



Matis

Impurities induced dephasing in solid state qubits

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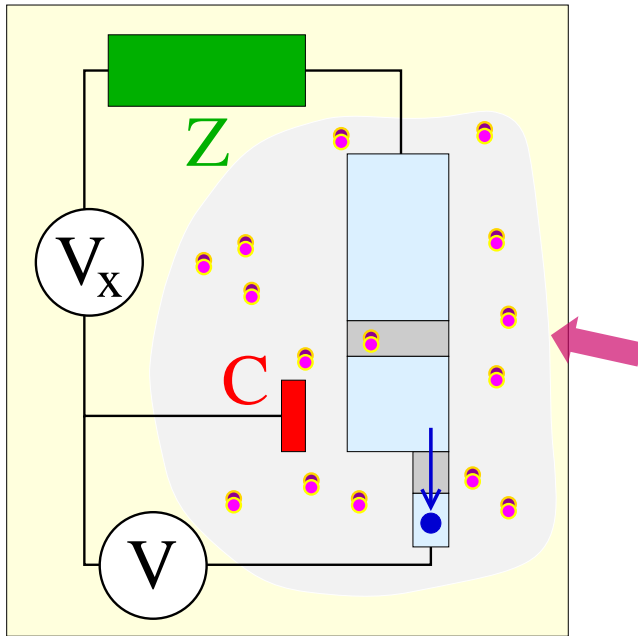
Solid-state qubits...

- ☺ Scalability, integration, easy tunable
- ☹ BUT solid-state noise, crosstalk, ...

Tasks of present research on SC nanostructures

- ❖ Identification of major noise sources and quantitative estimate of the ensuing decoherence
- ❖ Measurement of decoherence rate of single qubits
 - Major activity of all experimental groups
- ❖ Measurement/investigation of decoherence rates in two-qubit circuits
 - Emerging activity
- ❖ Quantum control
 - Major activities of all theory + experimental groups

Noise sources in SC qubits



$Z(\omega)$ electromagnetic fluctuations of the circuit (**gaussian**)

discrete noise due to fluctuating **background charges (BC)** trapped in the substrate or in the junction

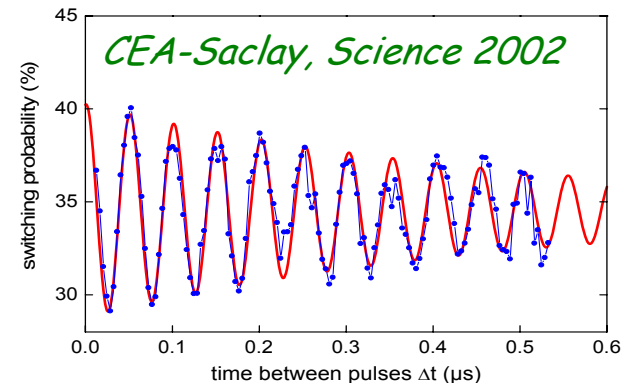
- trapped flux in mesoscopic SQUIDs
- trapped flux at the junctions in smaller SQUIDs

quasiparticles/measurement

charge noise (1/f)

↔ switching impurities close to the device
Paladino, Faoro, Falci, Fazio, PRL 2002

