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Efficient generation of CV entanglement by triply resonant non-degenerate OPA

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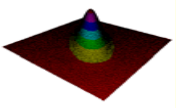
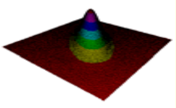


Outline

- Generation of a CV entangled state
  - *Non degenerate OPO hamiltonian and output states (seeded and un-seeded)*
  - *Experimental implementation*
  - *Preliminary measurements*
- Covariance matrix (σ) for a bipartite state of *e.m.* field
  - *Definition*
  - *Properties*
- Measuring the CM



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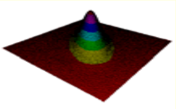
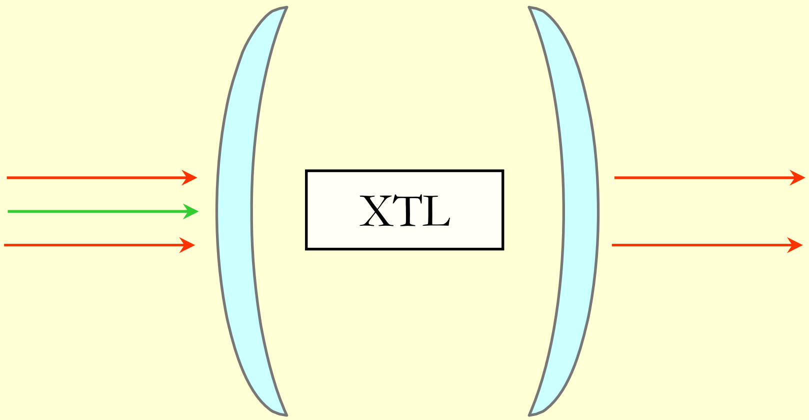


Non-degenerate *seeded* OPA



Weak coherent signal and idler beams injected into an “OPO” cavity

Phase Dependent Amplification

$\phi = 0$
amplification

$\phi = \pi/2$
de-amplification

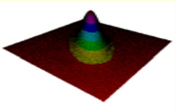
$$H_{int} = \hbar\chi^{(2)}(a^{+}b^{+}a_{p} - a_{p}^{+}a\,b)$$

