$$

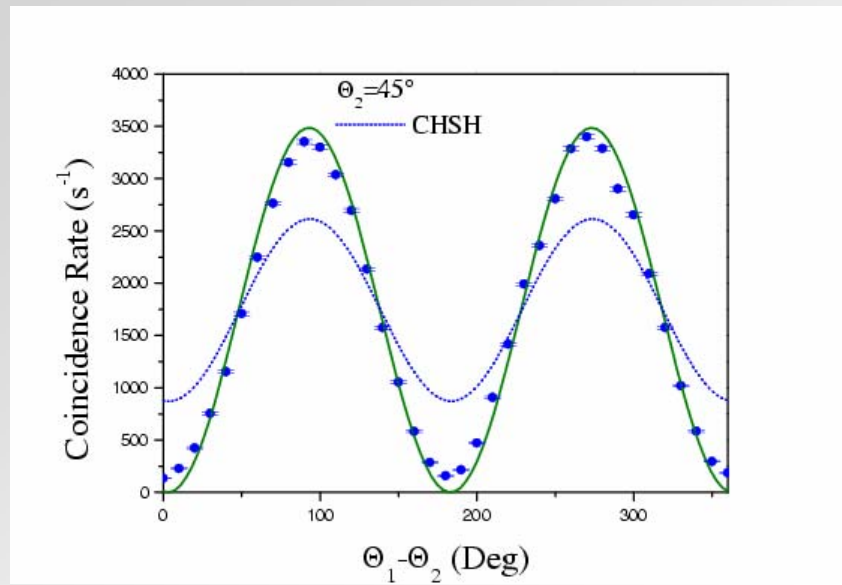
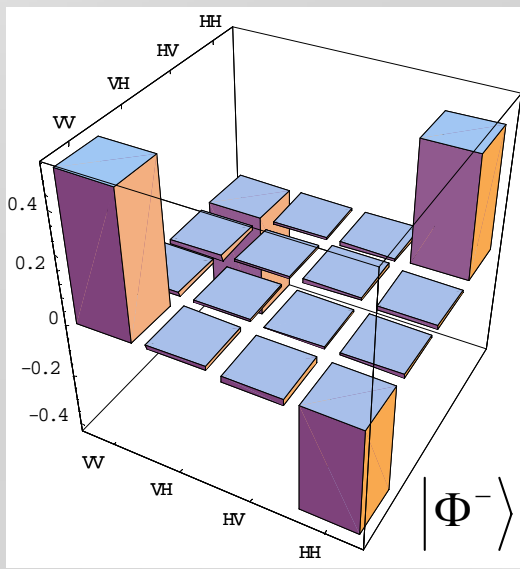
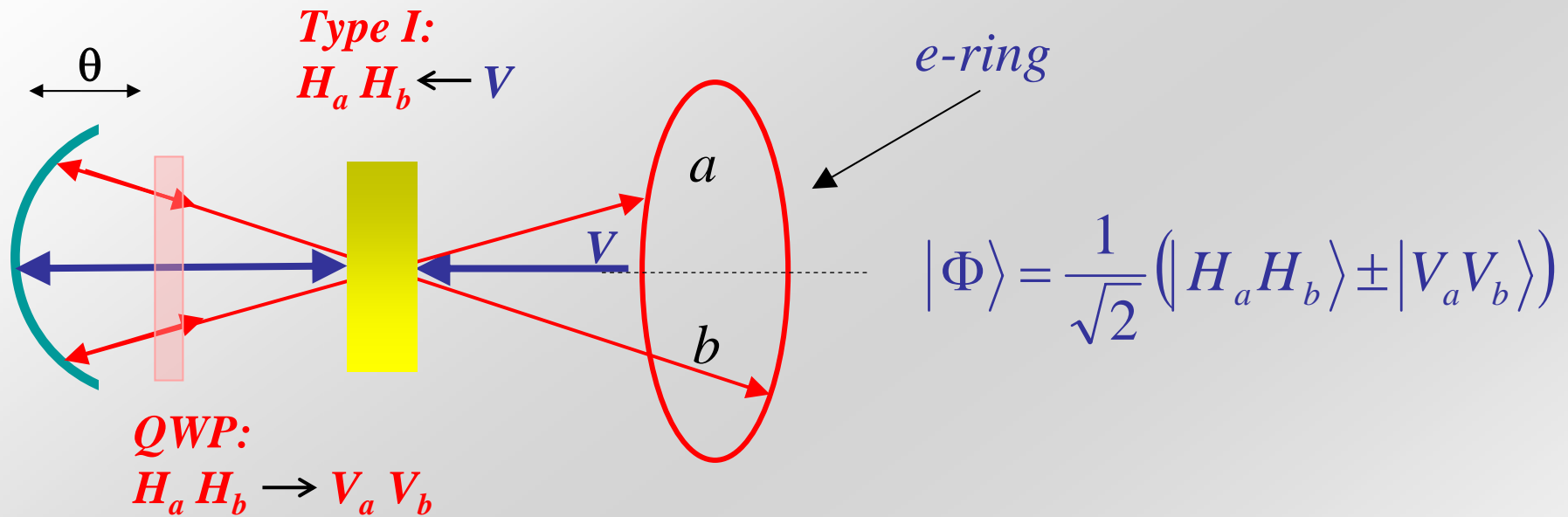
$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} [|01\rangle \pm |10\rangle]$$

Motivations:

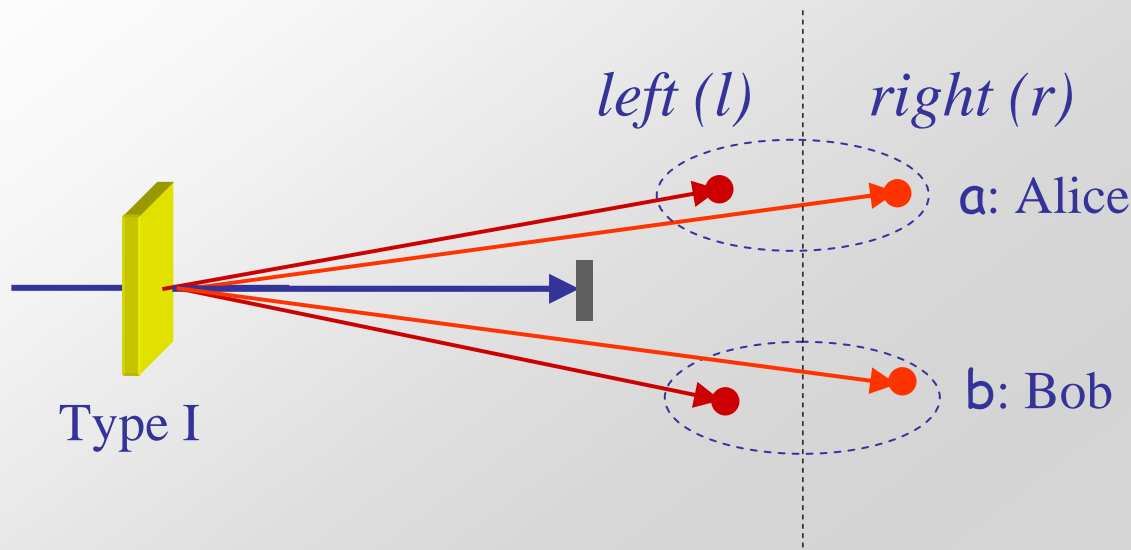
By expanding the Hilbert space to $2^N \times 2^N$ dimensions, nonlocality tests which are more resistant to noise and present more significant deviations from classical bounds can be performed

$N = 2$: hyperentanglement allows to codify two qubits on the same physical carrier, allows complete Bell State Analysis \rightarrow no need for multiparticle states

