Entanglement Localization by a single defect in a spin chain

T. J. G. Apollaro, F. Plastina

quant-ph/0610055 (2006), Phys. Rev. A to appear

UNIVERSITÀ DELLA CALABRIA
DIPARTIMENTO DI FISICA

Aim:

1) To describe the effect of a diagonal impurity in both the static and dynamical properties of entanglement;
2) To characterize the entanglement localization and the mirror effect induced by the defect.

Outline:

1) The model and its solution;
2) Entanglement localization in the ground state;
3) Entanglement distribution in the presence of the defect:
   a) localization
   b) reflection and transmission

Spin chain with one defect

We consider a 1-D XX spin ½ closed chain placed in an external magnetic field, homogeneous everywhere but for a single site (diagonal impurity)

$$H = h \sum_{i=-\frac{N}{2}}^{\frac{N}{2}} \sigma_z^i - J \sum_{i=-\frac{N}{2}}^{\frac{N}{2}} (\sigma_x^i \sigma_x^{i+1} + \sigma_y^i \sigma_y^{i+1}) + \epsilon\, \sigma_z^l$$

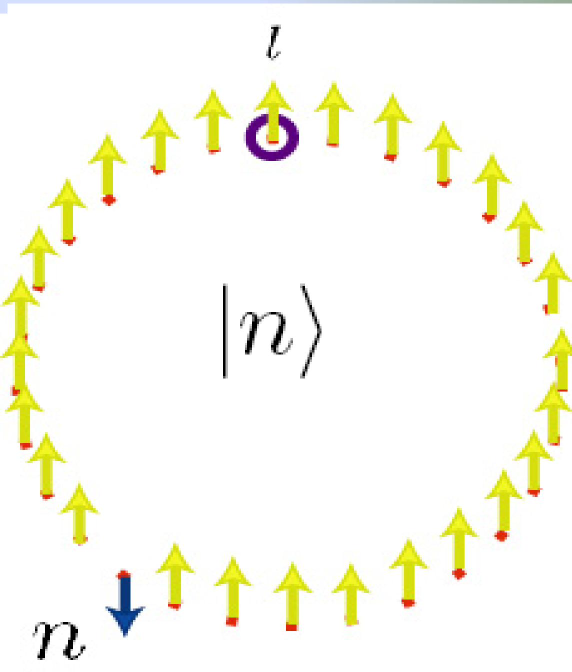
defect

We assume $2h > J$ → *unperturbed ground state:* $|0\rangle^{\otimes N+1}$

1-excitation sector

$$|k\rangle = \frac{1}{\sqrt{N+1}} \sum_n \exp\left[\frac{2\pi i k n}{N}\right] |n\rangle$$

$$E_k = 2h - J\cos\left(\frac{2\pi k}{N}\right)$$



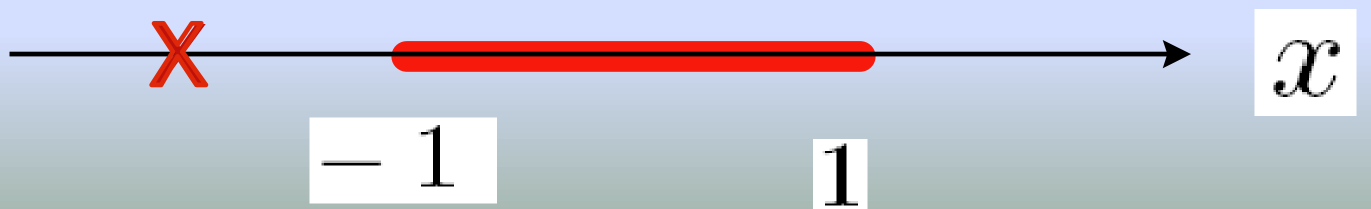
Green function approach (1-excitation sector)

$$G(z) = G_0(z) + G_0(z)|l\rangle \frac{2\epsilon}{1 - 2\epsilon G_0(l,l;z)} \langle l|G_0(z)$$