Non-Linear Hamiltonian

$$S(\zeta) = e^{\frac{\zeta}{2}(a\,b - a^{+}b^{+})}$$

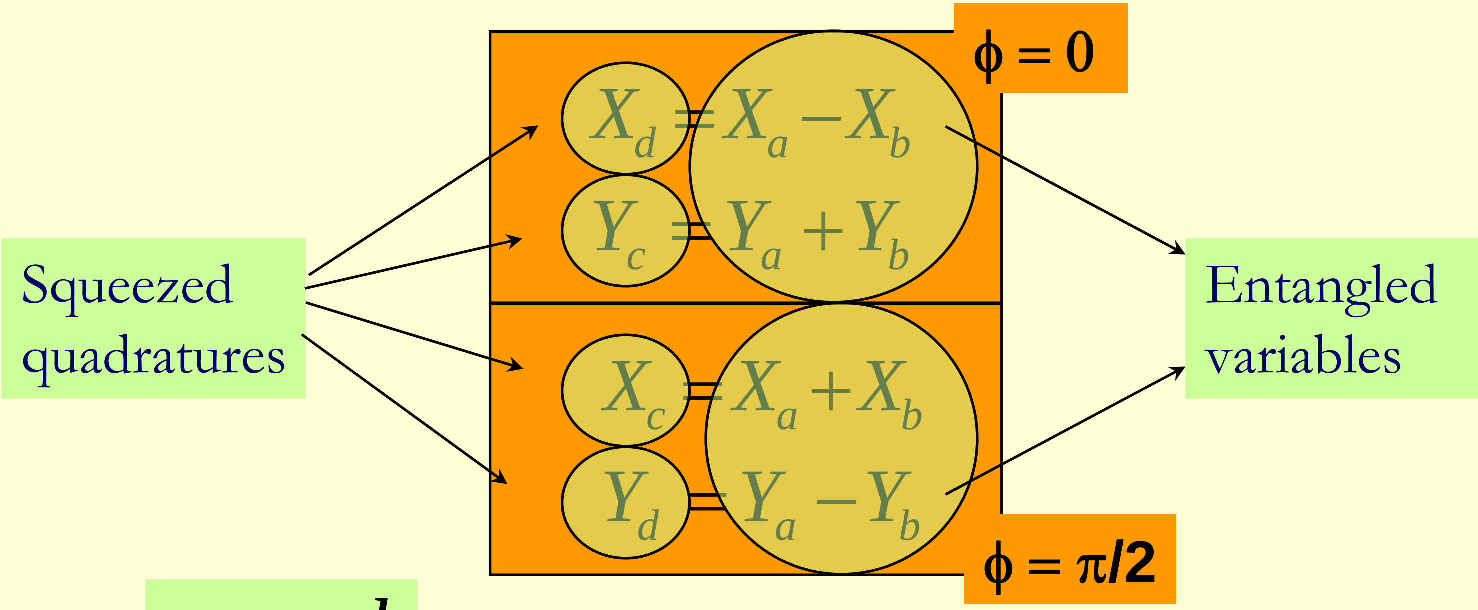
Two Photons
Squeezing Operator

Cavity resonating on both modes guarantees *a* and *b* signal phase locking at the cavity output



Seeded case: “bright CV entanglement”

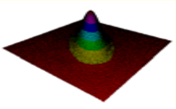
signal and idler (a,b) frequency degenerate ⟷ EPR correlated at the OPA output



$$c = a + b$$

$$d = a - b$$

⟷ **Entangled systems**



Un-seeded case: "the standard squeezer"

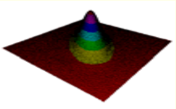
The DOPA operates on the vacua entering the cavity in the modes a and *b*

Modes *c* and *d* are nothing but in the squeezed vacuum state while *a* and *b* are purely thermal (entangled) states.

Non-Linear Hamiltonian

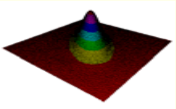
$$S(\zeta) = e^{\frac{\zeta}{2}(c^2 - c^{+2})}$$

Measurements in this condition are preliminary tests of the system. Iff the squeezing is >3dB mode *a* and *b* are entangled



Triply resonant OPO cavity

- A triply resonant OPO guarantees low threshold (i.e. high non-classicality)
  - *Generally obtained by multiple cavity schemes (dual cavities exploiting polarization or dual plano-concave cavities sharing the same crystal)*
- The optical phase delay at the mirror is fixed by the optical lencht inside the crystal for the different modes
  - *Temperature tuning and tilting of the crystal makes possible to "adjust" the phases so to obtain triple resonance*
  - *Two degree of freedom acting independently on three beams (pump, signal, idler)*



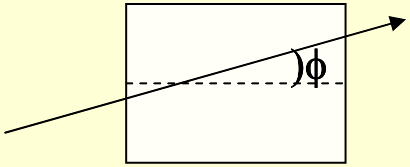
Triple Resonance

Triple resonance: $k_j L_{opt} = m_j \pi; \quad j = p, s, i$ simultaneously

Relative phase mismatches are acquired inside the crystal and depend on the crystal tilt angle ϕ and the temperature T

