Dynamical Formulation of (Anti-)Bunching

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Identical Particles in Quantum Mechanics

$$\psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = \pm\psi(\boldsymbol{r}_2, \boldsymbol{r}_1)$$

"+" : Bose-Einstein statistics ——— "bosons" (photons, …)

"−" : Fermi-Dirac statistics ——— "fermions" (electrons, neutrons, protons, …)

Two fermions (in the same spin states)
at the same position $\boldsymbol{r}_1 = \boldsymbol{r}_2 = \boldsymbol{r}$

$$\psi(\boldsymbol{r}, \boldsymbol{r}) = -\psi(\boldsymbol{r}, \boldsymbol{r}) = 0$$ —— antibunching

Fermions avoid each other.

The spin degree of freedom is omitted in this page.

Motivations

- (Anti-)Bunching
  - HBT effects in Astronomy, Heavy-Ion Collisions
  - Generation of Entangled Photons

- Coherence Properties of Multi-Particle Systems
  - Coherence Lengths (Longitudinal/Lateral)
  - Entanglement, …

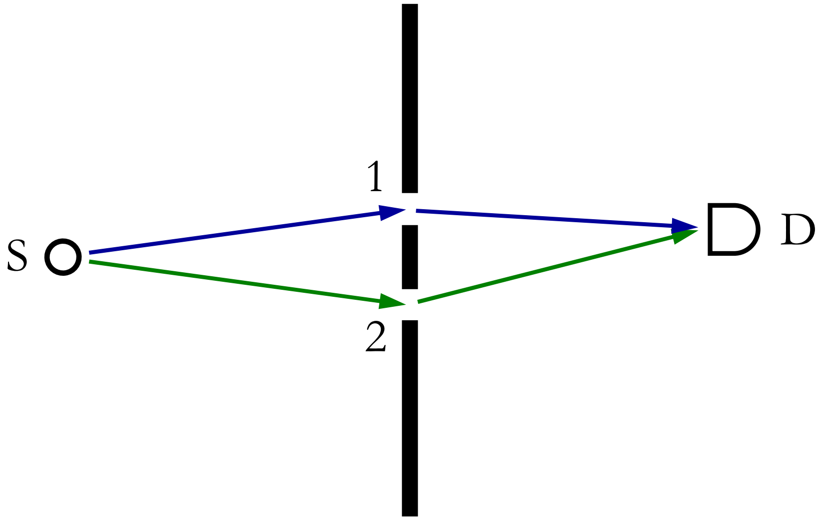
- Dynamical Model for Coincidence Exp.
  - Particle Source as a Dynamical System
  - Characteristics of the Source
    - Temperature, Size, …

The spin degree of freedom is omitted in this page.

First-/Second-Order Interference

Double-Slit Experiment

single detector

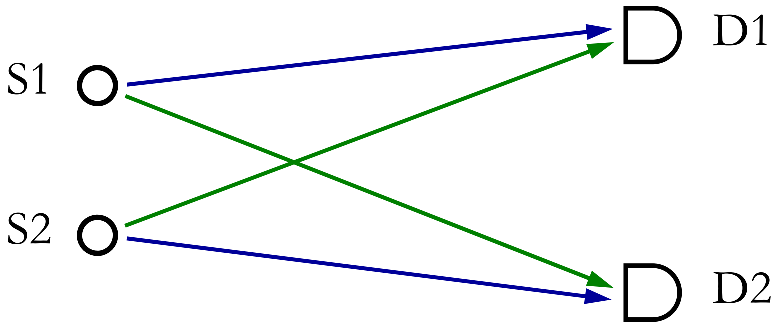


$$|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1^*\psi_2|\cos(\phi_1 - \phi_2)$$

phase difference

Coincidence Experiment

two detectors



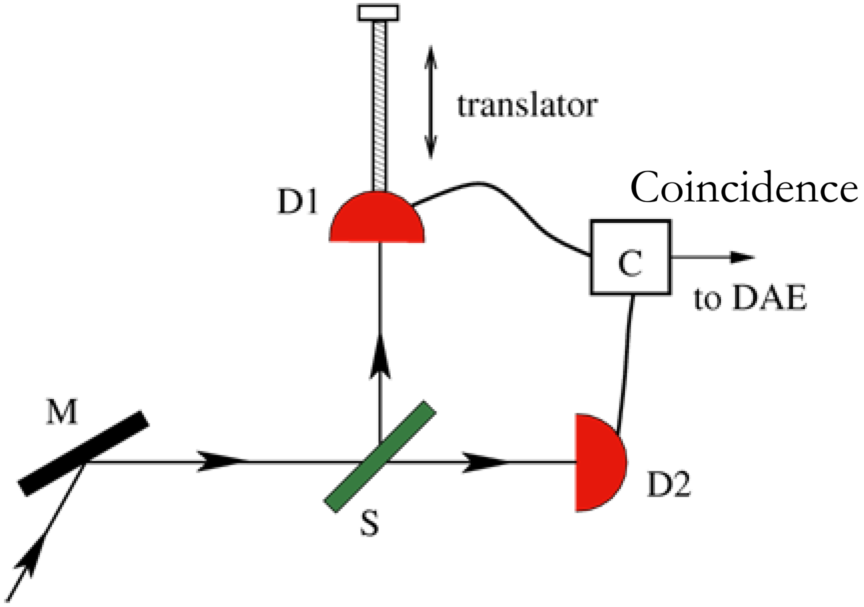
fermion/boson

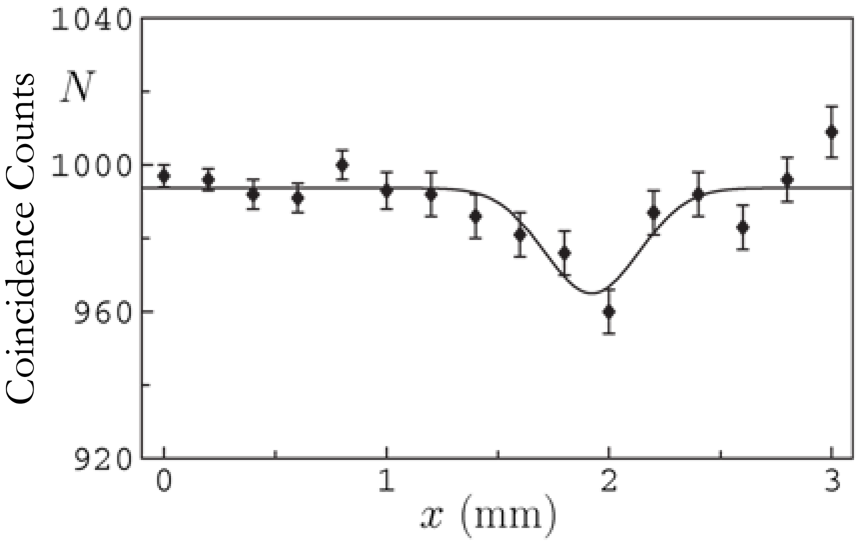
$$\frac{1}{\sqrt{2}}\Big(\psi_{S1}(1)\psi_{S2}(2) \mp \psi_{S1}(2)\psi_{S2}(1)\Big)$$