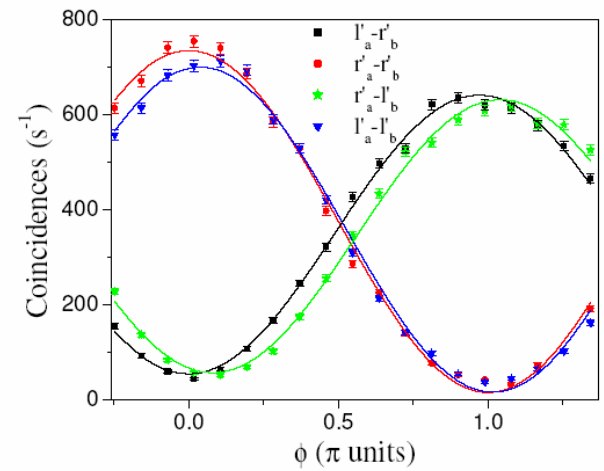
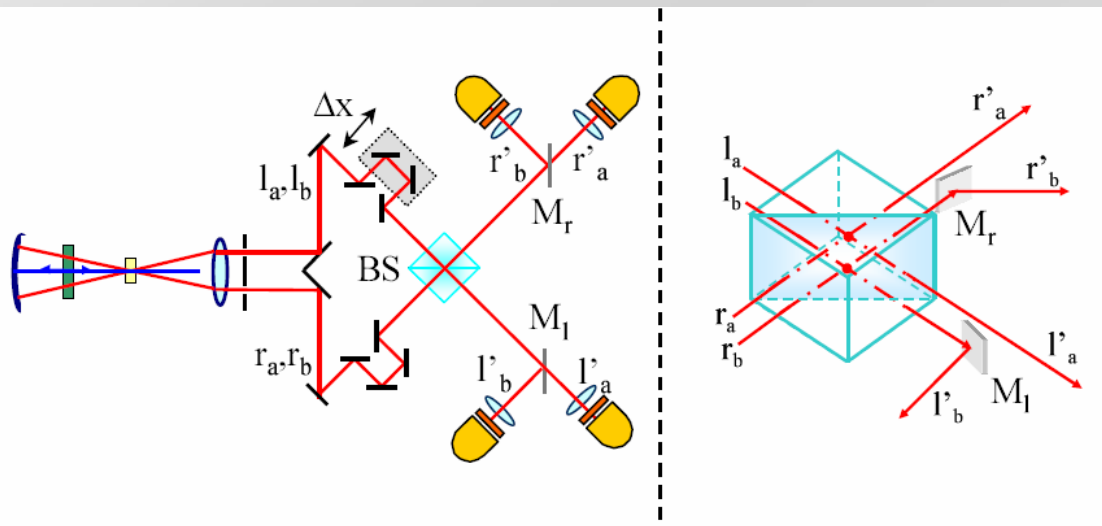
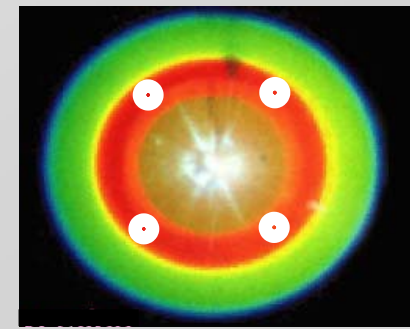
1) Polarization qubits



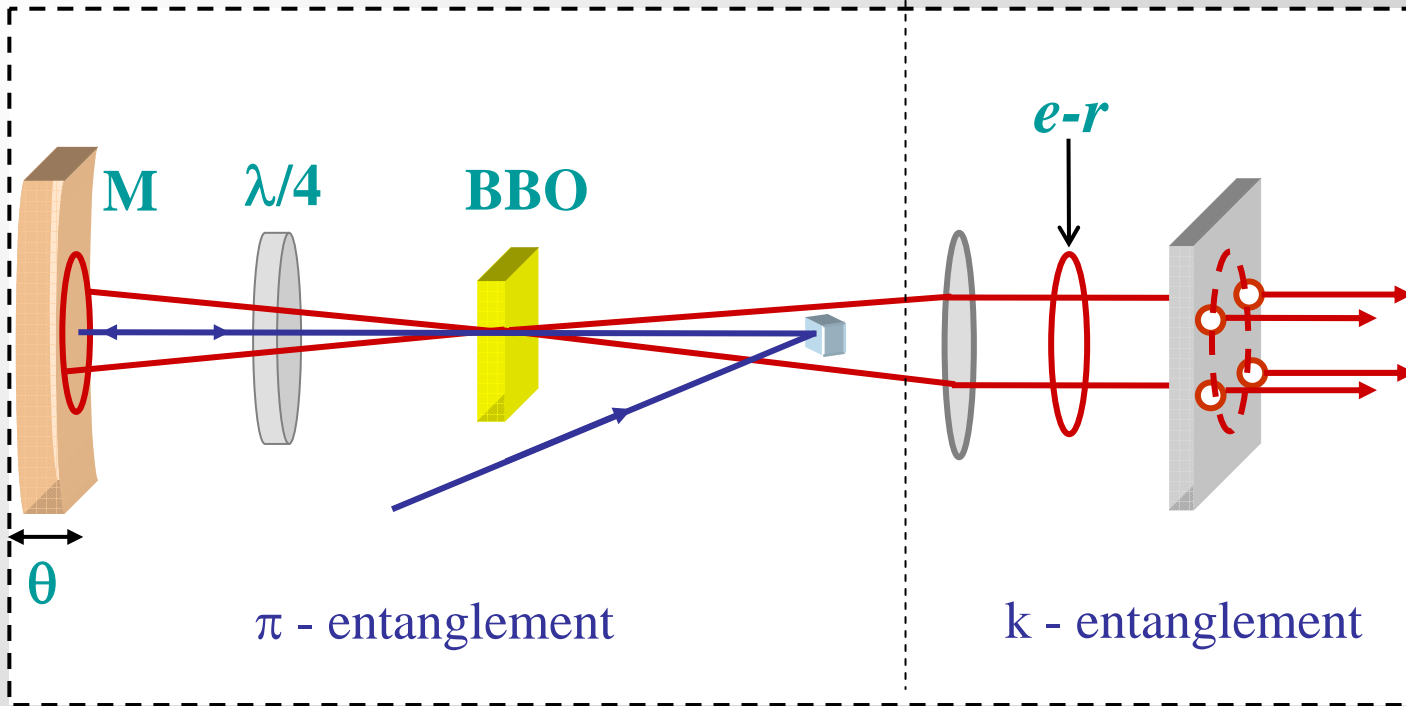
2) linear momentum qubits



$$|\psi\rangle = \frac{1}{\sqrt{2}} (|l_a r_b\rangle \pm |r_a l_b\rangle)$$



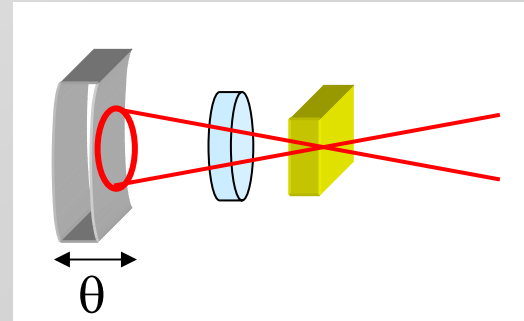
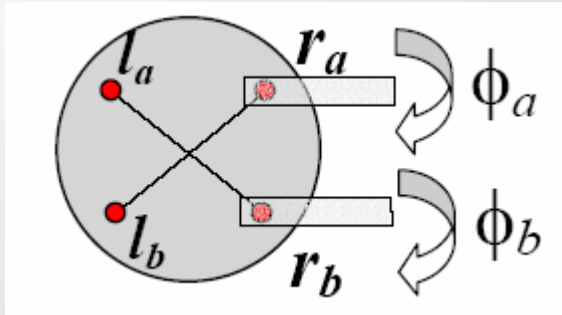
3) hyperentangled photons



$$|\Pi\rangle = \frac{1}{\sqrt{2}} \left[|H_a H_b\rangle + e^{i\theta} |V_a V_b\rangle \right] \otimes \frac{1}{\sqrt{2}} \left[|l_a r_b\rangle + e^{i\phi} |r_a l_b\rangle \right]$$