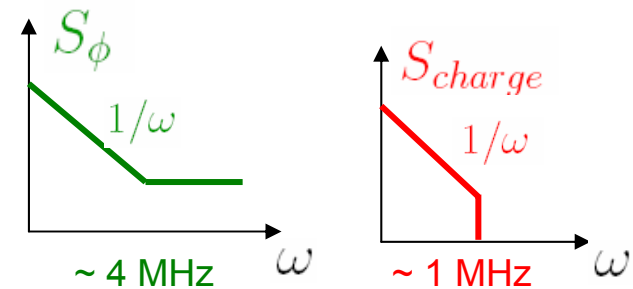
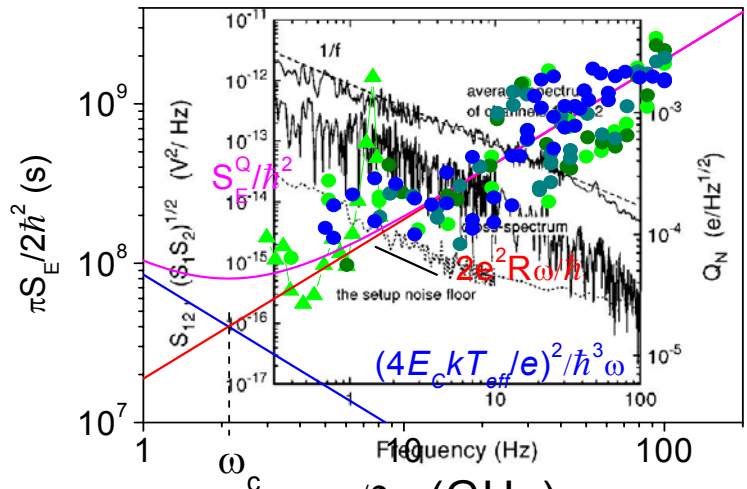
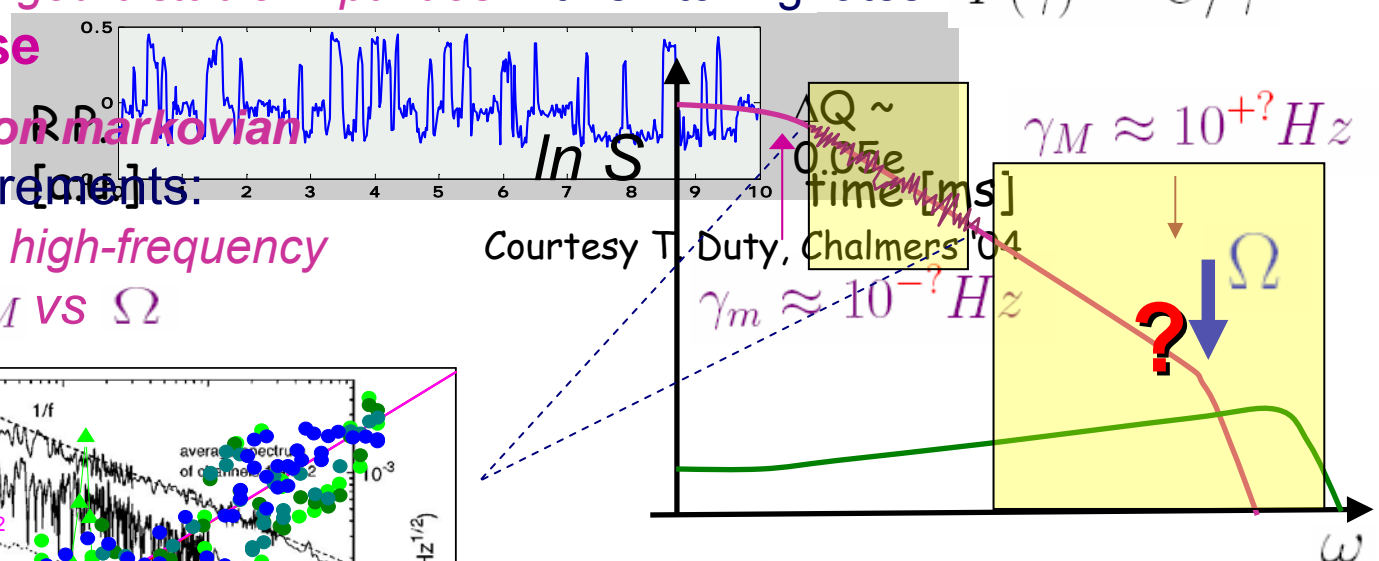
- **THE** problem in high Q charge based qubits
- Affects two-qubit operations in spin-qubits



Noise characterization

- Noise due to *charged bistable impurities* → RTN
- Distribution of *charged bistable impurities* with switching rates $P(\gamma) = C/\gamma$ leads to **1/f noise**

non gaussian, non markovian
 1/f noise measurements:
uncertainty on all high-frequency properties, eg. γ_M vs Ω

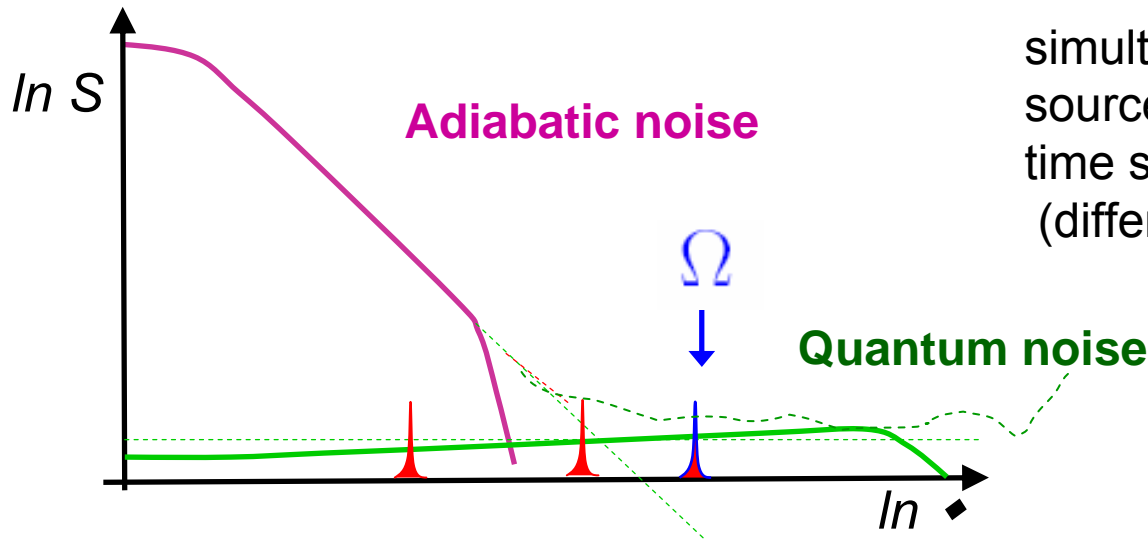


Experiments in SET: *Zorn et al., PRB (1996)*
 Experiments in charge qubit: *Astafiev et al., PRL 04*

Saclay qubit
Ithier et al., PRB 05

Noise characterization

Variety of observations, material & device dependent



simultaneous presence of noise sources with different characteristic time scales
(different approximation schemes)