$$\xrightarrow[\text{D1=D2}]{} \begin{cases} 0 & \ldots\text{ antibunching} \\ \sqrt{2}\,\psi_{S1}(\text{D})\psi_{S2}(\text{D}) & \\ & \ldots\text{ bunching} \end{cases}$$

Neutron Antibunching Experiment

M. Iannuzzi, A. Orecchini, F. Sacchetti, P. Facchi, and S. Pascazio, PRL **96**, 080402 (2006).



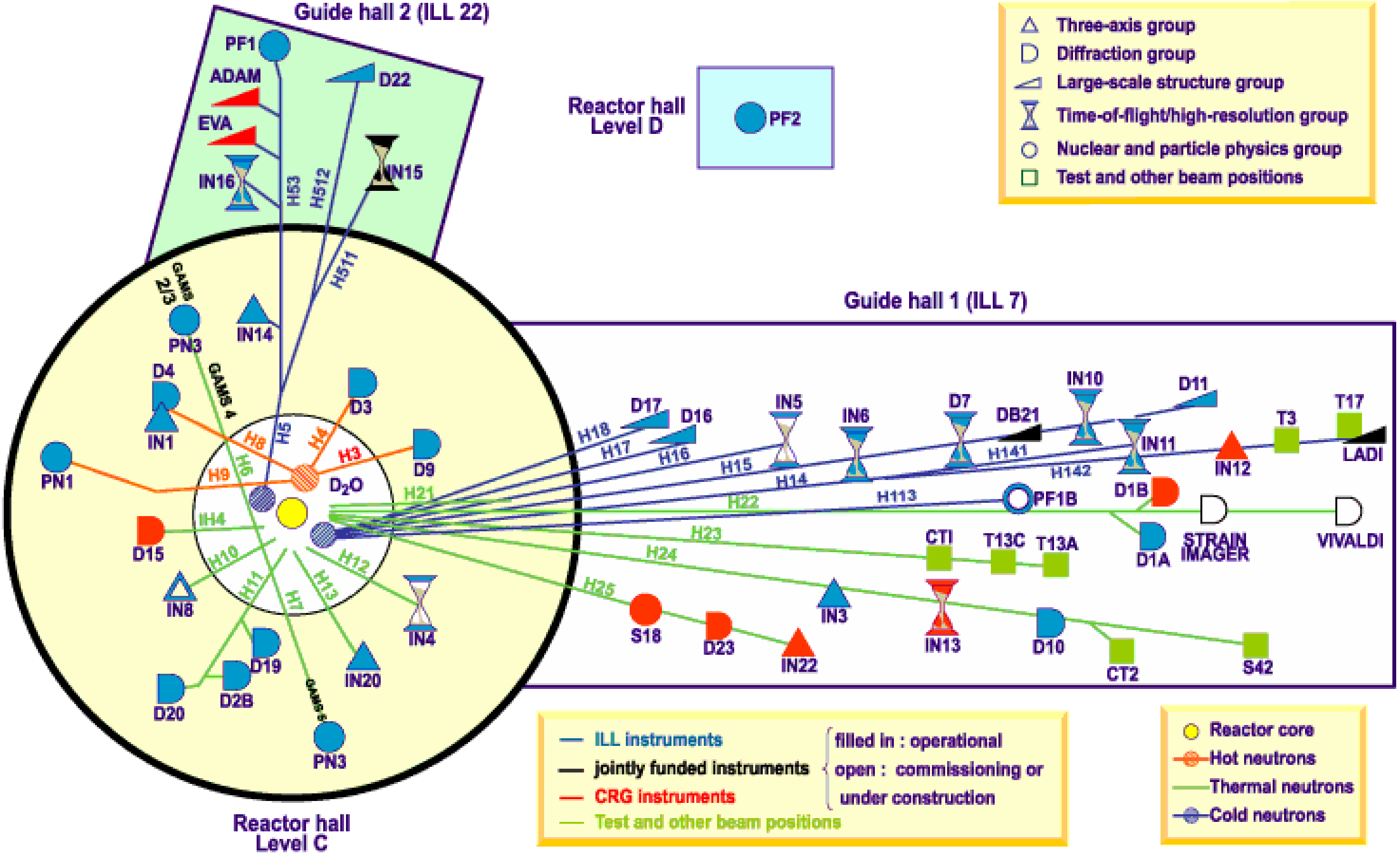


Several experiments with electrons:

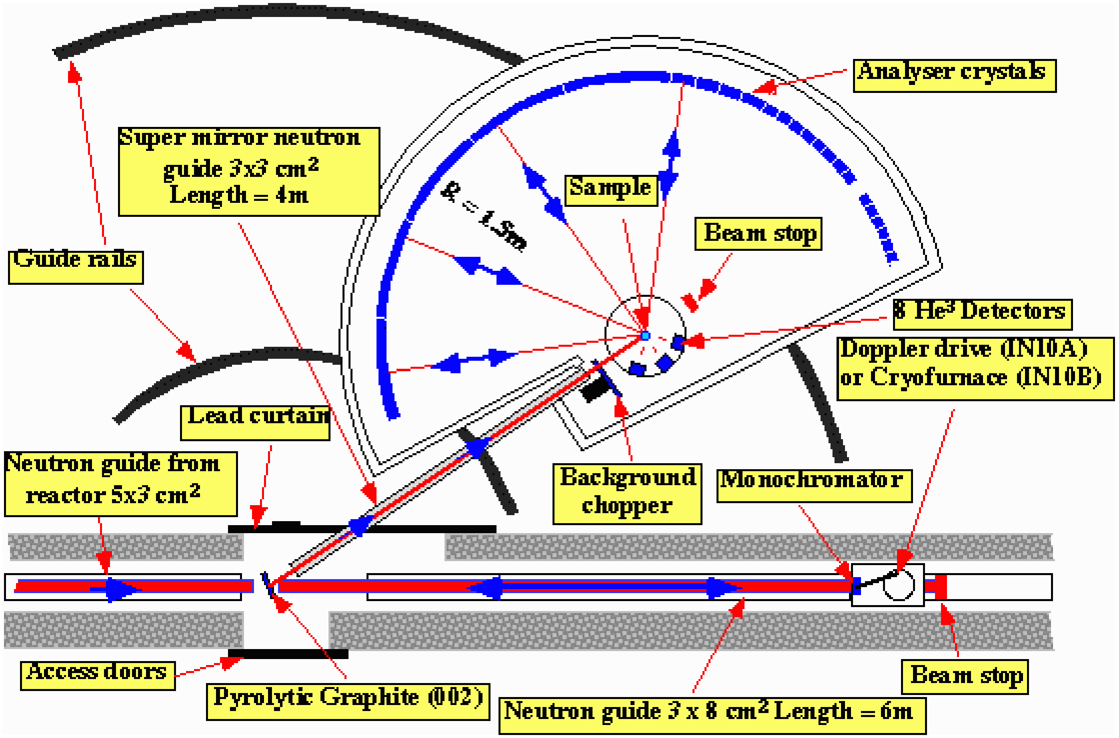
Liu *et al.* (1998), Henny *et al.* (1999), Oliver *et al.* (1999), Kiesel *et al.* (2002), Ji *et al.* (2003).

Institut Laue-Langevin (ILL), Grenoble

Instruments Map

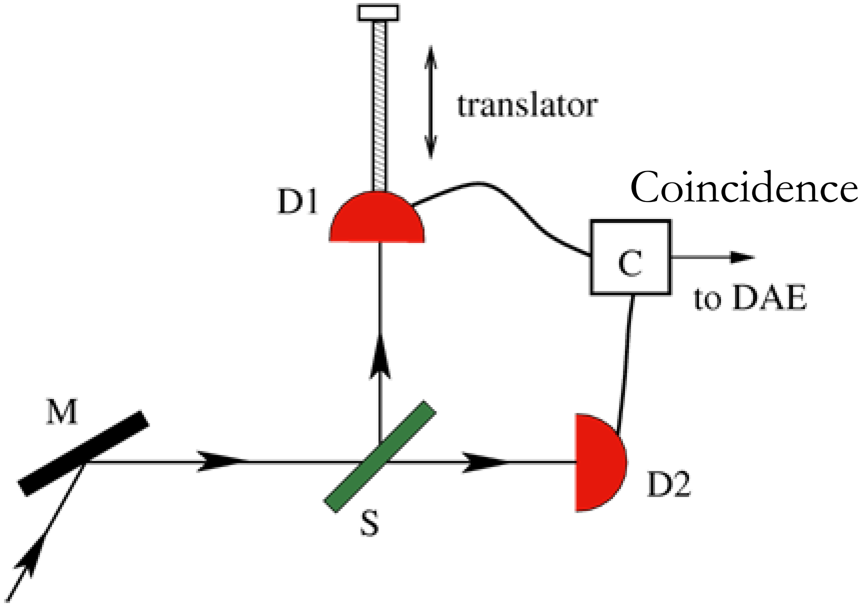


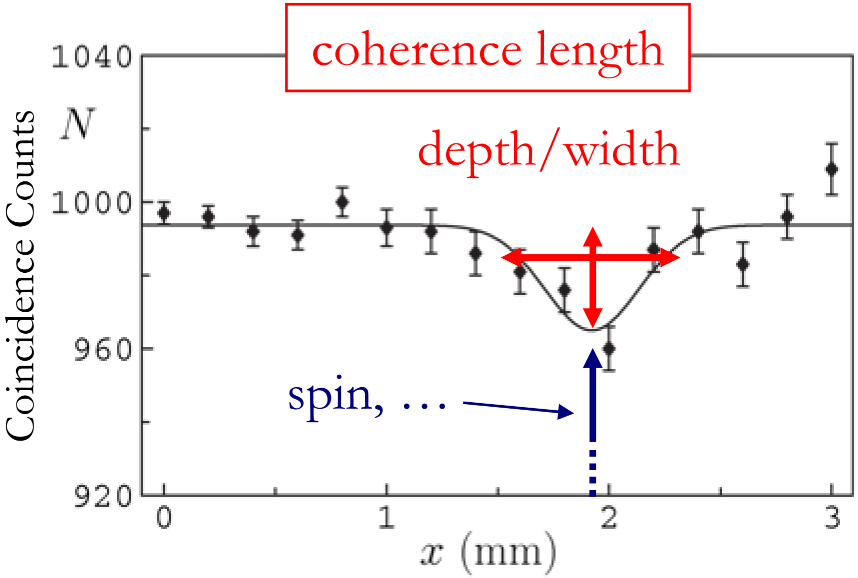
IN10@ILL


Analyser crystals
Super mirror neutron guide 3x3 cm²
Length = 4m
Sample
R = 1.5m
Beam stop
Guide rails
8 He³ Detectors
Doppler drive (IN10A) or Cryofurnace (IN10B)
Lead curtain
Neutron guide from reactor 5x3 cm²
Background chopper
Monochromator
Access doors
Pyrolytic Graphite (002)
Neutron guide 3 x 8 cm² Length = 6m
Beam stop

Neutron Antibunching Experiment

M. Iannuzzi, A. Orecchini, F. Sacchetti, P. Facchi, and S. Pascazio, PRL **96**, 080402 (2006).





coherence length

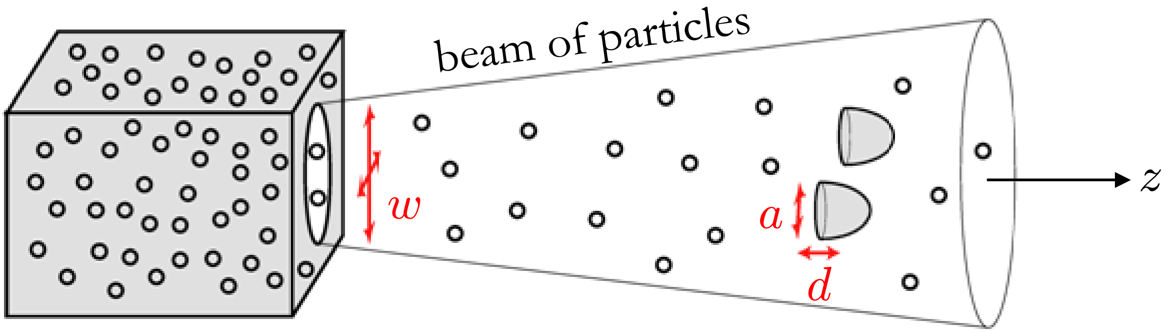
depth/width

spin, …

Several experiments with electrons:

Liu *et al.* (1998), Henny *et al.* (1999), Oliver *et al.* (1999), Kiesel *et al.* (2002), Ji *et al.* (2003).