Dyson

$$G_0^{\pm}(r,s;z) = \frac{(-x \pm i\sqrt{1-x^2})^{|r-s|}}{\pm iJ\sqrt{1-x^2}}, \quad z \in [2h - J, 2h + J]$$

$$G_0(r,s;z) = \frac{(-x + \sqrt{x^2-1})^{|r-s|}}{J\sqrt{x^2-1}}, \quad \text{otherwise}$$



$$x = \frac{z - 2h}{J}$$

Energy band

distortion due to the defect

$$|\Psi(E)\rangle = \sum_n a_n(E)|n\rangle$$

$$a_n(E) = \frac{1}{\sqrt{N+1}}\left(e^{i\theta n} + \frac{\alpha\, e^{i|\theta||n-l|}}{i|\sin\theta| - \alpha}\, e^{i\theta l}\right)$$

where $\cos\theta = (2h - E)/J$

$\alpha = \frac{2\epsilon}{J}$

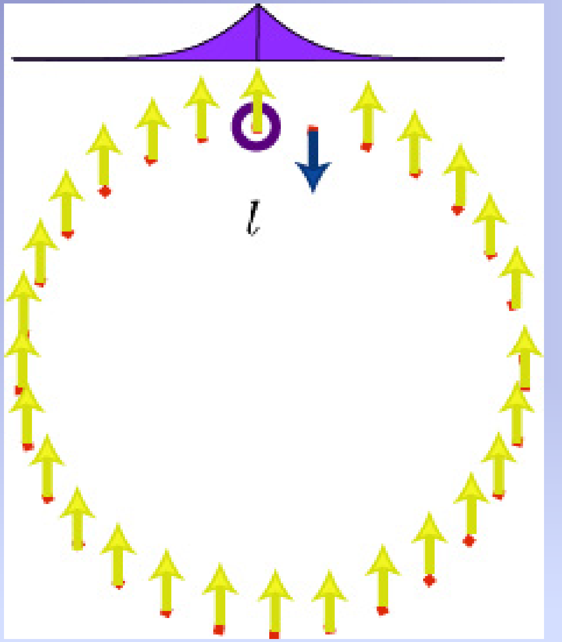
Localized state

$$|\Psi_{loc}\rangle = \sum_n b_n\ |n\rangle \qquad E_{loc} = 2h \mp J\sqrt{1+\alpha^2}$$

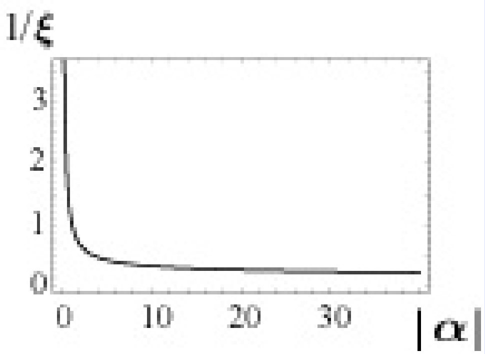
$$b_n = -\frac{\sqrt{|\alpha|}}{(1+\alpha^2)^{\frac{1}{4}}}\exp[-\xi(\alpha)|n-l|]$$