Strongly coupled noise

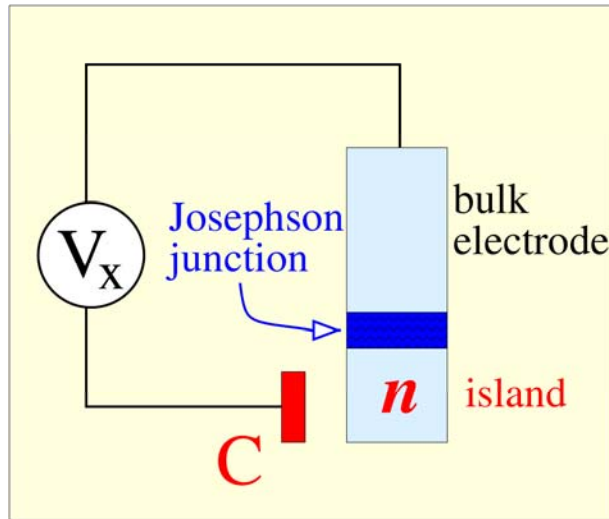
Falci, D'Arrigo, Mastellone, Paladino, PRL 2005

FOCUS: single impurity model

- Collection of impurities may originate $1/f + f$ noise
- Uncontrollable *dynamical impurities*:
expected relevant limitation for *multi-qubit on-chip* devices

Qubit-impurity model

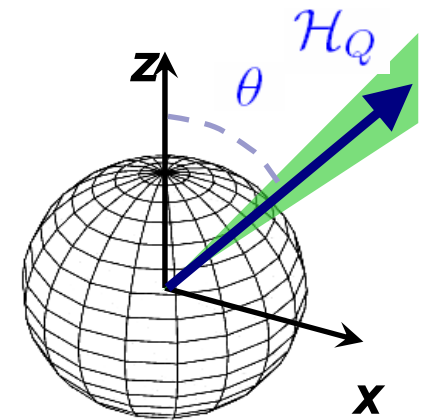
Experiments: V. Bouchiat et al., Journal of Superconductivity (1999)



$$\mathcal{H}_{qb} = -\frac{\epsilon_q}{2} \sigma_z - \frac{E_j}{2} \sigma_x$$

extra charge on the island

$$\{|0\rangle, |1\rangle\} \longleftrightarrow \sigma_z$$



Charged impurity couples to σ_z

$$\mathcal{H}_{qb-imp} = -\frac{\epsilon_q}{2} \sigma_z - \frac{E_j}{2} \sigma_x - \frac{v}{2} \sigma_z \tau_z \pm \mathcal{H}_{imp}$$

Single impurity model

$$\mathcal{H}_{imp} = -\frac{\varepsilon}{2} \tau_z - \frac{\Delta}{2} \tau_x - \frac{1}{2} \hat{X} \tau_z + \mathcal{H}_E \quad \text{spin-boson model}$$

$$\mathcal{H}_E = \sum \omega_\alpha a_\alpha^\dagger a_\alpha \quad \hat{X} = \sum \lambda_\alpha (a_\alpha + a_\alpha^\dagger)$$

$$\mathcal{H}_{qb-imp} = -\frac{\epsilon_q}{2} \sigma_z - \frac{E_j}{2} \sigma_x - \frac{v}{2} \sigma_z \tau_z \pm \mathcal{H}_{imp}$$

$$S(\omega) = \pi G(|\omega|) \coth \frac{\mu|\omega|}{2}$$

Qubit σ

Idle qubit τ

target σ

Qubit σ

Two qubits, σ

and bistable impurity

$$\rho(t) = \text{Tr}_{\tau, osc} \{ W(t) \}$$

τ id control

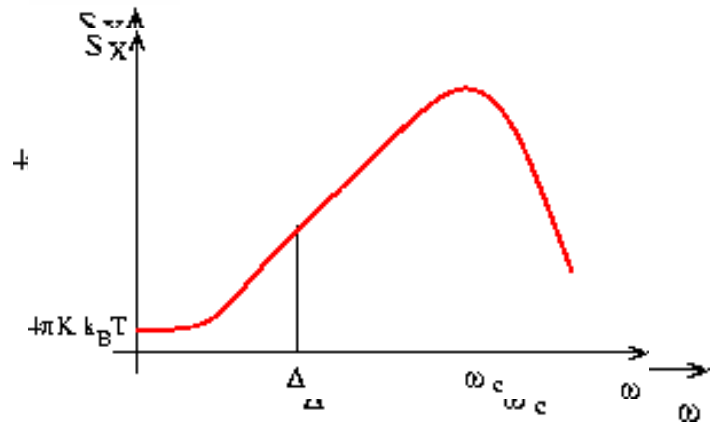
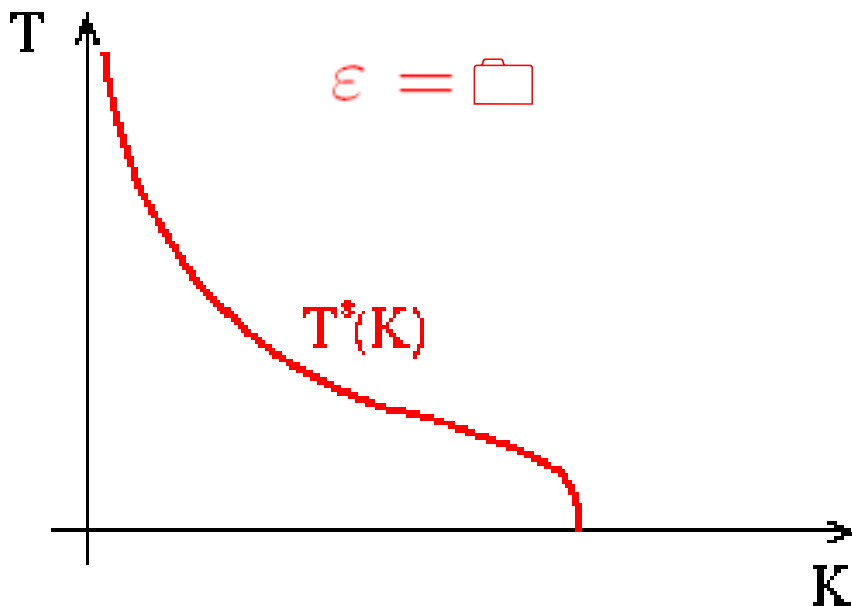
as measurement device