Dynamical Formulation of Coincidence Exp.



Reservoir
(temperature T)

Detectors
(resolution a, d)

- Hamiltonian $H = H_{\text{beam}} + H_{\text{reservoir}} + \lambda V$ in 3D

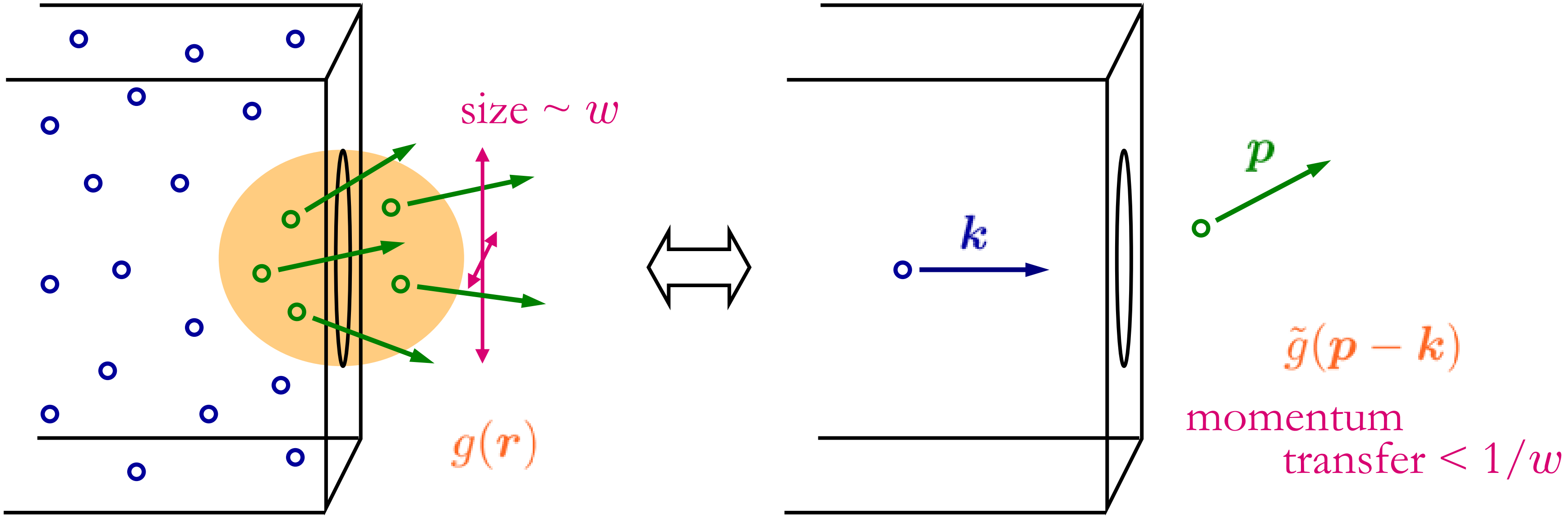
$$\begin{cases} H_{\text{beam}} = \sum_{\sigma} \int \mathrm{d}^3\boldsymbol{p}\, \varepsilon_{\boldsymbol{p}} c^{\dagger}_{\boldsymbol{p}\sigma} c_{\boldsymbol{p}\sigma}, \quad H_{\text{reservoir}} = \sum_{\sigma} \int \mathrm{d}^3\boldsymbol{k}\, \omega_{\boldsymbol{k}} a^{\dagger}_{\boldsymbol{k}\sigma} a_{\boldsymbol{k}\sigma} \\ V = \sum_{\sigma} \int \mathrm{d}^3\boldsymbol{p} \int \mathrm{d}^3\boldsymbol{k} \left(T_{\boldsymbol{p}\boldsymbol{k}} c^{\dagger}_{\boldsymbol{p}\sigma} a_{\boldsymbol{k}\sigma} + \text{h.c.} \right) \end{cases}$$

$$\{c_{\boldsymbol{p}\sigma}, c^{\dagger}_{\boldsymbol{p}'\sigma'}\} = \delta_{\sigma\sigma'} \delta^3(\boldsymbol{p} - \boldsymbol{p}')$$

$$\{a_{\boldsymbol{k}\sigma}, a^{\dagger}_{\boldsymbol{k}'\sigma'}\} = \delta_{\sigma\sigma'} \delta^3(\boldsymbol{k} - \boldsymbol{k}')$$

- Initial State (Temp. $T \otimes$ Vacuum) $\xrightarrow[\uparrow \text{ Hamiltonian}]{t \to \infty}$ Stationary Beam

Emission Hamiltonian



Configuration Space

Momentum Space

$$V = \sum_{\sigma} \int \mathrm{d}^3\boldsymbol{p} \int \mathrm{d}^3\boldsymbol{k}\, c^{\dagger}_{\boldsymbol{p}\sigma} \langle \boldsymbol{p} | f(\underset{\uparrow\text{ monochromator}}{p_z}) g(\boldsymbol{r}) \theta(k_z) | \boldsymbol{k} \rangle a_{\boldsymbol{k}\sigma} + \mathrm{h.c.}$$

Window Size ⟺ Beam Divergence

Correlation Functions

- First-Order Correlation (in interaction picture)

$$\langle \psi_\sigma^\dagger(\boldsymbol{r},t)\psi_{\sigma'}(\boldsymbol{r}',t)\rangle = \delta_{\sigma\sigma'}\rho_t^{(1)}(\boldsymbol{r}|\boldsymbol{r}'), \quad \psi_\sigma(\boldsymbol{r},t) = \int \frac{\mathrm{d}^3\boldsymbol{p}}{\sqrt{(2\pi)^3}} c_{\boldsymbol{p}\sigma} \mathrm{e}^{\mathrm{i}(\boldsymbol{p}\cdot\boldsymbol{r}-\varepsilon_{\boldsymbol{p}}t)}$$