2 photons \rightarrow 4 qubits

Manipulate hyperentangled states



$$\phi = 0 \rightarrow \pi$$

$$\theta = 0 \rightarrow \pi$$

Half wave plate) on modes l_a, l_b :

$$|\Pi^{\pm\pm}\rangle = |\Phi^\pm\rangle \otimes |\psi^\pm\rangle = \frac{1}{\sqrt{2}} [|H_a H_b\rangle \pm |V_a V_b\rangle] \otimes \frac{1}{\sqrt{2}} [|l_a r_b\rangle \pm |r_a l_b\rangle]$$

$$|\Xi^{\pm\pm}\rangle = |\Psi^\pm\rangle \otimes |\psi^\pm\rangle = \frac{1}{\sqrt{2}} [|H_a V_b\rangle \pm |V_a H_b\rangle] \otimes \frac{1}{\sqrt{2}} [|l_a r_b\rangle \pm |r_a l_b\rangle]$$

$$\begin{aligned} |\Pi^{++}\rangle &\Rightarrow |\Xi^{++}\rangle \\ |\Pi^{--}\rangle &\Rightarrow |\Xi^{--}\rangle \\ |\Pi^{+-}\rangle &\Rightarrow |\Xi^{+-}\rangle \\ |\Pi^{-+}\rangle &\Rightarrow |\Xi^{-+}\rangle \end{aligned}$$

Nonlocality tests with hyperentangled states

Separate Bell-CHSH inequality tests:

$$|S_\pi| = |P(\theta_1, \theta_2) - P(\theta_1, \theta'_2) + P(\theta'_1, \theta_2) + P(\theta'_1, \theta'_2)|$$

$$P(\theta_1, \theta_2) = \frac{[C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) - C(\theta_1, \theta_2^\perp) - C(\theta_1^\perp, \theta_2)]}{[C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) + C(\theta_1, \theta_2^\perp) + C(\theta_1^\perp, \theta_2)]}$$

$$\theta_1 = 0, \theta'_1 = \pi/4,$$

$$\theta_2 = \pi/8, \theta'_2 = 3\pi/8$$

$$|S_k| = |E(\phi_a, \phi_b) - E(\phi_a, \phi^*_b) + E(\phi^*_a, \phi_b) + E(\phi^*_a, \phi^*_b)|$$

$$E(\phi_a, \phi_b) = \frac{[C(a'_1, b'_1) + C(a'_2, b'_2) - C(a'_1, b'_2) - C(a'_2, b'_1)]}{[C(a'_1, b'_1) + C(a'_2, b'_2) + C(a'_1, b'_2) + C(a'_2, b'_1)]}$$

$$\phi_a = 0, \phi^*_a = \pi/2,$$

$$\phi_b = \pi/4, \phi^*_b = 3\pi/4$$



Polarization: 130- σ violation

Linear momentum: 102- σ violation

“All-versus nothing” (AVN) proof of Quantum Mechanics

Different approach: one observes that assigning to certain Pauli observables a value which pre-exists to the measurement leads to logical inconsistency

Generalization of the GHZ argument to the case of two observers. Test performed over *all* the quantum particles involved in the experiment

Polarization observables

$$Z_i \equiv \sigma_{Z_i} = |H_i\rangle\langle H_i| - |V_i\rangle\langle V_i|$$

$$X_i \equiv \sigma_{X_i} = |H_i\rangle\langle V_i| + |V_i\rangle\langle H_i|$$

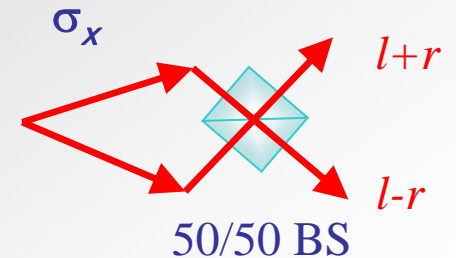
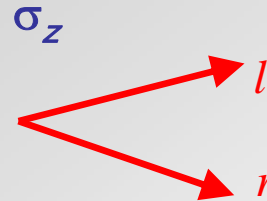
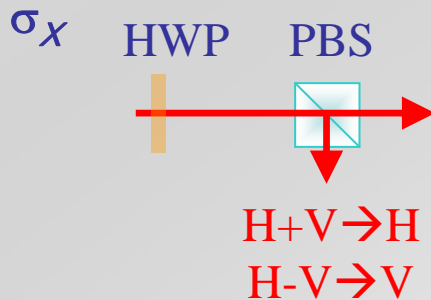
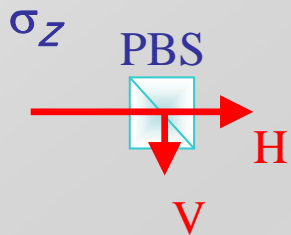
Momentum observables

$$z_i \equiv \sigma_{z_i} = |l_i\rangle\langle l_i| - |r_i\rangle\langle r_i|$$

$$x_i \equiv \sigma_{x_i} = |l_i\rangle\langle r_i| + |r_i\rangle\langle l_i|$$

$$i = a, b$$

easily performed by linear optics



Double singlet state:
$$|\Xi^{--}\rangle = \frac{1}{\sqrt{2}} \left[|H_a V_b\rangle - |V_a H_b\rangle \right] \otimes \frac{1}{\sqrt{2}} \left[|l_a r_b\rangle - |r_a l_b\rangle \right]$$

Quantum Mechanics:

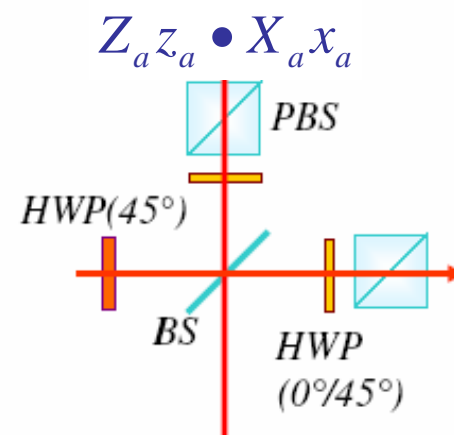
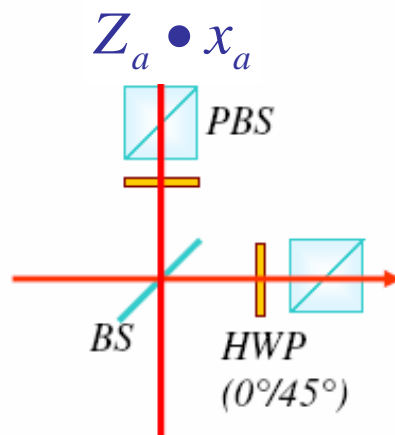
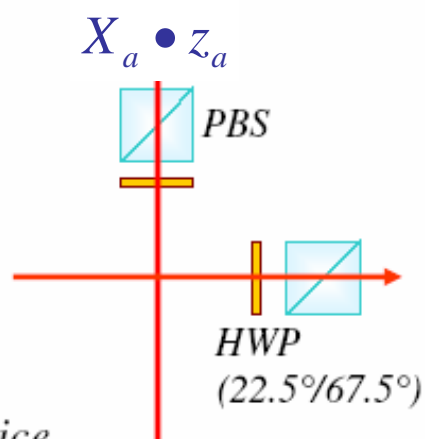
assigning to some observables a pre-definite value v :