τ and

coupled to a oscillator

Impurity dynamical regimes

$$\mathcal{H}_{imp} = -\frac{\varepsilon}{2} \tau_z - \frac{\Delta}{2} \tau_x - \frac{1}{2} \hat{X} \tau_z + \mathcal{H}_E$$



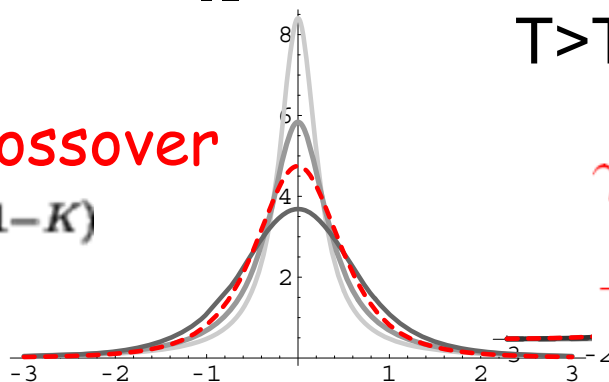
$T < T^*$ time scale

$$\Delta_T = \Delta_r (2\pi k_B T / \Delta_r)^K$$

$T > T^*$ time scale

Large T
coherent-incoherent crossover

$$k_B T^*(K) \approx \Delta_r (\Delta_r / \omega_c)^{K/(1-K)}$$



$$\gamma(T) = 2\pi K k_B T$$

$$\rightarrow \frac{\Delta_r}{K} \left(\frac{\Delta_r}{2\pi k_B T} \right)^{1-2K}$$

Kondo/Zeno

Impurity time scales

$$\mathcal{H}_{qb-imp} = -\frac{\epsilon_q}{2} \sigma_z - \frac{E_j}{2} \sigma_x - \frac{v}{2} \sigma_z \tau_z \pm \mathcal{H}_{imp}$$

$$S_r(\omega) = v^2 \int_{-\infty}^{\infty} dt \frac{1}{2} (\langle \tau_r(t) \tau_r(0) + \tau_r(0) \tau_r(t) \rangle - \langle \tau_r \rangle_{\infty}^2) e^{i\omega t}$$

Impurity "correlation time"

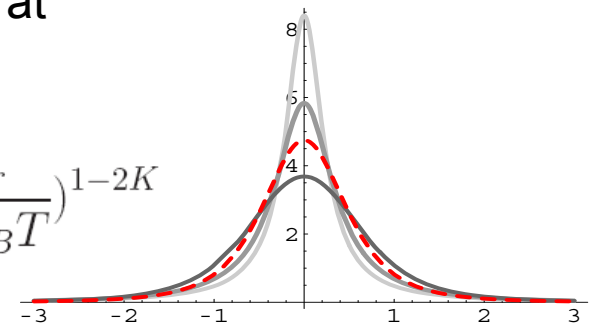
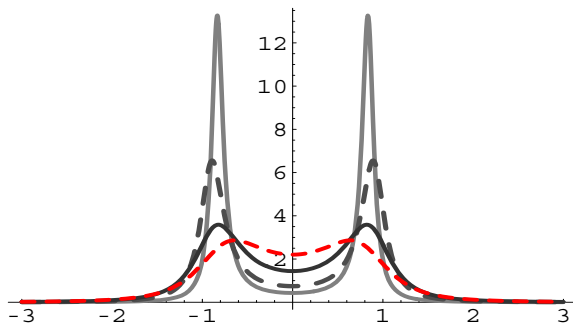
$$\tau_c^{-1} \approx \Delta_T$$

$$\tau_c^{-1} \approx \gamma(T)$$

Kondo behavior at

$$T \gg T^*(K)$$

$$\gamma(T) \approx \frac{\Delta_r}{K} \left(\frac{\Delta_r}{2\pi k_B T} \right)^{1-2K}$$



Expected dynamical regimes for the qubit

$$\mathcal{H}_{qb-imp} = -\frac{\epsilon_q}{2} \sigma_z - \frac{E_j}{2} \sigma_x - \frac{v}{2} \sigma_z \tau_z \pm \mathcal{H}_{imp}$$

Weak-coupling $v \tau_c^{-1} \ll 1$

$$\Gamma_\tau = \frac{1}{2} \sin^2 \theta S_\tau(\Omega_1)$$

$$\Gamma_\phi = \frac{1}{4} \sin^2 \theta S_\tau(\Omega_1) + \frac{1}{2} \cos^2 \theta S_\tau(0)$$

$$\Omega_1 = \sqrt{\epsilon_q^2 + E_j^2} \rightarrow \Omega_1 + \text{shift}$$

$$\delta \Omega_1 = -\frac{1}{4} \sin^2 \theta \mathcal{P} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{S_\tau(\omega)}{\omega - \Omega_1}$$