- Spin-Summed Two-Particle Distribution

$$\rho_t^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \sum_{\sigma_1,\sigma_2} \langle \psi_{\sigma_1}^\dagger(\boldsymbol{r}_1,t)\psi_{\sigma_2}^\dagger(\boldsymbol{r}_2,t)\psi_{\sigma_2}(\boldsymbol{r}_2,t)\psi_{\sigma_1}(\boldsymbol{r}_1,t)\rangle$$

$$= 4\rho_t^{(1)}(\boldsymbol{r}_1|\boldsymbol{r}_1)\rho_t^{(1)}(\boldsymbol{r}_2|\boldsymbol{r}_2) - 2\rho_t^{(1)}(\boldsymbol{r}_1|\boldsymbol{r}_2)\rho_t^{(1)}(\boldsymbol{r}_2|\boldsymbol{r}_1)$$

Wick's Theorem

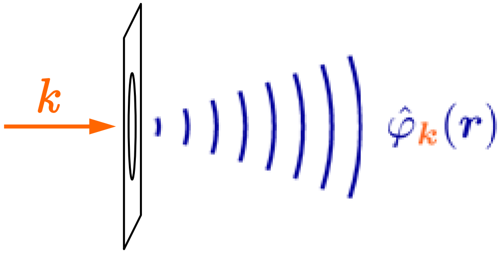
- Detector Resolution

$$\bar{\rho}_t^{(2)}(\bar{\boldsymbol{r}}_1,\bar{\boldsymbol{r}}_2) = \int \mathrm{d}^3\boldsymbol{r}_1\, R_{\bar{\boldsymbol{r}}_1,\boldsymbol{d}}(\boldsymbol{r}_1) \int \mathrm{d}^3\boldsymbol{r}_2\, R_{\bar{\boldsymbol{r}}_2,\boldsymbol{d}}(\boldsymbol{r}_2)\rho_t^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2)$$

In the following, we consider the Gaussian resolution $R_{\bar{\boldsymbol{r}},\boldsymbol{d}}(\boldsymbol{r})$ centered at $\bar{\boldsymbol{r}}$ with the width $\boldsymbol{d} = (a,a,d)$

Structure of the Correlation Functions

- Mixture over the Reservoir Modes



$$\rho_t^{(1)}(\boldsymbol{r}_1|\boldsymbol{r}_2) \xrightarrow{t\to\infty} \lambda^2 \int \mathrm{d}^3\boldsymbol{k}\, N(\omega_{\boldsymbol{k}}) \underbrace{\hat{\varphi}_{\boldsymbol{k}}^*(\boldsymbol{r}_1)\hat{\varphi}_{\boldsymbol{k}}(\boldsymbol{r}_2)}$$

"pure" wave from mode $\boldsymbol{k}$

mixture over the Fermi distribution

$$\hat{\varphi}_{\boldsymbol{k}}(\boldsymbol{r}) = \int \frac{\mathrm{d}^3\boldsymbol{p}}{\sqrt{(2\pi)^3}\,\mathrm{i}} \frac{T_{\boldsymbol{pk}}\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}}{\varepsilon_{\boldsymbol{p}} - \omega_{\boldsymbol{k}} - \mathrm{i}0^+}$$

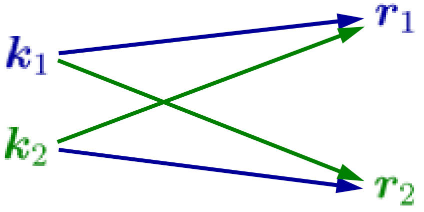
- Triplet/Singlet Contributions

$$\rho_t^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2) = 4\rho_t^{(1)}(\boldsymbol{r}_1|\boldsymbol{r}_1)\rho_t^{(1)}(\boldsymbol{r}_2|\boldsymbol{r}_2) - 2\rho_t^{(1)}(\boldsymbol{r}_1|\boldsymbol{r}_2)\rho_t^{(1)}(\boldsymbol{r}_2|\boldsymbol{r}_1)$$

$$\xrightarrow{t\to\infty} \lambda^4 \int \mathrm{d}^3\boldsymbol{k}_1 \int \mathrm{d}^3\boldsymbol{k}_2\, N(\omega_{\boldsymbol{k}_1})N(\omega_{\boldsymbol{k}_2}) \left(3\left|\Psi_{\boldsymbol{k}_1\boldsymbol{k}_2}^{(-)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\right|^2 + \left|\Psi_{\boldsymbol{k}_1\boldsymbol{k}_2}^{(+)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\right|^2\right)$$

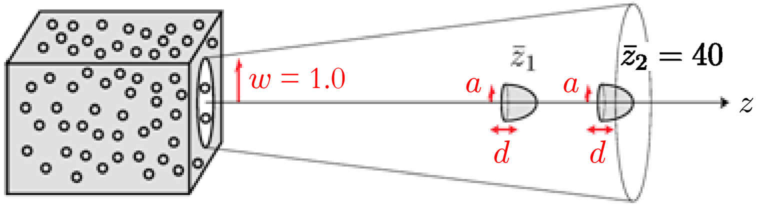
$$\Psi_{\boldsymbol{k}_1\boldsymbol{k}_2}^{(\pm)}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{1}{\sqrt{2}}\left(\hat{\varphi}_{\boldsymbol{k}_1}(\boldsymbol{r}_1)\hat{\varphi}_{\boldsymbol{k}_2}(\boldsymbol{r}_2) \pm \hat{\varphi}_{\boldsymbol{k}_1}(\boldsymbol{r}_2)\hat{\varphi}_{\boldsymbol{k}_2}(\boldsymbol{r}_1)\right)$$

$$\begin{cases} \text{Anti-Symmetric Wave Func.} \longleftrightarrow \text{Triplet Spin} \longrightarrow 3 \\ \text{Symmetric Wave Func.} \longleftrightarrow \text{Singlet Spin} \longrightarrow 1 \end{cases}$$