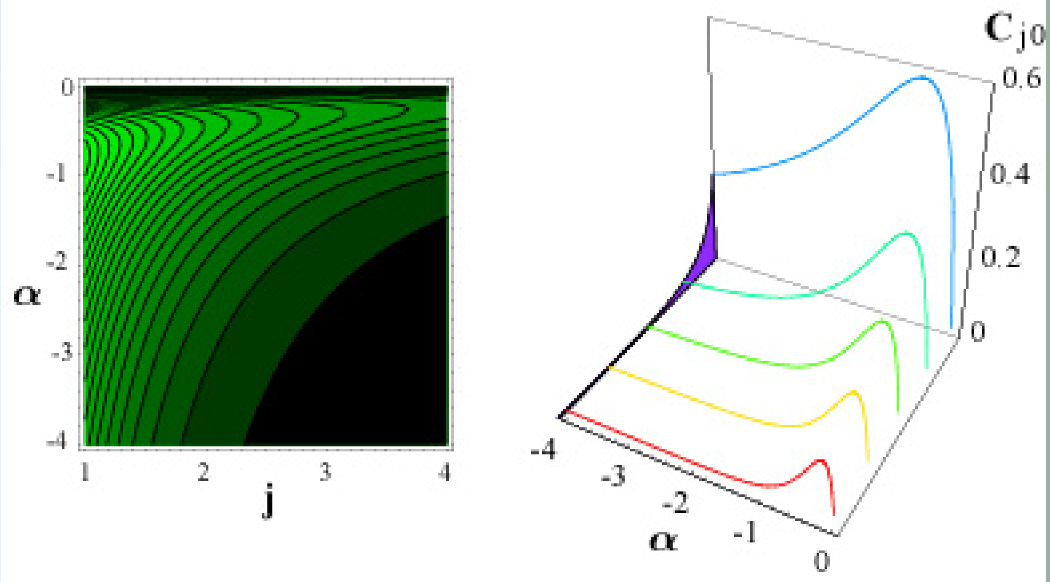
Inverse localization lenght

Localization



$$\xi(\alpha) = -\ln\left(\sqrt{1+\alpha^2} - |\alpha|\right)$$



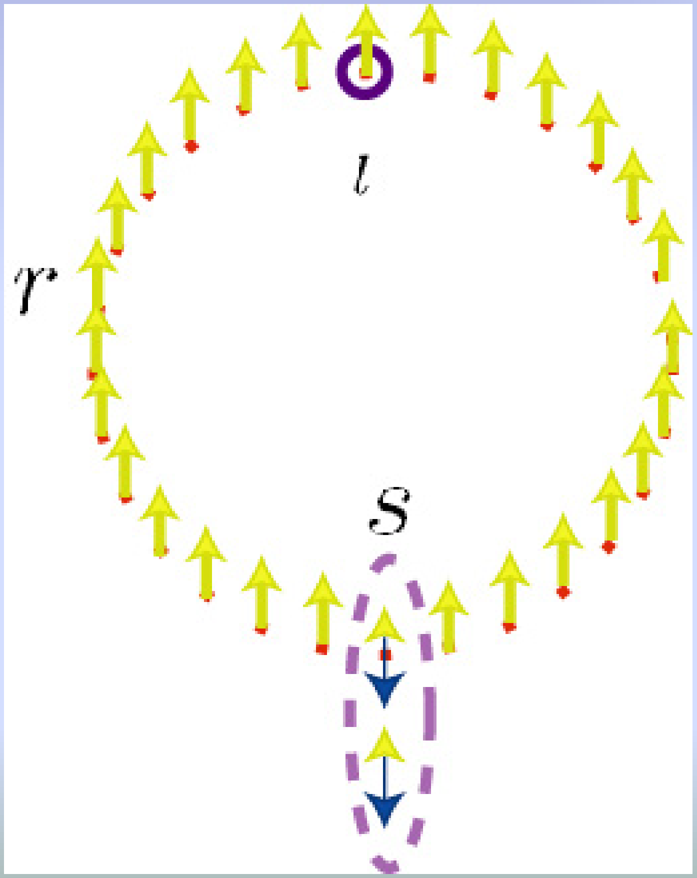


Entanglement

$$C_{ij} = 2Res[G(i,j;E_{loc})] = \frac{2|\alpha|}{\sqrt{1+\alpha^2}}\, e^{-\xi\,(|i|+|j|)}$$