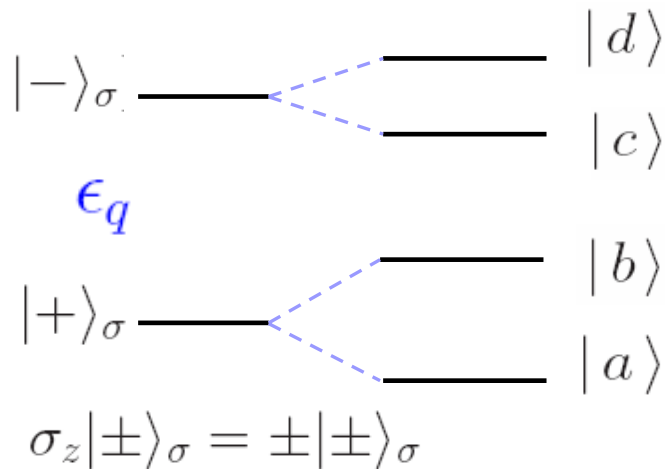
Valid only if the Environment is fast

$$\tau_c \ll \Delta t \ll \Gamma_\phi^{-1}$$

Modulable correlation time $\tau_c(T) \longrightarrow v \tau_c^{-1} \geq 1$
 qubit dynamics ??

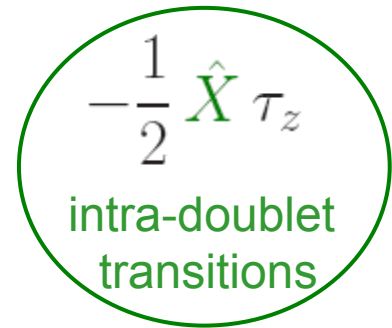
Extended Hilbert space qubit+impurity

Pure dephasing $\mathcal{H}_0 = -\frac{\epsilon_q}{2} \sigma_z - \frac{\epsilon}{2} \tau_z - \frac{\Delta}{2} \tau_x - \frac{v}{2} \sigma_z \tau_z$



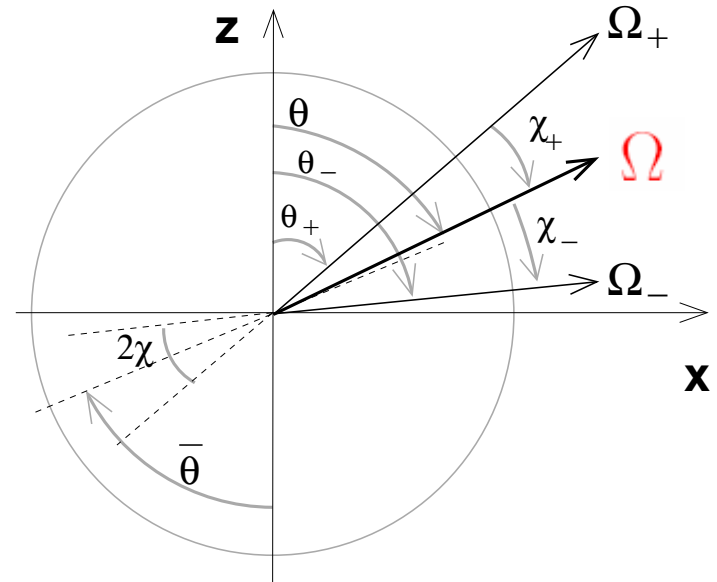
$$E_{-\pm} = \frac{1}{2}(\epsilon_q \mp \Omega_-)$$

$$E_{+\pm} = -\frac{1}{2}(\epsilon_q \pm \Omega_+)$$



conditional level splittings

$$\Omega_\pm = \sqrt{(\epsilon \pm v)^2 + \Delta^2}$$



Theory for general T and coupling v

$\rho(t)$ qubit+impurity density matrix

At pure dephasing $[\sigma_z, \mathcal{H}] = 0$

relevant dynamical quantities

$$\langle \sigma_- \rangle = \text{Tr}(\rho \sigma_-) = \cos \chi (\rho_{ac} + \rho_{bd}) + \sin \chi (\rho_{ad} - \rho_{bc})$$

coherences

Master equation in the enlarged Hilbert space

$$\begin{aligned} \partial_t \rho(t) = & -i[\mathcal{H}_0, \rho(t)] - \int_0^\infty dt' \left\{ \frac{1}{4} S(t') [\tau_z, [\tau_z(t'), \rho(t)]] \right. \\ & \left. + \frac{i}{2} \chi(t') [\tau_z, [\tau_z(t'), \rho(t)]_+] \right\} \end{aligned}$$

with factorised initial condition

$$\rho(0) = \rho_\sigma(0) \otimes \rho_\tau(0) \quad \rho_\tau(0) = \frac{1}{2} \hat{I} + \frac{1}{2} \delta p(0) \tau_z$$

Coherences (secular approximation)