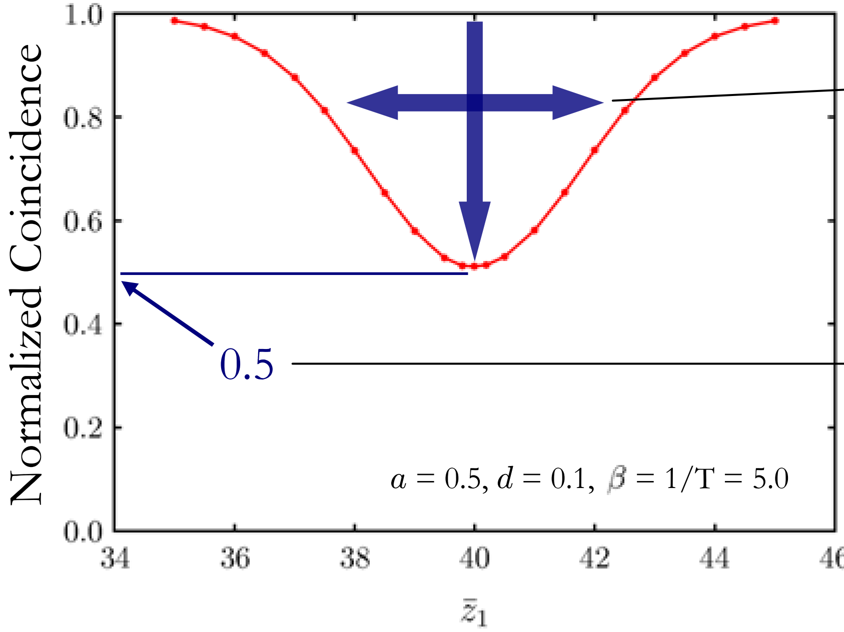


Depth/Width

(with fine resolution)



monochromator $k_0 = 5.0,\ \delta k_z = 0.5$
Fermi level $E_{\mathrm{F}} = 15 \quad (\hbar = m = 1)$



width of the spectrum

$$\rho_t^{(1)}(\boldsymbol{r}_1|\boldsymbol{r}_2) \sim \int \mathrm{d}^3\boldsymbol{k}\, N(\omega_{\boldsymbol{k}}) f^2(k_z) \cdots \mathrm{e}^{-\mathrm{i}k_z(\bar z_1 - \bar z_2)}$$

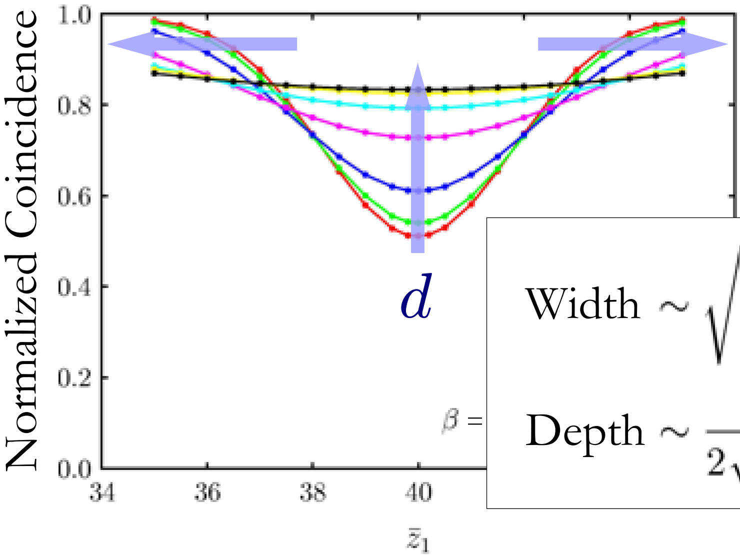
0.5

$$= 1 - \frac{3}{4} + \frac{1}{4}$$

singlet … bunching

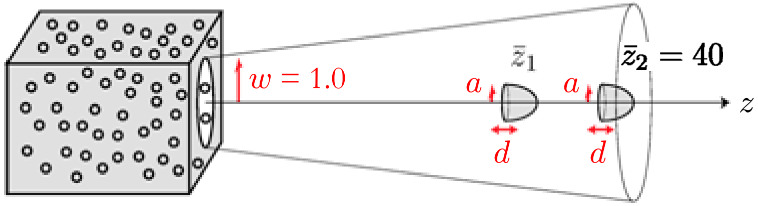
triplet … antibunching

Detector Resolution



$$\text{Width} \sim \sqrt{\frac{1}{2(\delta k_z)^2} + 2d^2}$$

$$\text{Depth} \sim \frac{1}{2\sqrt{1 + 4(\delta k_z)^2 d^2}}$$



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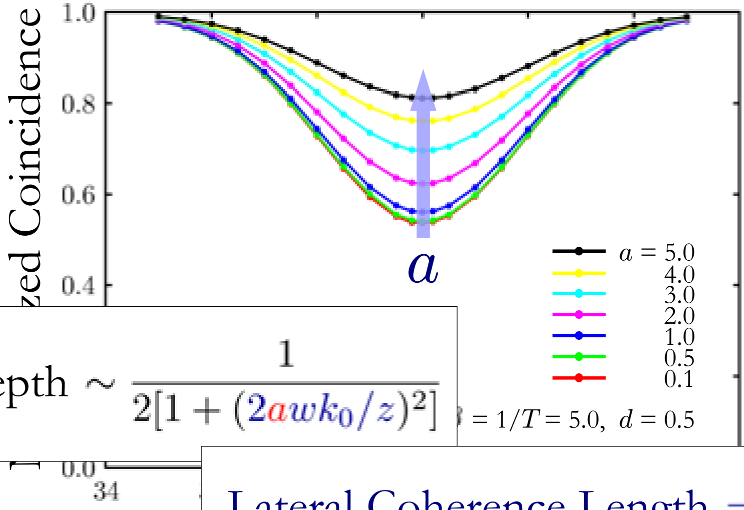
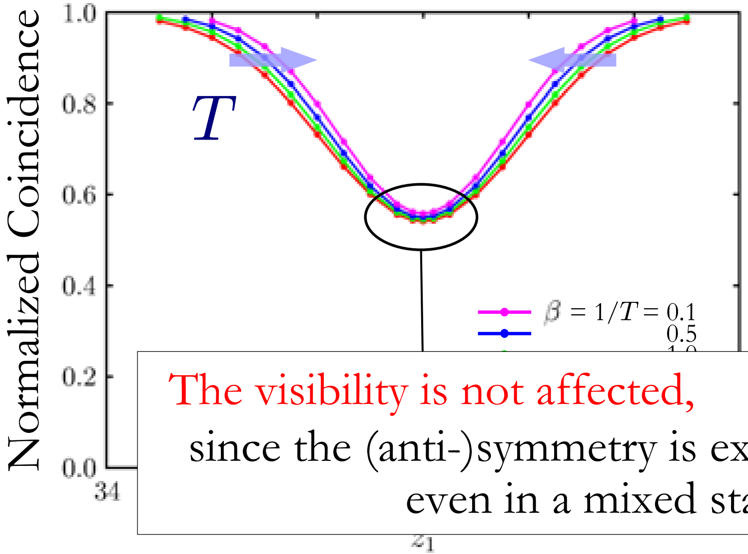