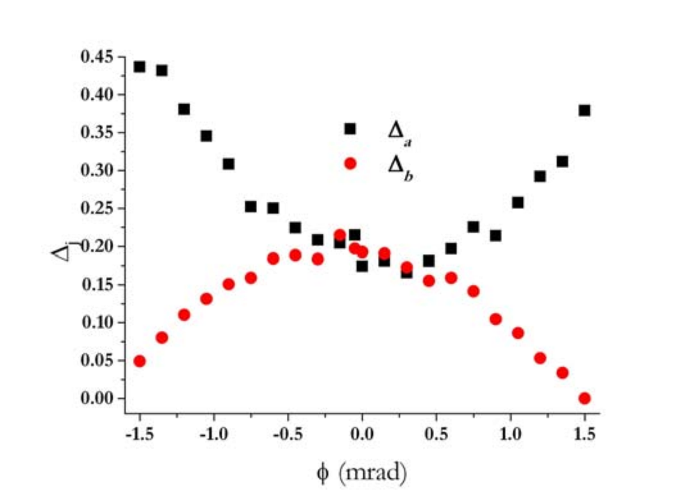
$$\Delta_j(\phi, T) = \frac{C_p - C_{IR\,j}}{FSR_p}$$

C_k represents the position of the resonance inside a given FSR as measured by scanning the cavity lenght



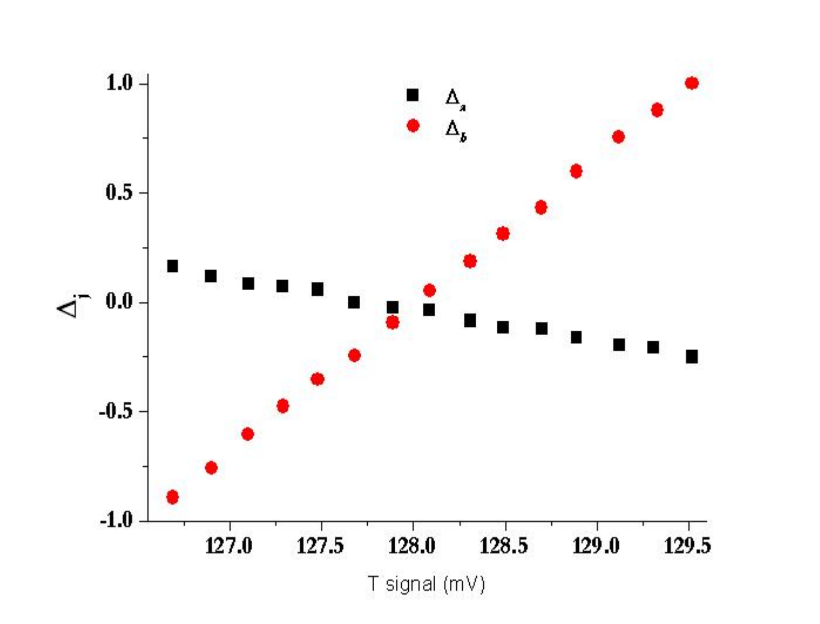
It is necessary to know how the system behaves tuning the temperature and tilting the crystal

Crystal characterization



Varying ϕ, *a* and *b* move in opposite directions with respect to the pump resonance
N.B. Vertex location depends on the temperature T
N.B. max walk-off for an angle of 1mrad < 0.7μm