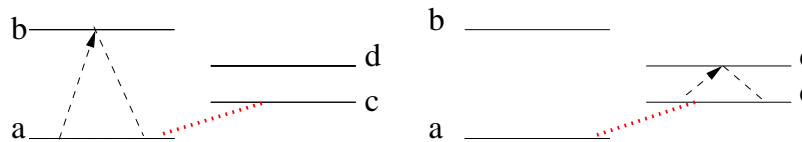
$$\begin{pmatrix} \dot{\rho}_{ac} \\ \dot{\rho}_{bd} \end{pmatrix} = \begin{pmatrix} -i\delta - \Gamma_1 & \Gamma_{12} \\ \Gamma_{21} & i\delta - \Gamma_2 \end{pmatrix} \begin{pmatrix} \rho_{ac} \\ \rho_{bd} \end{pmatrix} \quad \delta = (\Omega_+ - \Omega_-)/2$$

$$\begin{pmatrix} \dot{\rho}_{ad} \\ \dot{\rho}_{bc} \end{pmatrix} = \begin{pmatrix} -i\Omega - \Gamma_3 & \Gamma_{34} \\ \Gamma_{43} & i\Omega - \Gamma_4 \end{pmatrix} \begin{pmatrix} \rho_{ad} \\ \rho_{bc} \end{pmatrix} \quad \Omega = (\Omega_+ + \Omega_-)/2$$

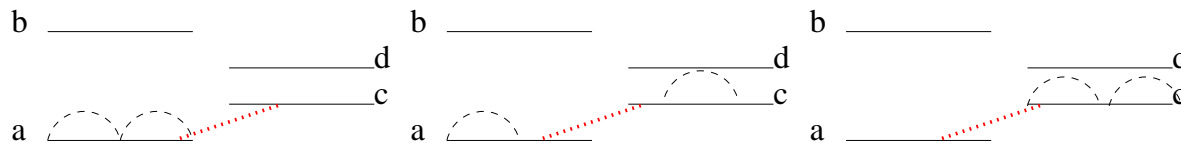
$$\Gamma_{21} = \frac{1}{2} \tau_{ab} \tau_{cd} [\Gamma_-(\Omega_+) + \Gamma_-(\Omega_-)]$$

absorption/emission rates

$$\Gamma_{\pm}(\omega) = \pi G(|\omega|) \text{sgn}(\omega) [\coth(\beta\omega/2) \pm 1]$$



Γ_1



Coherences

$$\langle \sigma_- \rangle_t = \frac{e^{i\epsilon_q t}}{2} \left[\cos \chi^2 (e^{\lambda_1 t} + e^{\lambda_2 t} - \frac{\Gamma_{12} + \Gamma_{21}}{\Lambda} (e^{\lambda_1 t} - e^{\lambda_2 t})) \right. \\ \left. + \sin \chi^2 (e^{\lambda_3 t} + e^{\lambda_4 t} + \frac{\Gamma_{34} + \Gamma_{43}}{\Sigma} (e^{\lambda_3 t} - e^{\lambda_4 t})) \right]$$

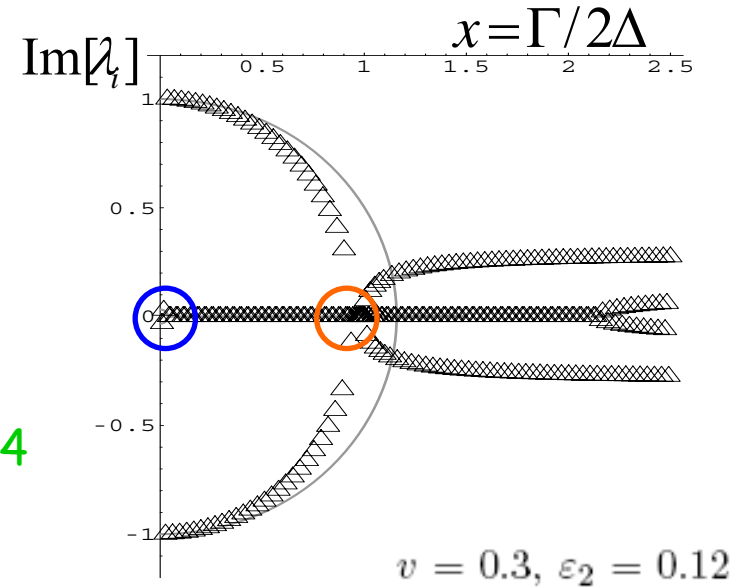
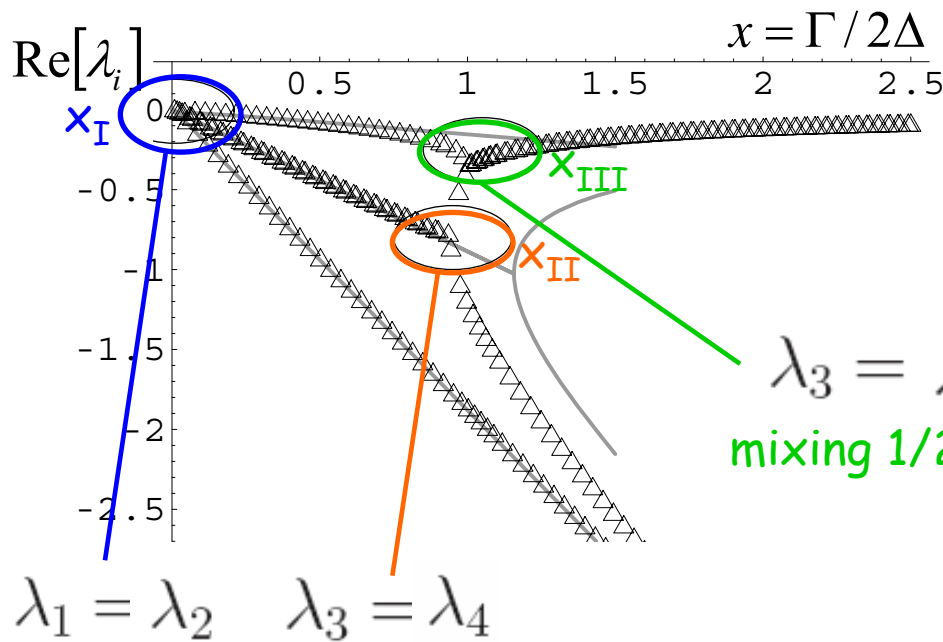
$$\lambda_{1/2} = -\frac{\Gamma_1 + \Gamma_2}{2} \pm \frac{\sqrt{(2i\delta + \Gamma_2 - \Gamma_1)^2 + 4\Gamma_{12}\Gamma_{21}}}{2}$$