$$\begin{aligned} Z_a \bullet Z_b |\Xi^{--}\rangle &= -|\Xi^{--}\rangle & X_a \bullet X_b |\Xi^{--}\rangle &= -|\Xi^{--}\rangle \\ z_a \bullet z_b |\Xi^{--}\rangle &= -|\Xi^{--}\rangle & x_a \bullet x_b |\Xi^{--}\rangle &= -|\Xi^{--}\rangle \\ Z_a z_a \bullet Z_b z_b |\Xi^{--}\rangle &= |\Xi^{--}\rangle \\ X_a x_a \bullet X_b x_b |\Xi^{--}\rangle &= |\Xi^{--}\rangle \\ X_a \bullet z_a \bullet X_b z_b |\Xi^{--}\rangle &= |\Xi^{--}\rangle \\ Z_a \bullet x_a \bullet Z_b x_b |\Xi^{--}\rangle &= |\Xi^{--}\rangle \\ Z_a z_a \bullet X_a x_a \bullet X_b z_b \bullet Z_b x_b |\Xi^{--}\rangle &= -|\Xi^{--}\rangle \end{aligned}$$

$$\begin{aligned} v(Z_a)v(Z_b) &= -1 & v(X_a)v(X_b) &= -1 \\ v(z_a)v(z_b) &= -1 & v(x_a)v(x_b) &= -1 \\ v(Z_a z_a)v(Z_b z_b) &= 1 \\ v(X_a x_a)v(X_b x_b) &= 1 \\ v(X_a)v(z_a)v(X_b z_b) &= 1 \\ v(Z_a)v(x_a)v(Z_b x_b) &= 1 \\ v(Z_a z_a)v(X_a x_a)v(X_b z_b)v(Z_b x_b) &= -1 \end{aligned}$$

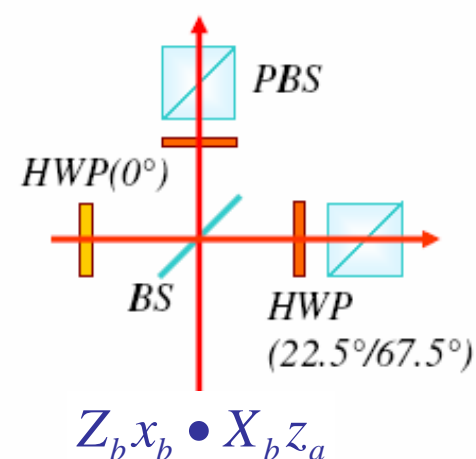
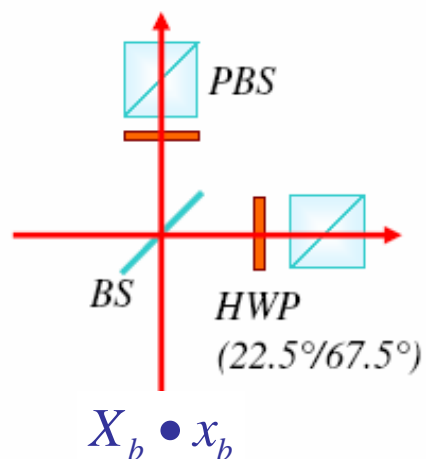
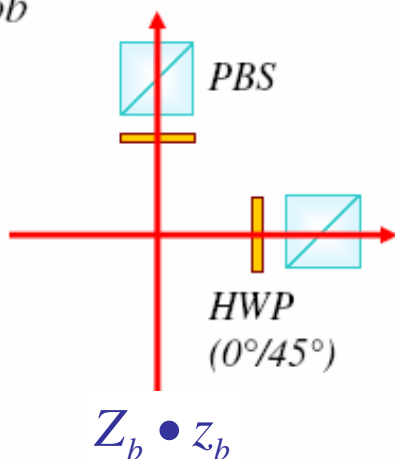
By multiplying all the equalities but the last, one gets the inconsistency:

$$v(Z_a z_a)v(X_a x_a)v(X_b z_b)v(Z_b x_b) = 1$$



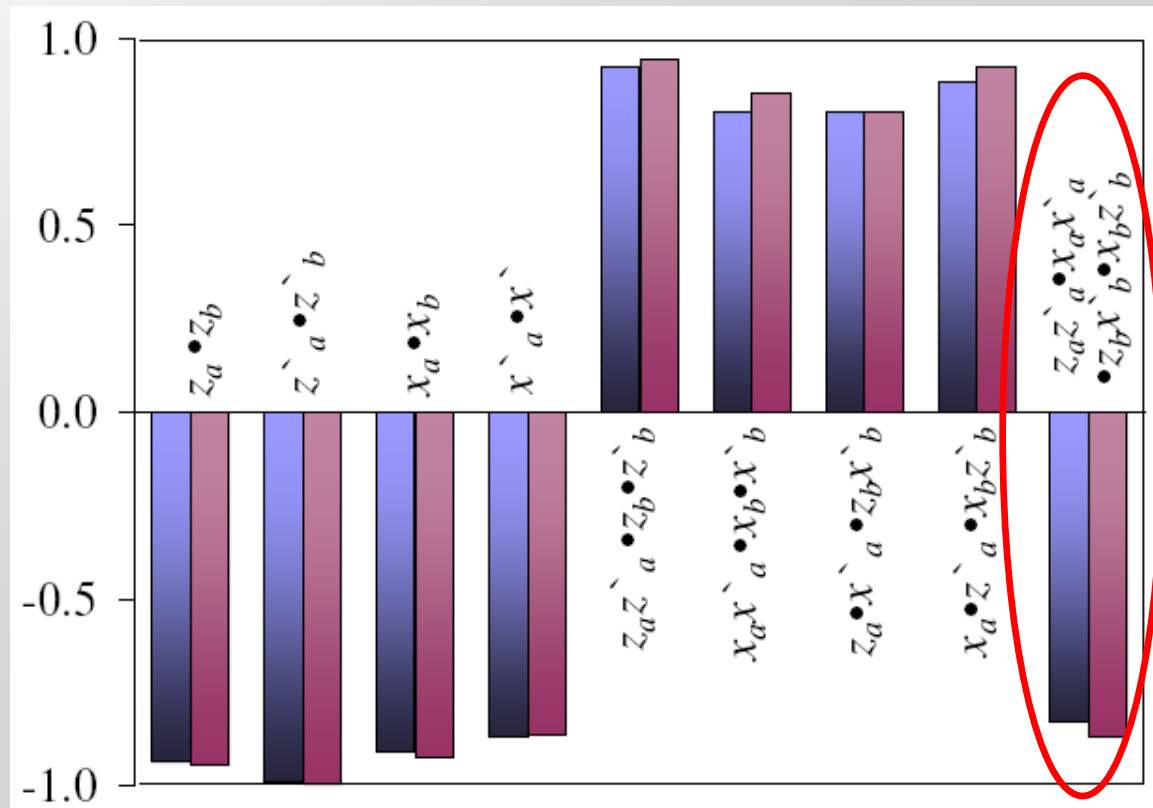
Alice

Bob



Check all possible combinations between Alice and Bob

Experimental results:



define the operator:

$$\hat{O} = -Z_a \bullet Z_b - X_a \bullet X_b - z_a \bullet z_b - x_a \bullet x_b + Z_a z_a \bullet Z_b \bullet z_b + X_a x_a \bullet X_b \bullet x_b + X_a \bullet z_a \bullet X_b z_b + Z_a \bullet x_a \bullet Z_b x_b - Z_a z_a \bullet X_a x_a \bullet X_b z_b \bullet Z_b x_b$$

Perfect correlations are impossible \rightarrow inequality is needed:

local realism bound: $\langle \hat{O} \rangle \leq 7$

QM bound: $\langle \hat{O} \rangle = 9$

experiment: $\langle \hat{O} \rangle = 8.114 \pm 0.01$

Inequality violated by 101- σ

Growing-with-size quantum nonlocality

- The magnitude of the violation of local realism can grow exponentially with the number n of particles (qubits) [Mermin: PRL. (1990)]
- Hyper-entanglement allows to replace n two-level systems by two N -level systems (# qubits = $2N$) [Aravind: Found. Phys. Lett. (2002), Cabello, PRL, (2006)]

$$|\Pi^{-+}\rangle = \frac{1}{\sqrt{2}} [|H_a V_b\rangle - |V_a H_b\rangle] \otimes \frac{1}{\sqrt{2}} [|l_a r_b\rangle + |r_a l_b\rangle]$$