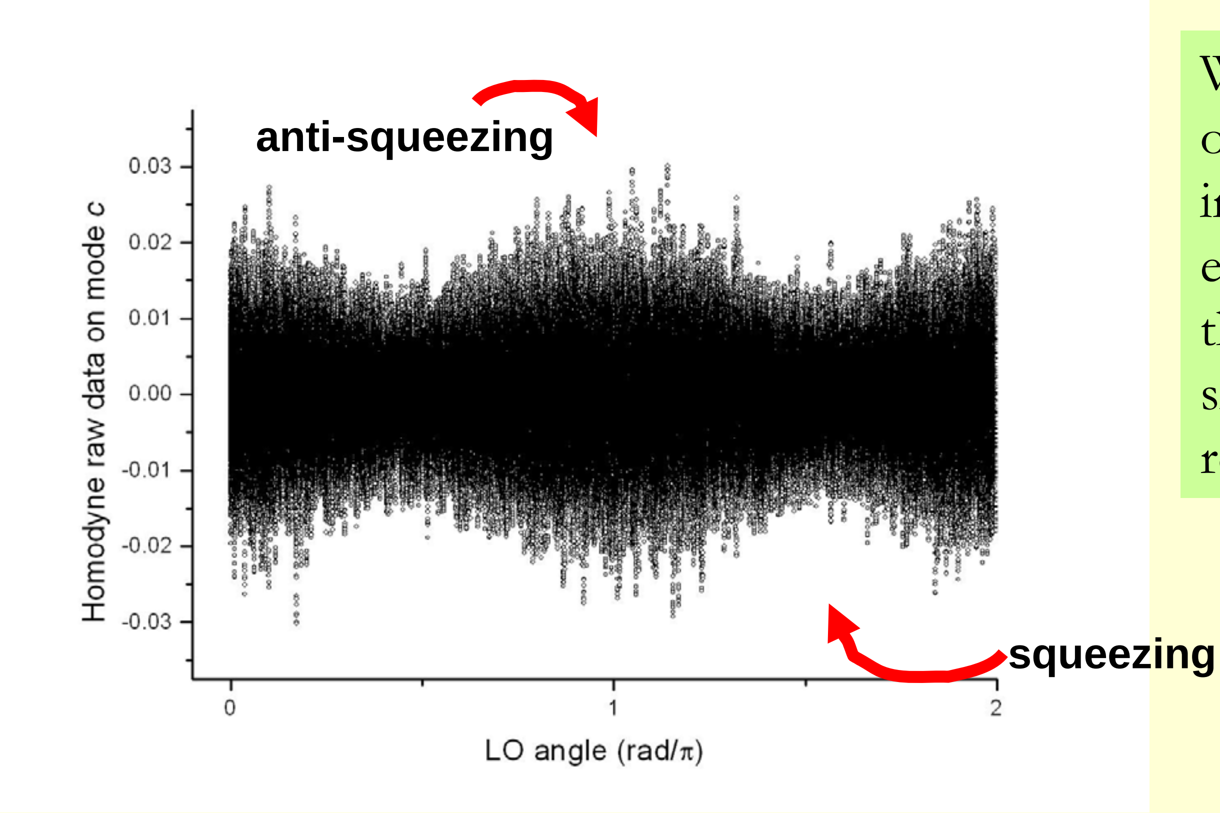
a moves (in *T)* 3 times faster than *b* and in opposite directions



Combining tilt and temperature tuning it is possible to obtain repeatedly triple resonance operation.
Experimentally it is possible to check this condition by interfering the two seeds
Only for simultaneous resonance they interfere with a good visibility (while the cavity is locked on the pump).

Preliminary measurement

As a preliminary test we have operated the NOPA as a "regular" squeezer. If the seed beam is missed mode *c* and *d* are nothing but in the squeezed vacuum state while *a* and *b* are purely thermal (entangled).



We have measured ~2 dB of bare squeezing. Taking into account detection efficiency the squeezing at the cavity mirror is >3dB signaling that *entanglement* is realized.

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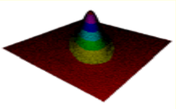


The Covariance matrix σ

field’s quadrature

$$x_{k,\phi} = \frac{k^{+}e^{i\phi} + ke^{-i\phi}}{\sqrt{2}}$$

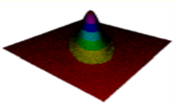
$$X_k = x_{k,0} = \frac{k^{+} + k}{\sqrt{2}}$$

$$Y_k = x_{k,\pi/2} = i\frac{k^{+} - k}{\sqrt{2}}$$

2 separate field modes → a, b

Mutual correlations

$$\begin{pmatrix} \Delta X_a^2 & \Delta X_a Y_a & \Delta X_a X_b & \Delta X_a Y_b \\ \Delta Y_a X_a & \Delta Y_a^2 & \Delta Y_a X_b & \Delta Y_a Y_b \\ \Delta X_b X_a & \Delta X_a Y_b & \Delta X_b^2 & \Delta X_b Y_b \\ \Delta Y_b X_a & \Delta Y_b Y_a & \Delta Y_b X_b & \Delta Y_b^2 \end{pmatrix}$$



