RELATIVISTIC QUANTUM MECHANICS AND PATH INTEGRALS (AA 20-21)

Fiorenzo Bastianelli

Program

lecture 1 Introduction. Relativistic quantum mechanics (first quantization). Review of special relativity: Lorentz group, Poincaré group, Minkowski spacetime, tensors.
lecture 2 Review of the Schrödinger equation. The Klein-Gordon equation. Plane wave solutions. Conserved current. Failure of probabilistic interpretation.
lecture 3 Yukawa potential.
lecture 4 Green functions, propagator and particle interpretation. Retarded and advanced green functions. Action principle.
lecture 5 Noether theorem. Action for the Klein-Gordon field. Symmetries: U(1) symmetry and conserved current, space-time translations and the energy-momentum tensor.
lecture 6 Historical derivation of the Dirac equation. The Dirac equation in hamiltonian and covariant form. Continuity equation. Gamma matrices.
lecture 7 Plane wave solutions. Non-relativistic limit, Pauli equation and gyromagnetic ratio.
lecture 8 Conservation of angular momentum and spin operator. Spectrum of the hydrogen atom with the Schrödinger, Klein-Gordon and Dirac equations. Covariance of the Dirac equation. Lorentz transformation of spinors: infinitesimal form and generators.
lecture 9 Finite Lorentz transformation on spinors and examples. General form of plane wave solutions.
lecture 10 Pseudounitarity. Transformations of fermionic bilinears. Spinorial representations. Parity and chiral fermions.
lecture 11 Time reversal. Hole theory, antiparticles and charge conjugation. CPT invariance.
lecture 12 Action for Dirac fermions, $U(1)$ symmetry, non-abelian symmetries, action for chiral fermions, Majorana mass. Propagator of the Dirac field. Wave equations for massive particles of arbitrary spin (Pauli-Fierz).
lecture 13 Massive particles of spin 1: Proca equations, plane waves, degrees of freedom, and propagator. Massless particles of spin 1: Maxwell equations and gauge symmetry. Massless particles of spin 2: linearized Einstein equations and gauge symmetries.
lecture 14 Classical action and path integrals. Lagrangian and hamiltonian formalisms, canonical quantization.
lecture 15 Classical mechanics of non-relativistic particles and global symmetries. Particle in a magnetic field and in curved space (nonlinear sigma model).
lecture 16 Classical mechanics of relativistic particles, global and local symmetries.
lecture 17 Canonical quantization of relativistic particle. Path integrals in phase space.
lecture 18 Path integrals in configuration space. The free non-relativistic particle. Wick rotation and statistical mechanics.
lecture 19 Correlation functions, generating functionals, gaussian formulae, hypercondensed notation, free theories. Wick's theorem.
lecture 20 Two-point function (propagator) of the harmonic oscillator and of the Klein-Gordon field.
lecture 21 Perturbative expansion of the path integral. Perturbative corrections to the ground state energy of the anharmonic oscillator and Feynman diagrams. Grassmann variables.
lecture 22 The fermionic harmonic oscillator. Hamiltonian formulation of systems with Grassmann variables and canonical quantization.
lecture 23 Examples and representation of Clifford algebras. Coherent states.
lecture 24 Path integral for fermions. Propagators and loops.