- 1) Verify that this state violates two Bell's inequalities, for polarization (π) and momentum (\mathbf{k}), separately
- 2) Demonstrate that Bell's inequalities violation grows exponentially with the number N of degrees of freedom

Define CHSH the Bell operators β_π and β_k for *polarization* and *momentum* observables:

$$\beta_\pi = -A_\pi \otimes B_\pi + A_\pi \otimes b_\pi + a_\pi \otimes B_\pi + a_\pi \otimes b_\pi$$

$$A_\pi = |H\rangle\langle H| - |V\rangle\langle V|$$

$$a_\pi = |V\rangle\langle H| + |H\rangle\langle V|$$

$$B_\pi = \frac{1}{\sqrt{2}} [|H\rangle\langle H| - |V\rangle\langle V| + |V\rangle\langle H| + |H\rangle\langle V|]$$

$$b_\pi = \frac{1}{\sqrt{2}} [|V\rangle\langle V| - |H\rangle\langle H| + |V\rangle\langle H| + |H\rangle\langle V|]$$

$$\beta_k = A_k \otimes B_k - A_k \otimes b_k + a_k \otimes B_k + a_k \otimes b_k$$

$$A_k = |l\rangle\langle r| + |r\rangle\langle l|$$

$$a_k = i[|r\rangle\langle l| - |l\rangle\langle r|]$$

$$B_k = \frac{1}{\sqrt{2}} [(i+1)|r\rangle\langle l| - (i-1)|l\rangle\langle r|]$$

$$b_k = \frac{1}{\sqrt{2}} [(i-1)|r\rangle\langle l| - (i+1)|l\rangle\langle r|]$$

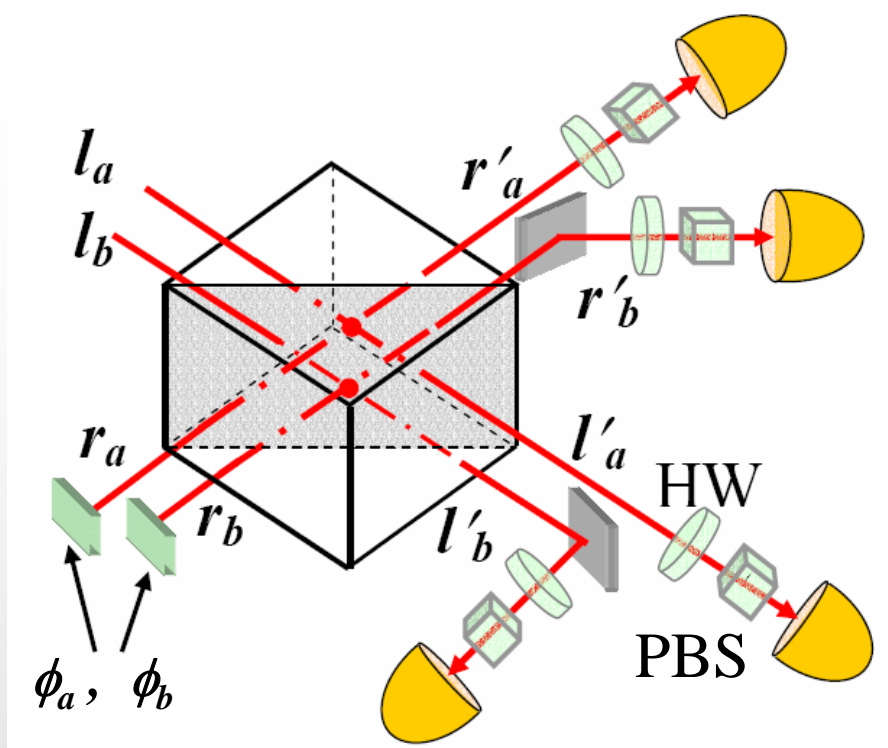
→ LR theory: $|\langle \beta_\pi \rangle| \leq 2$
 $|\langle \beta_k \rangle| \leq 2$

$$\beta = \beta_\pi \otimes \beta_k$$



LR theory: $\beta \leq 4$

QM: $\beta = 8$



Photon a : $A_\pi A_k, A_\pi a_k, a_\pi A_k, a_\pi a_k$

Photon b : $B_\pi B_k, B_\pi b_k, b_\pi B_k, b_\pi b_k$

(16 experimental configurations)

$$|\langle \beta_\pi \rangle| = 2.5762 \pm 0.0068 \quad (85\text{-}\sigma \text{ violation})$$

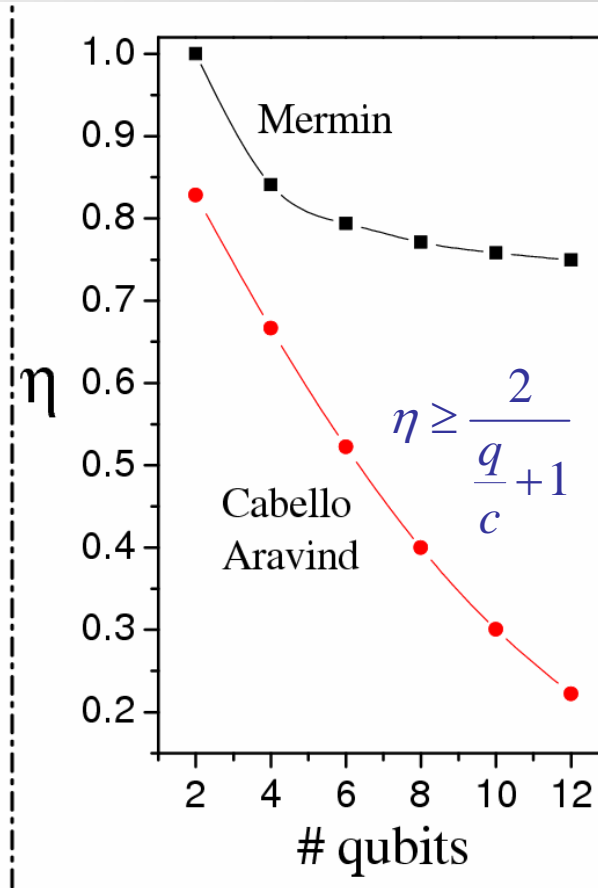
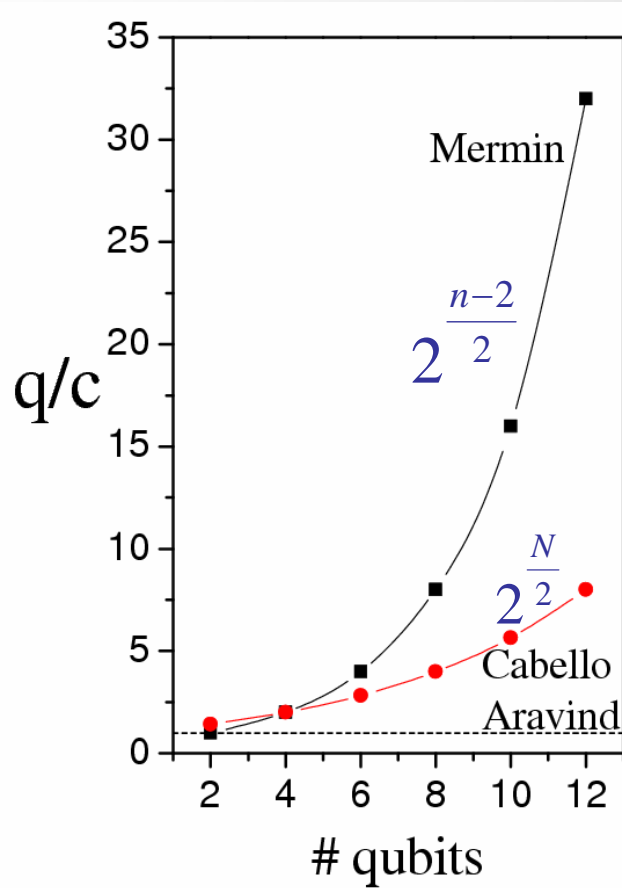
$$|\langle \beta_k \rangle| = 2.5658 \pm 0.0067 \quad (84\text{-}\sigma \text{ violation})$$

$$|\langle \beta \rangle| = 7.019 \pm 0.015 \quad (196\text{-}\sigma \text{ violation})$$

$$\frac{|\langle \beta \rangle|}{4} > \frac{|\langle \beta_\pi \rangle|}{2}$$

Note: $\frac{|\langle \beta \rangle|}{4} > \frac{|\langle \beta_k \rangle|}{2}$

$$|\langle \beta \rangle| > |\langle \beta_\pi \rangle| |\langle \beta_k \rangle| = 6.610$$



$$N = 1 \Rightarrow \frac{q}{c} = \sqrt{2}$$

$$N = 2 \Rightarrow \frac{q}{c} = 2$$

$$N = 3 \Rightarrow \frac{q}{c} = 2\sqrt{2}$$



No *fair sampling* assumption

Bell states analysis for quantum communication protocols

More secure protocols (QKD, dense coding) and larger robustness to noise achievable by using d -level quantum systems (qu-dits, $d = 3,4$)