$$\text{Depth} \sim \frac{1}{2[1 + (2awk_0/z)^2]}$$

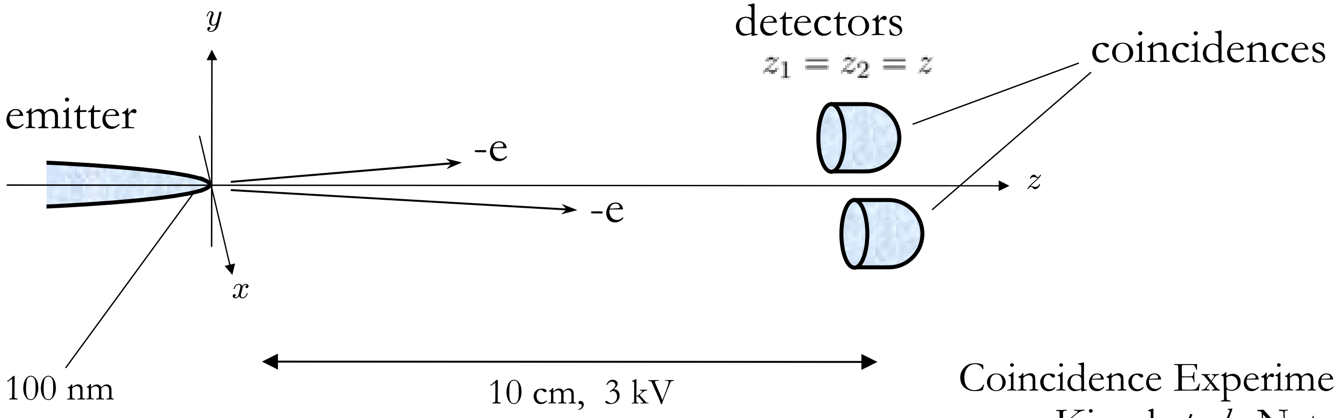
Lateral Coherence Length $\dfrac{z}{2wk_0}$

Temperature



The visibility is not affected,
since the (anti-)symmetry is exact
even in a mixed state.

Field Emission



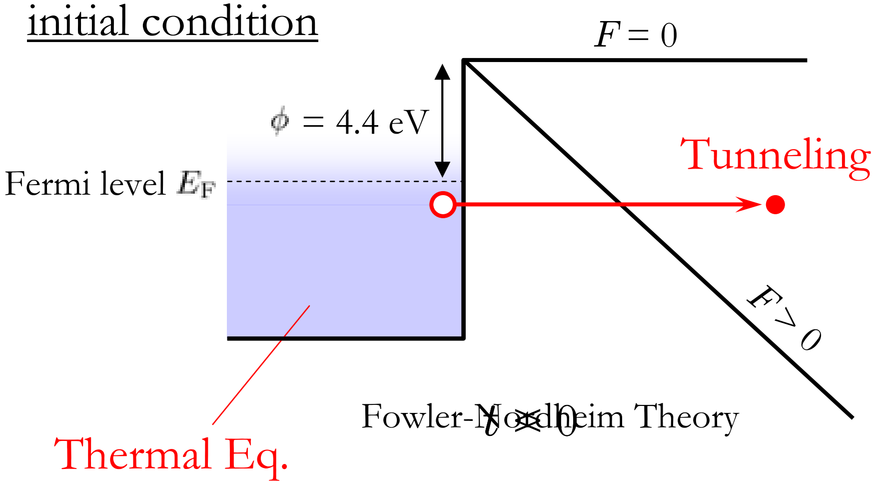
R = 100 nm

work func. ϕ = 4.4 eV

T = 300 K

Coincidence Experiment:
Kiesel *et al.*, Nature **418**, 392 (2002)
(with a slightly different setup)

initial condition



- tunneling Hamiltonian*
- initial-value problem
- 3D
- acceleration
- (Coulomb force neglected)

*J. Bardeen, PRL **6**, 57 (1961); etc.

$$\bar{\rho}^{(2)}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2)=\bar{\rho}^{(1)}(\boldsymbol{r}_1)\bar{\rho}^{(1)}(\boldsymbol{r}_2)\left[1-\frac{\pi}{4}\frac{t_c}{\tau}e^{-|t_1-t_2|/\tau}\frac{1}{1+a^2(w^2/\ell_c^2)/[a^2+\Delta^2(z)]}\times\exp\left(-\frac{(x_1-x_2)^2+(y_1-y_2)^2}{4[1+a^2/\Delta^2(z)]\{a^2+(\ell_c^2/w^2)[a^2+\Delta^2(z)]\}}\right)\right]$$

$z_1=z_2=z$
$T=0$

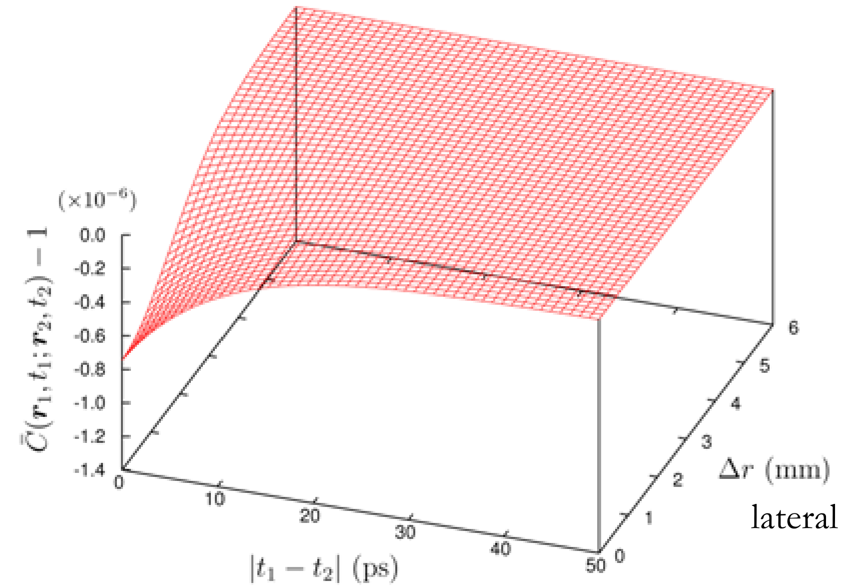
response time of the detectors

$$\begin{cases}\ell_c^2=3S_\mathrm{F}/2\kappa_\mathrm{F}^2,\\ t_c=3\hbar S_\mathrm{F}/\phi,\end{cases}\qquad \Delta^2(z)=w^2+(1+\ell_c^2/w^2)\frac{(z+z_\mathrm{D})^2}{4\kappa_\mathrm{F}z_\mathrm{D}},$$