$$\lambda_{3/4} = -\frac{\Gamma_3 + \Gamma_4}{2} \pm \frac{\sqrt{(2i\Omega + \Gamma_4 - \Gamma_3)^2 + 4\Gamma_{34}\Gamma_{43}}}{2}$$

- The qubit dynamics is conditioned by **crossings** of λ_i
- ✓ **NOTE:** ME interpolates between systematic expansions valid for $T \rightarrow 0$ (systematic weak damping approximation) and in the high-T limit (NIBA) changing T and v

High-temperatures

White noise $S(\omega) = 2\Gamma$ (with $\Gamma = 2\pi K k_B T$)



$\langle \sigma_- \rangle$

$$\langle \sigma_- \rangle \approx e^{i\Omega_1 t - \Gamma_Z t} \{ A e^{i\Omega_Z t} + B e^{-i\Omega_Z t} \}^{1/2 t}$$

slow modulation & beatings at δ

$$- \sin \chi \left| \rho_{ad}(0) e^{-i\Omega_{2\text{eff}} t} - \rho_{bc}(0) e^{i\Omega_{2\text{eff}} t} \right| e^{-\mathcal{K} e \lambda_{3/4} t}$$

Counterintuitive: τ slowly relaxing with $\Gamma_Z \approx \Delta^2 / (2\Gamma)$ modulates the phase

of σ at $\Omega_Z \approx \bar{\Omega} \sin \bar{\theta} \sin \chi + \delta \cos \bar{\theta} \cos \chi$

independent on damping!!!!

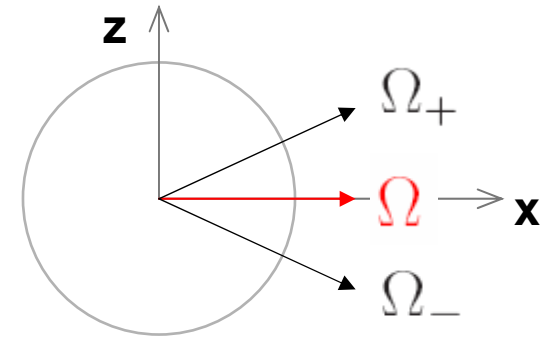
Coherences at $\epsilon = 0$

Additional symmetries

$$\delta = 0 \quad \Omega_+ = \Omega_- = \Omega = \sqrt{v^2 + \Delta_2^2}$$

Three complex λ_i

$$\lambda_2 = -\gamma_\phi \quad \lambda_{3/4} = -\frac{\gamma_r}{2} \pm \sqrt{\left(\frac{\gamma_r}{2}\right)^2 - \Omega^2}$$

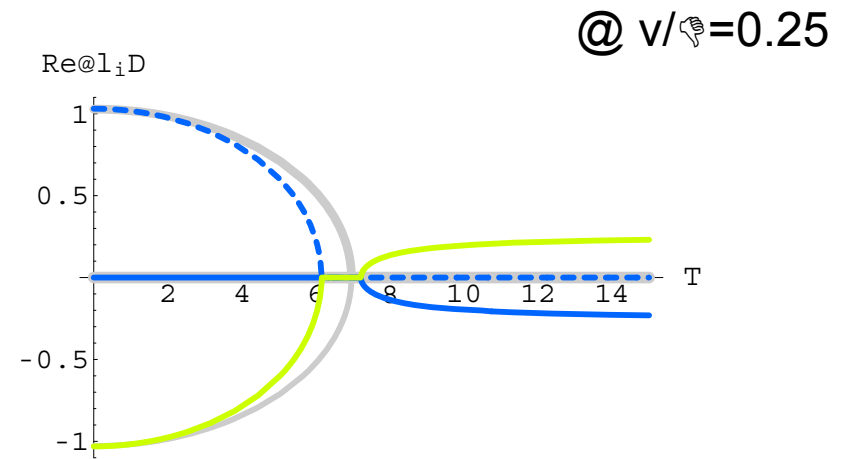
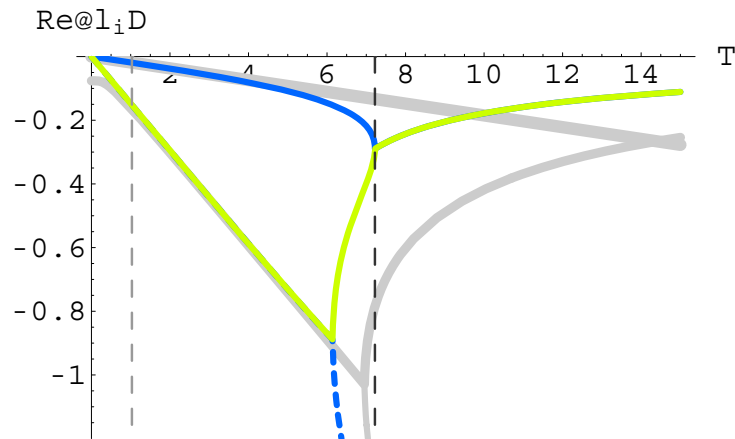


