RELATIVISTIC QUANTUM MECHANICS AND PATH INTEGRALS (AA 21-22)

Prof. Fiorenzo Bastianelli

lecture 1 Introduction. Relativistic quantum mechanics (first quantization). Review of special relativity: 4-vectors and the Lorentz group.
lecture 2 Review of special relativity: Minkowski spacetime, Lorentz and Poincaré groups, tensors. Review of the Schrödinger equation.
lecture 3 Properties of the Schrödinger equation. The Klein-Gordon equation. Plane-wave solutions, conserved current, failure of the probabilistic interpretation.
lecture 4 Yukawa potential. Green's functions, propagator, particle interpretation. Retarded and advanced Green's functions.
lecture 5 Action principle. Noether's theorem. Action for the Klein-Gordon field. Symmetries: U(1) symmetry and conserved current, space-time translations, energy-momentum tensor.
lecture 6 Historical derivation of the Dirac equation. The Dirac equation in hamiltonian and covariant form. Continuity equation. Gamma matrices.
lecture 7 Plane wave solutions. Non-relativistic limit, Pauli equation and gyromagnetic ratio.
lecture 8 Spin operator and conservation of angular momentum. Spectrum of the hydrogen atom with the Schrödinger, Klein-Gordon and Dirac equations.
lecture 9 Covariance of the Dirac equation. Lorentz transformation of spinors: infinitesimal form, generators, finite transformations, examples. General form of plane wave solutions.
lecture 10 Pseudounitarity. Transformations of fermionic bilinears. Spinorial representations.
lecture 11 Parity and chiral fermions. Time reversal. Hole theory, antiparticles, charge conjugation.
lecture 12 Charge conjugation. CPT invariance. Action for Dirac fermions, $U(1)$ symmetry, non-abelian symmetries, action for chiral fermions, Majorana mass.
lecture 13 Propagator of the Dirac field. Wave equations for massive particles of arbitrary spin (Pauli-Fierz). Massive spinning particles: Proca equations, plane waves, degrees of freedom, propagator.
lecture 14 Massless particles of spin 1: Maxwell equations and gauge symmetry. Massless particles of spin 2: linearized Einstein equations and gauge symmetries.
lecture 15 Classical mechanics of non-relativistic particles and global symmetries.
lecture 16 Particle in a magnetic field and in curved space (nonlinear sigma model). Classical mechanics of relativistic particles, global and local symmetries.
lecture 17 Canonical quantization of relativistic particle. Path integrals in phase space.
lecture 18 Path integrals in configuration space. The free non-relativistic particle. Wick rotation and statistical mechanics.
lecture 19 Correlation functions, generating functionals, gaussian formulae, hypercondensed notation, free theories. Wick's theorem.
lecture 20 Two-point functions (propagator) of the harmonic oscillator and Klein-Gordon field.
lecture 21 Perturbative expansion of the path integral. Perturbative corrections to the ground state energy of the anharmonic oscillator, Feynman diagrams.
lecture 22 Grassmann variables. The fermionic harmonic oscillator. Hamiltonian formulation of bosonic systems and canonical quantization.
lecture 23 Hamiltonian formulation of systems with Grassmann variables and canonical quantization. Examples and representation of Clifford algebras. Coherent states.
lecture 24 Path integral for fermions.