σ Properties

$$\begin{pmatrix} a & 0 & c_1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & 0 & 0 & 0 \end{pmatrix}$$

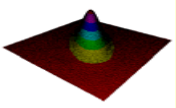
$$\begin{pmatrix} a & 0 & c_1 & 0 \end{pmatrix}$$

$$\sigma = \begin{pmatrix} a & 0 & c_1 & 0 \\ 0 & a & 0 & c_2 \\ c_1 & 0 & b & 0 \\ 0 & c_2 & 0 & b \end{pmatrix}$$

For a Gaussian bipartite system (two mode state)

If the modes **a** and en

If $c_1 \neq 0$ or $c_2 \neq 0$ the two mode are correlated (nothing at this level can be said about the type of correlation)



Uncertainty relation for σ

Once in normal form it is possible to write an uncertainty relation for a state described by σ by considering the determinants $I_1\, I_2\, I_3$ of the four 2x2 sub-matrices and I_4 determinant of σ itself

$$\sigma = \begin{pmatrix} I_4 \end{pmatrix}$$

$$I_1 + I_2 + 2I_3 \leq 4I_4 + \frac{1}{4}$$

Pure bipartite states (MUS) → $I_4 = \frac{1}{16}$ → $I_1 + I_2 + 2I_3 = \frac{1}{2}$

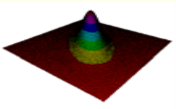
σ for an entangled state

For a separable state the partial transposition of **σ** (obtained by transposing the **b** variables) must obey to the uncertainty relation. This properties give stronger condition that allows to fix a separability criterion.

Separability criterion → $$I_1 + I_2 + 2|I_3| \leq 4I_4 + \frac{1}{4}$$

If σ does not complain with this inequalities the state is

ENTANGLED



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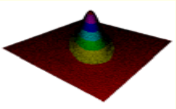

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How to measure σ

In general for measuring σ it is necessary to have a measuring system able to perform simultaneous measurements on couples of independent quadratures so to recover their mutual correlation

σ elements are combination of first and second order statistical momenta of field quadratures → $\sigma = \mathbf{V} - \mathbf{M}$

M=First Statistical Momenta
(not requiring simultaneous measurements)

V=Second Statistical Momenta
(requiring simultaneous measurements)

In case of cross polarized frequency degenerate modes σ can be rewritten in terms of a set of 5 (of 6) independent modes made by suitable combinations of the original ones