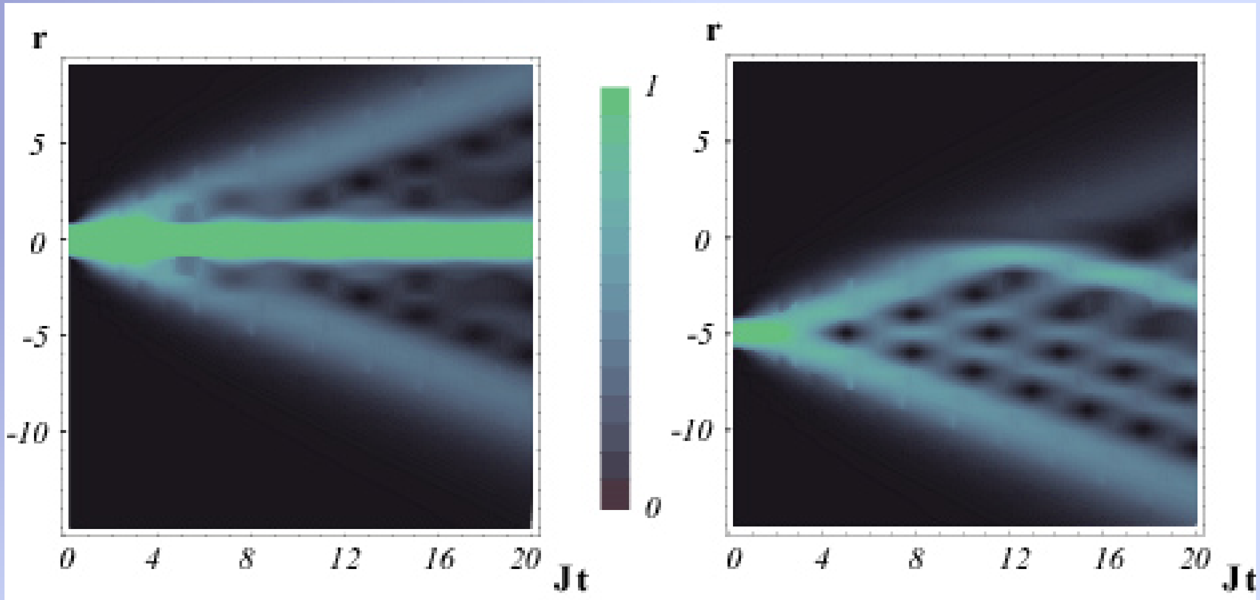
Entanglement distribution

$$C_r(t) = \left| \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} \left\{ e^{i\theta(r-s)} + e^{i\theta(l-s)} g_{r,l}^{(+)} + e^{i\theta(r-l)}\, g_{l,s}^{(-)} \;\; + g_{r,l}^{(+)} g_{l,s}^{(-)} \right\} e^{-iEt} + Res[G(r,s;E_{loc})]\, e^{-iE_{loc}t} \right|$$



$$g_{i,j}^{(\pm)}(E) = \frac{\alpha G_0^{(\pm)}(i,j;E)}{1-\alpha G_0^{(\pm)}(l,l;E)} = \frac{\alpha\, e^{\pm i|\theta||i-j|}}{\pm i \sin|\theta| - \alpha}$$

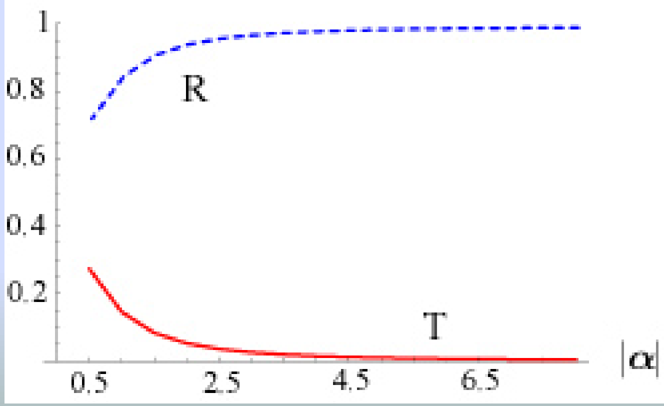
$$|\psi, t=0\rangle = \frac{1}{\sqrt{2}} \Big(|0_{ext}, 1_s\rangle - |1_{ext}, 0_s\rangle \Big)$$



$$C_r \simeq \begin{cases} 1 - \frac{1}{2\alpha^2} & r = 0 \\ \frac{r}{|\alpha|\,\tau} J_r(\tau) & r \neq 0 \end{cases}$$

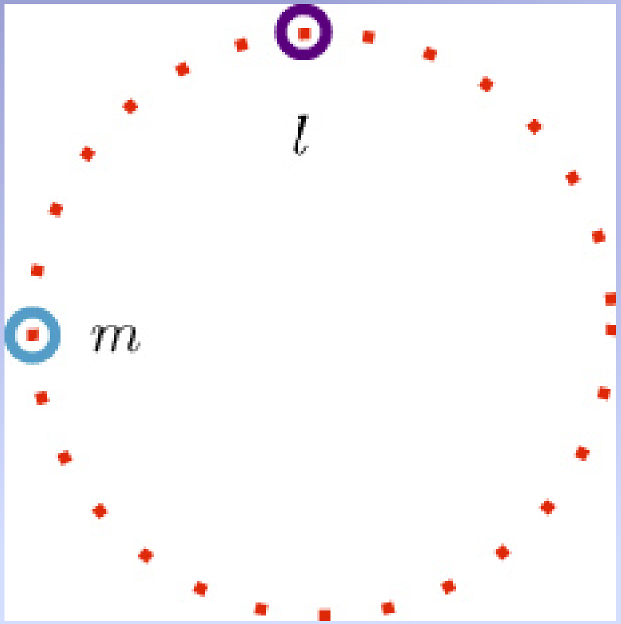
$$C_r = \left| (-1)^s J_{r-s}(\tau) - J_{r+s} - i \frac{J_{r+s+1} + J_{r+s-1}}{2\alpha} \right|$$

Reflection and transmission



$$T = \lim_{t\to\infty} \sum_{r>0} C_r^2(t), \quad R = \lim_{t\to\infty} \sum_{r<0} C_r^2(t)$$

Two defects (in progress….)



$$|\psi_{loc}\rangle = \frac{1}{N}\sum_n \left(e^{-\xi|l-n|} + K\, e^{-\xi|m-n|}\right)\, |n\rangle$$