beam expansion

$$\begin{cases}S_\mathrm{F}=\dfrac{2\sqrt{2m\phi^3}}{3\hbar eF}\\ \kappa_\mathrm{F}=\sqrt{\dfrac{2m\phi}{\hbar^2}}\end{cases}$$

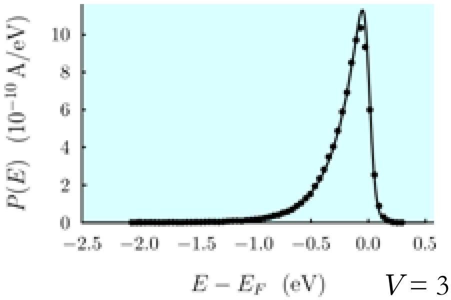
—— tunnel action



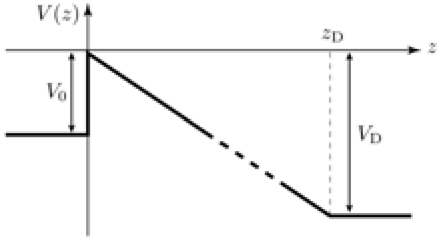
lateral

$$\bar{C}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2)=\frac{\bar{\rho}^{(2)}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2)}{\bar{\rho}^{(1)}(\boldsymbol{r}_1,t_1)\bar{\rho}^{(1)}(\boldsymbol{r}_2,t_2)},\quad \Delta r=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Energy Spectrum*



$V=3.8$ kV
$S_\mathrm{F}\sim 6.81$



$$\begin{cases}S_\mathrm{F}=7\\ w=10\,\mathrm{nm},\ T=0\\ z=z_1=z_2=10\,\mathrm{cm}\\ z_\mathrm{D}=0.1\,\mu\mathrm{m}\end{cases}$$

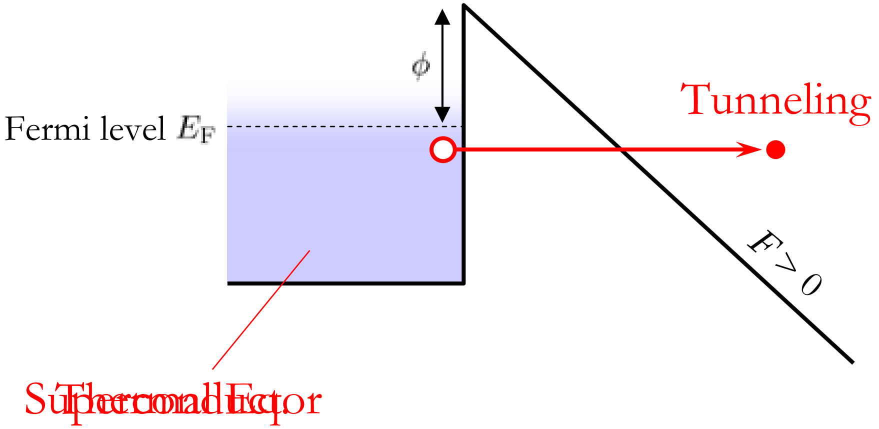
$$\begin{cases}t_c=3.1\,\mathrm{fs}\\ \ell_c=0.30\,\mathrm{nm}\\ \Delta(z)=1.5\,\mathrm{mm}\end{cases}$$

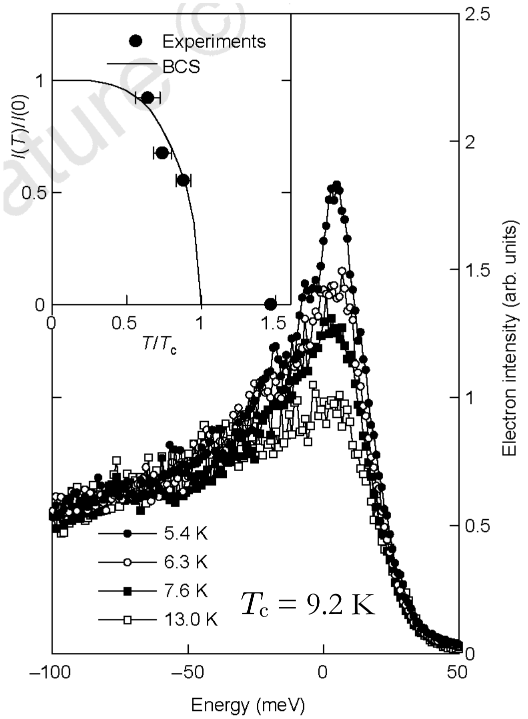
$$\begin{cases}a=1\,\mathrm{mm}\\ \tau=10\,\mathrm{ps}\end{cases}$$

- Magnification of the beam would be necessary.
- Coulomb force ?

*K. Nagaoka *et al.* (Waseda, Tokyo), Appl. Surf. Sci. **182**, 12 (2001).

Field Emission from a Superconductor





K. Nagaoka *et al.* (Waseda, Tokyo)
Nature **396**, 557 (1998).

- sharp energy spectrum
  - ⇨ long coherence time/length expected
- Cooper pairs
  - ⇨ resource of entanglement ?

To be explored …

Summary

- Dynamical Formulation of Coincidence Exp.
  - The reservoir is treated as a quantized dynamical system.

    ⇨ The temperature is naturally introduced.
  - The emission process is described dynamically.

    ⇨ The stationary beam is prepared naturally.
  - Detector Resolution / Lateral Effect / Temperature Effect

- An Interesting Future Subject
  - Thermal Reservoir ⇨ Superconductor*

    Cooper-Pair Correlation ⟶ Entanglement ??

*Nagaoka *et al.* (Waseda, Tokyo), Nature **396**, 557 (1998).