[Bechmann-Pasquinucci, A. Peres, PRL (2000); Bruss, Macchiavello, PRL (2002)]

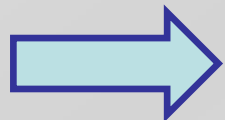
Quantum cryptographic schemes realized with ququarts ($d = 4$) based on mutually unbiased bases made by two-photon Bell states:

$$|\xi\rangle = \alpha|00\rangle + \beta|11\rangle + \gamma|01\rangle + \delta|10\rangle$$

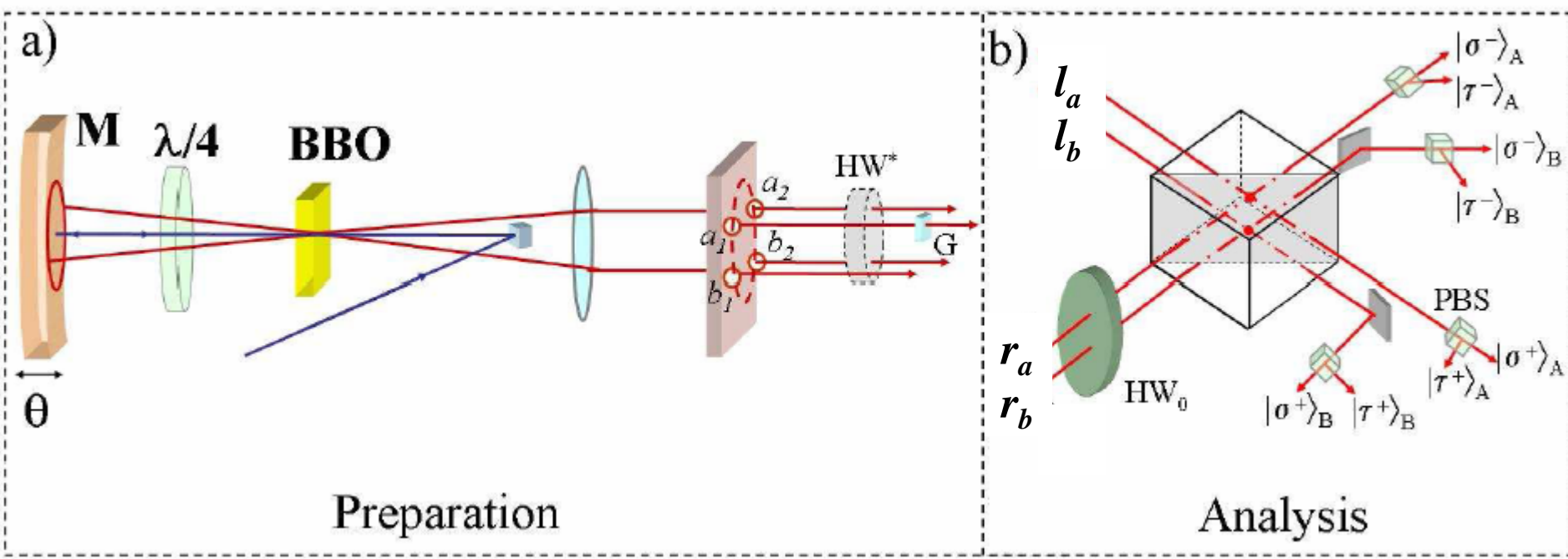
$$|0\rangle \equiv |H\rangle$$

$$|1\rangle \equiv |V\rangle$$

Complete Bell state analysis not achievable by a single joint measurement on the two particles (max. 50%).



Further deg. of freedom besides polarization are needed
(linear momentum assisted measurement)



Analysis based on single photon Bell states:

$$|\sigma^\pm\rangle = (|Hl\rangle \pm |Vr\rangle)$$

$$|\tau^\pm\rangle = (|Vl\rangle \pm |Hr\rangle)$$

$$|\Pi^{++}\rangle = |\Phi^+\rangle \otimes |\psi^+\rangle = \frac{1}{2} [|\sigma^+\rangle_a |\tau^+\rangle_b - |\sigma^-\rangle_a |\tau^-\rangle_b + |\tau^+\rangle_a |\sigma^+\rangle_b - |\tau^-\rangle_a |\sigma^-\rangle_b]$$

$$|\Pi^{-+}\rangle = |\Phi^-\rangle \otimes |\psi^+\rangle = \frac{1}{2} [-|\sigma^+\rangle_a |\tau^-\rangle_b + |\sigma^-\rangle_a |\tau^+\rangle_b + |\tau^+\rangle_a |\sigma^-\rangle_b - |\tau^-\rangle_a |\sigma^+\rangle_b]$$

$$|\Xi^{++}\rangle = |\Psi^+\rangle \otimes |\psi^+\rangle = \frac{1}{2} [|\sigma^+\rangle_a |\sigma^+\rangle_b - |\sigma^-\rangle_a |\sigma^-\rangle_b + |\tau^+\rangle_a |\tau^+\rangle_b - |\tau^-\rangle_a |\tau^-\rangle_b]$$

$$|\Xi^{-+}\rangle = |\Psi^-\rangle \otimes |\psi^+\rangle = \frac{1}{2} [-|\sigma^+\rangle_a |\sigma^-\rangle_b + |\sigma^-\rangle_a |\sigma^+\rangle_b - |\tau^+\rangle_a |\tau^-\rangle_b + |\tau^-\rangle_a |\tau^+\rangle_b]$$

Analysis:

1) HW_0 operation:

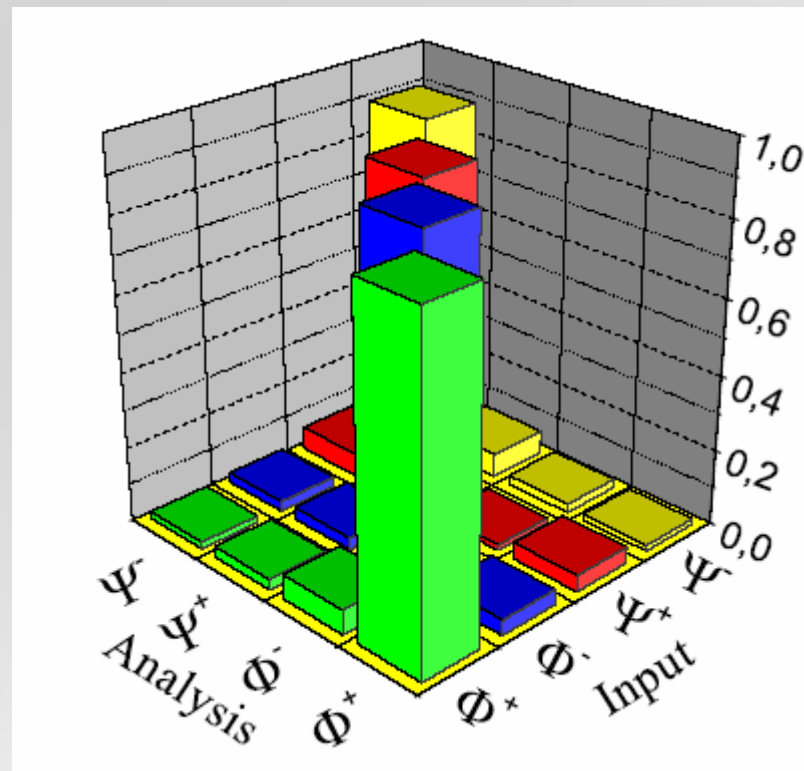
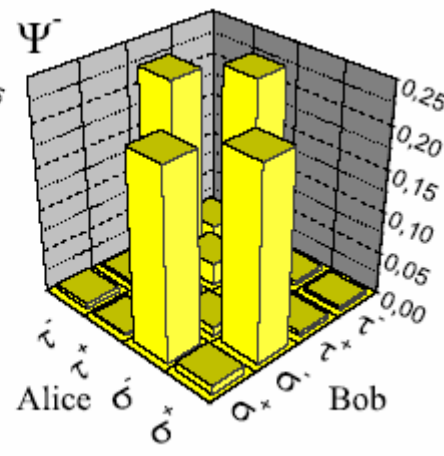
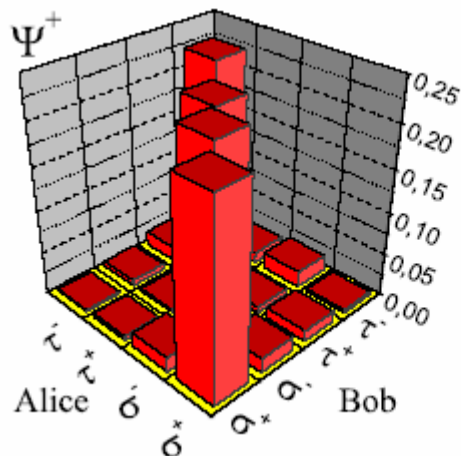
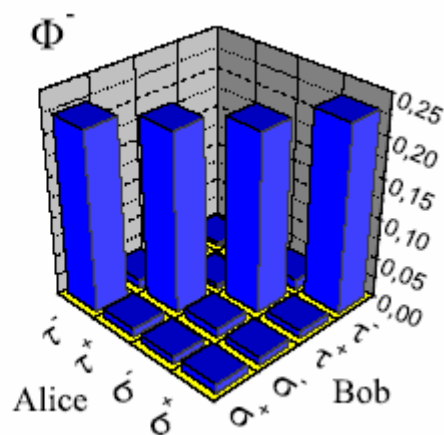
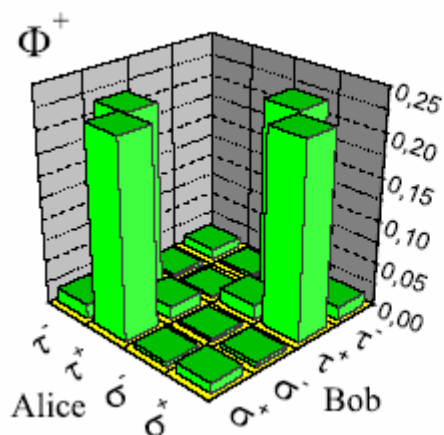
$$|\sigma^\pm\rangle \Rightarrow |H\rangle \otimes \frac{1}{\sqrt{2}}(|l\rangle \pm |r\rangle)$$

$$|\tau^\pm\rangle \Rightarrow |V\rangle \otimes \frac{1}{\sqrt{2}}(|l\rangle \pm |r\rangle)$$

2) BS discrimination:

$$|l\rangle + |r\rangle \text{ from } |l\rangle - |r\rangle$$

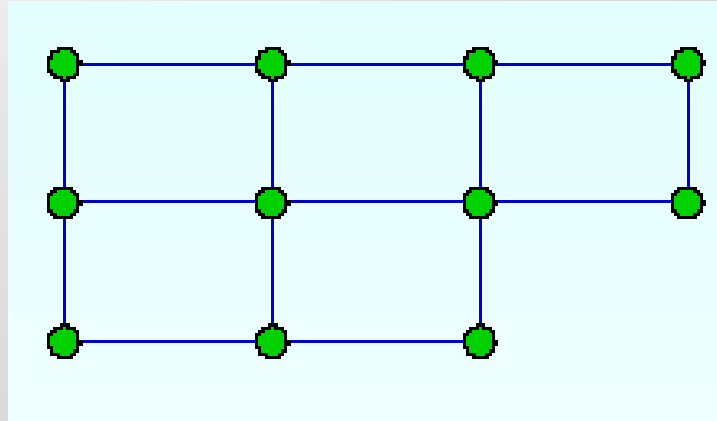
3) Pol. analysis performed by PBS's



average fidelity $F = 0.889 \pm 0.010$

Two photon linear cluster states

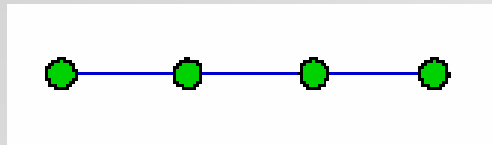
Cluster state: quantum state obtained by starting from a finite n-dimensional lattice



Each dot corresponds to the qubit: $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Each link corresponds to a Control Phase (CP) gate: $CP = |0\rangle_1\langle 0| \otimes \mathbb{1}_2 + |1\rangle_1\langle 1| \otimes \sigma_z^{(2)}$

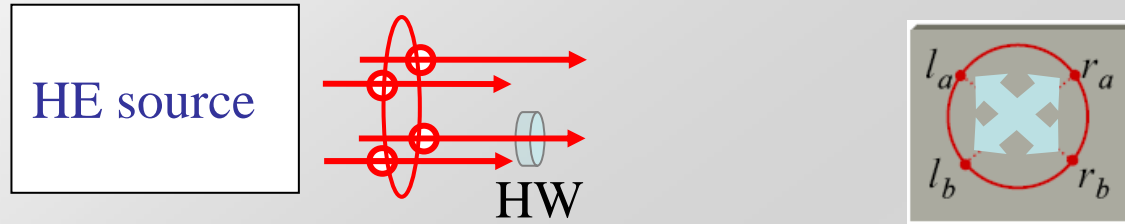
4-qubit linear cluster state:



4-photons:

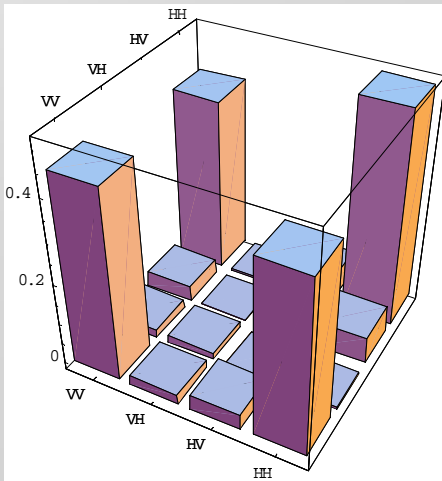
$$\frac{1}{4} \left[|H_a H_b H_c H_d\rangle + |H_a H_b V_c V_d\rangle + |V_a V_b H_c H_d\rangle - |V_a V_b V_c V_d\rangle \right]$$

Create 2-photons linear cluster states (starting by hyperentanglement):