$$a \qquad b$$

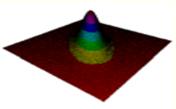
Original modes

$$c = \frac{a+b}{\sqrt{2}} \qquad d = \frac{a-b}{\sqrt{2}}$$

Linear polarized combinations

$$e = \frac{ia+b}{\sqrt{2}} \qquad f = \frac{ia-b}{\sqrt{2}}$$

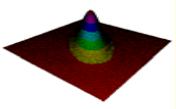
Circurarly polarized combinations



Determining **V**

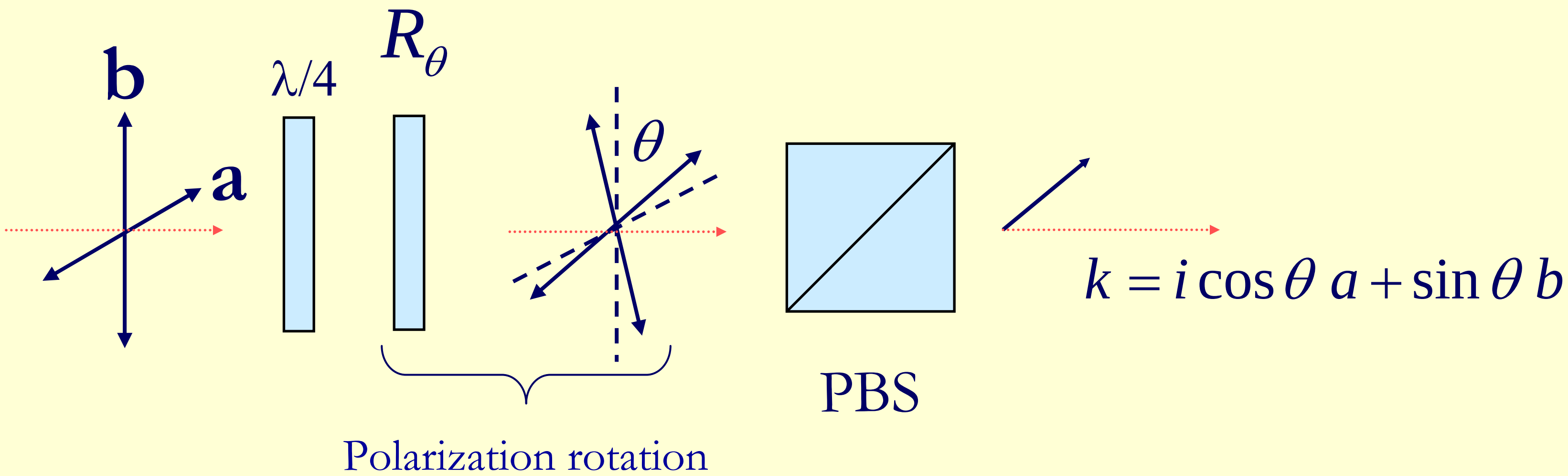
$$\mathbf{V}=\frac{1}{2}\begin{pmatrix} 2\langle x_a^2\rangle & \langle z_a^2\rangle-\langle t_a^2\rangle & \langle x_c^2\rangle-\langle x_d^2\rangle & \langle y_e^2\rangle-\langle y_f^2\rangle \\ \langle z_a^2\rangle-\langle t_a^2\rangle & 2\langle y_a^2\rangle & \langle x_f^2\rangle-\langle x_e^2\rangle & \langle y_c^2\rangle-\langle y_d^2\rangle \\ \langle x_c^2\rangle-\langle x_d^2\rangle & \langle x_f^2\rangle-\langle x_e^2\rangle & 2\langle x_b^2\rangle & \langle z_b^2\rangle-\langle t_b^2\rangle \\ \langle y_e^2\rangle-\langle y_f^2\rangle & \langle y_c^2\rangle-\langle y_d^2\rangle & \langle z_b^2\rangle-\langle t_b^2\rangle & 2\langle y_b^2\rangle \end{pmatrix}$$

Moreover
$$\begin{aligned}\langle y_e^2\rangle-\langle y_f^2\rangle &= 2\langle y_e^2\rangle-\left(\langle x_a^2\rangle+\langle y_b^2\rangle\right)\\ \langle x_f^2\rangle-\langle x_e^2\rangle &= \left(\langle x_b^2\rangle+\langle y_a^2\rangle\right)-2\langle x_e^2\rangle\end{aligned}$$



Realizing *a*, *b*, *c*, *d*, *e*, and *f* modes

Starting from the modes **a** and **b** (frequency degenerate, orthogonally polarized) it is possible to generate the mode **k** by inserting a polarization rotator (λ/2 retardation plate) and a λ/4 plate



M.G Raymer and A.C. Funk Phys. Rev. A **61**:015801 (1999)

Conclusions and Perspectives

- A non degenerate triply resonant OPA is used for generating CV entangled beams
- Entaglement can be characterized by the Covariance Matrix σ written in terms of quadratures relative to linear combination of the NOPA modes
- A single homodyne detector in the case of CV entangled frequency degenerate but cross polarized beams is sufficient to measure σ
- The experimental procedure is less complicate than standard techniques where simultaneous measurements are unavoidable
- The experimental measure of σ is … *in progress*