Simply related to GR relaxation and dephasing rate for τ

$$\gamma_r = \frac{1}{2} \sin^2 \theta S(\Omega) \quad \gamma_\phi = \frac{1}{2} \cos^2 \theta S(0)$$

Crossings: one real \rightarrow two complex conjugate dominant eigenvalues

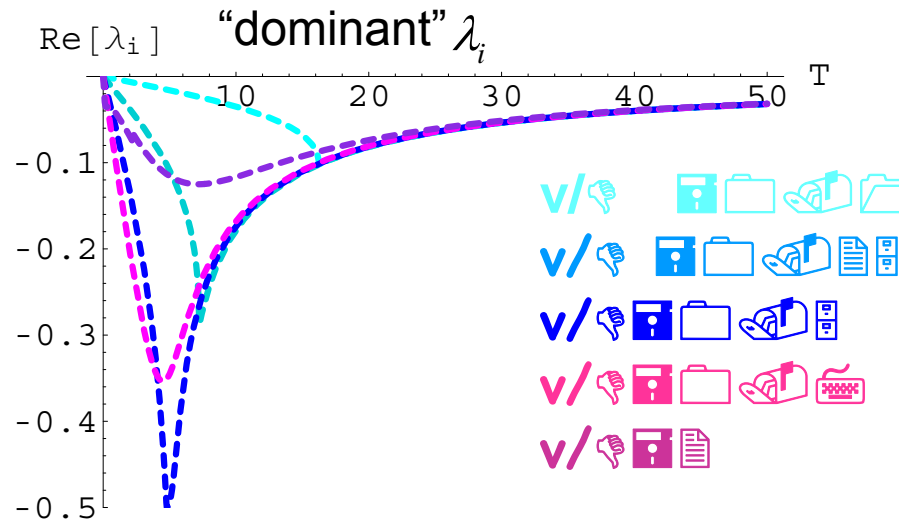
Coherences



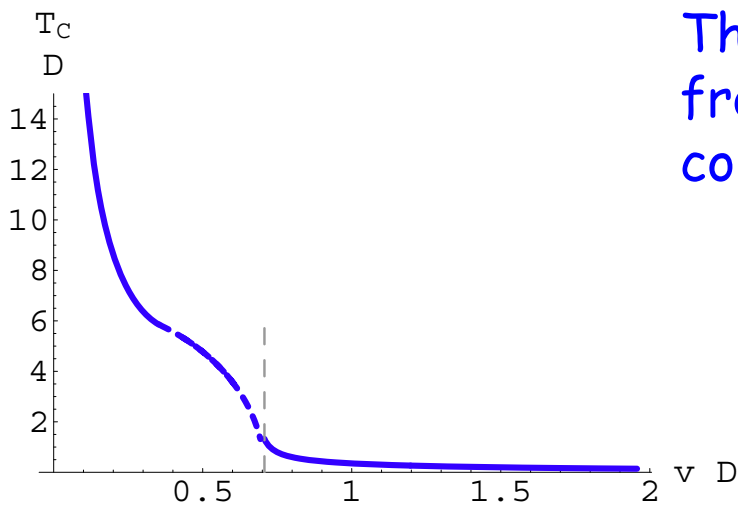
Small temperatures : single real dominant eigenvalues

High temperature: two complex conjugate eigenvalues

Crossover modulated by T and v



- Possible strong dephasing also at small T (from SMALL couplings v)
- Saturation of dephasing at high T (Kondo/Zeno behavior) independent on v

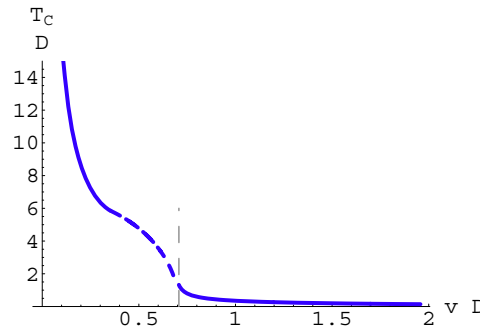


The crossover temperature follows from the condition on the impurity correlation time (if $v/\tau_c \ll 1$)

$$v \tau_c^{-1} = 1$$

Conclusions & Perspectives

- ✓ Impurity model showing a complex dynamics
- ✓ Crossover effects in the qubit dynamics



Hints:

- High temperatures saturation of dephasing → dynamical decoupling techniques for “dirty” qubits
- Very low temperatures → small dephasing for “clean” qubits

Paladino, Sassetti, Falci, Weiss, submitted 2006

- Effect of a distribution of impurities?
- Quantitative indication of the quantum/classical dynamics of the impurity? (purity, concurrence....) c.f. Experiment: Simmonds et.al. PRL 2004