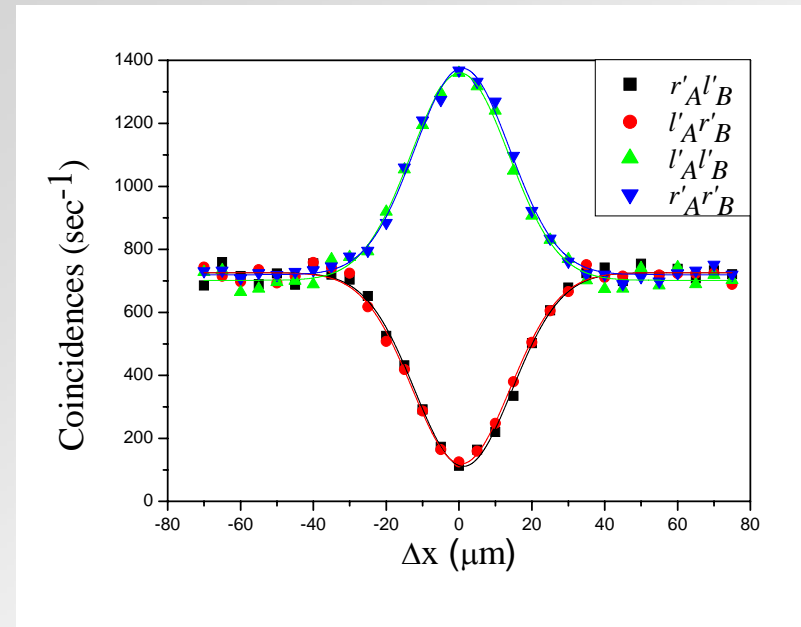
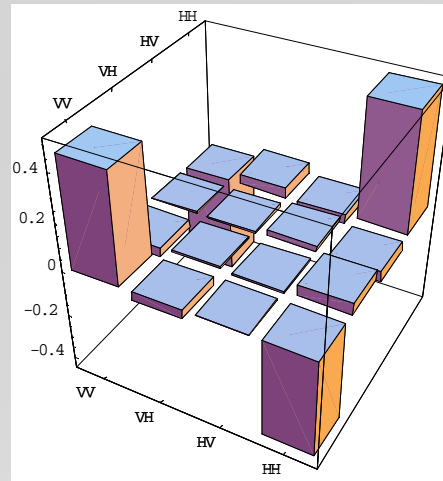


$$\frac{1}{4} \left[|H_a l_a\rangle |H_b r_b\rangle + |H_a r_a\rangle |H_b l_b\rangle + |V_a l_a\rangle |V_b r_b\rangle - |V_a r_a\rangle |V_b l_b\rangle \right]$$

Modes $r_a - l_b$



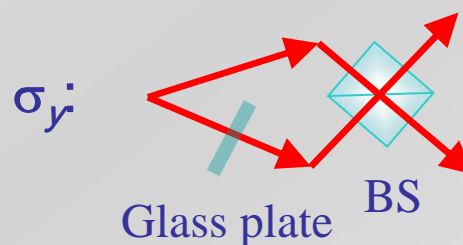
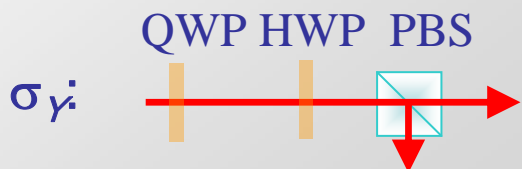
Modes $l_a - r_b$



Characterization and nonlocality test with linear cluster states:

- Entanglement Witness [Kiesel *et al.* PRL (2005)]
- Stronger two observer AVN violation of Local Realism (provides larger amount of LR violation) [Cabello, PRL (2005)]

Both require to measure further operators: $\sigma_Y (Y)$ and $\sigma_y (y)$



$$\hat{S} = X_a x_a Y_b y_b - Y_a x_a X_b y_b + X_a X_b z_b + Y_a Y_b z_b$$

$$2 \leq \langle S \rangle \leq 4$$



$$\langle S \rangle = 3.4145 \pm 0.0053$$

$$\hat{W} = \frac{1}{2} [4 - Z_a Z_b + z_a z_b - Z_a x_a x_b - Z_b x_a x_b - X_a X_b z_b + X_a X_b z_a]$$

$$\langle W \rangle_{Cluster} = -1$$

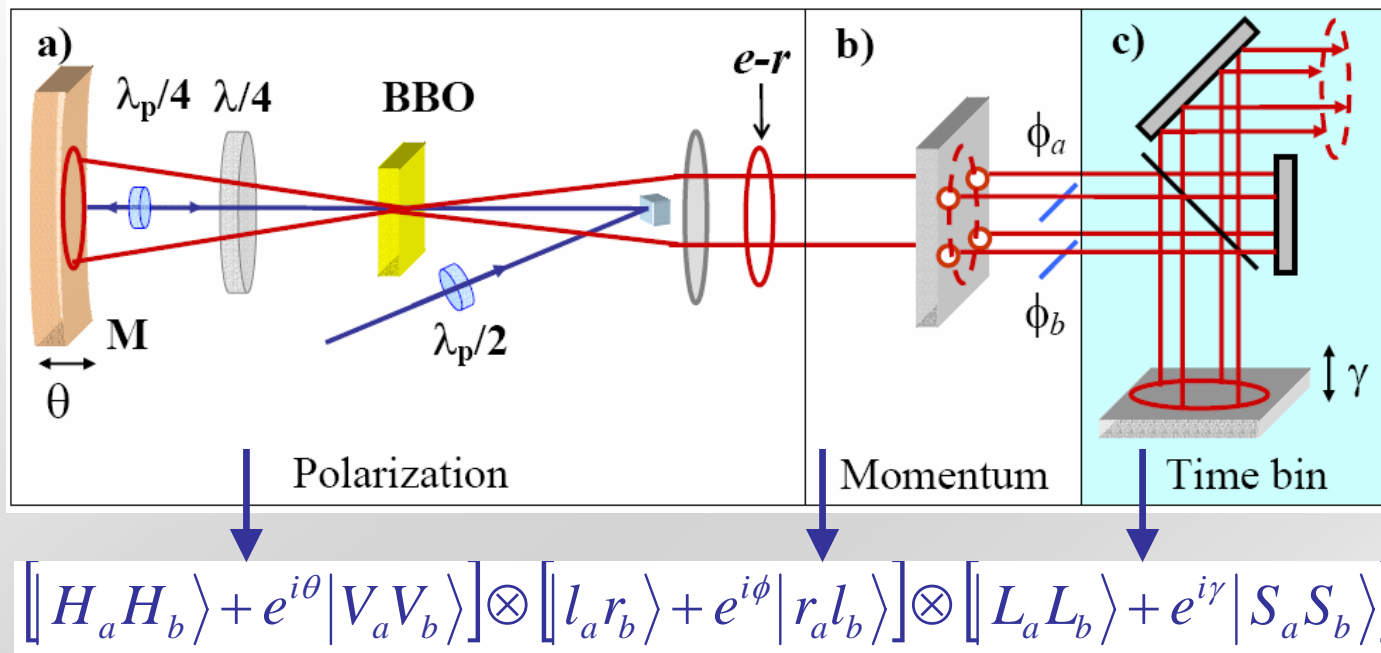
$$\langle W \rangle_{HE} = +1$$



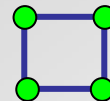
$$\langle W \rangle = -0.6814 \pm 0.0071$$

Perspectives

1) Enhancing quantum nonlocality by further degrees of freedom:



2) Realization of the box linear cluster states:



3) Use two photon cluster states for one-way quantum computation?

References:

- M. Barbieri, *et al.*, “*Polarization-momentum hyperentangled states: Realization and characterization*”, *Phys. Rev. A* **72**, 052110 (2005).
- C. Cinelli, *et al.*, “*All-Versus-Nothing Nonlocality Test of Quantum Mechanics by Two-Photon Hyperentanglement*”, *Phys. Rev. Lett.* **95**, 240405 (2005).
- M. Barbieri, *et al.*, “*Enhancing the Violation of the Einstein-Podolsky-Rosen Local Realism by Quantum Hyper-entanglement*”, *Phys. Rev. Lett.* **97**, 140707 (2006).
- M. Barbieri, *et al.*, “*Complete and Deterministic Bell state identification by two photon hyper-entanglement*”, [quant-ph/0609080](https://arxiv.org/abs/quant-ph/0609080).