

Relativity (AA 21-22)

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Introduction. Newtonian mechanics and Galilean relativity. Inertial frame of reference. Galileo's transformations and symmetries of Newtonian mechanics. Conflict with the symmetries of electrodynamics. Maxwell's equations and Lorentz transformations.

Principles of special relativity. Consequences of special relativity: addition of velocities, contraction of lengths, time dilation. Exercise on cosmic ray.

Four-vectors, invariant length, Minkowski spacetime.

Spacetime and the Minkowski metric. Analogies between the orthogonal group (rotations) and the Lorentz group. Matrix notation and tensor notation.

Tensors. Lorentz transformations of the electromagnetic field. Exercises.

Proper and orthochronous Lorentz group. Relativistic mechanics. Relativistic definitions of energy and momentum. Four-momentum and four-force.

Massless particles. Conservation of total 4-momentum. Exercises.

Maxwell's equations in covariant form. Continuity equation. Gauge symmetry and action principle.

Relativistic particle, particle action with local invariance. Lorentz force. Exercises

Analysis of Maxwell's equations: Coulomb gauge, Lorentz gauge, electromagnetic waves, energy-momentum tensor.

Klein-Gordon equation. Antiparticles. Electromagnetic waves and physical polarizations.

Introduction to General Relativity. The principle of equivalence and systems in free fall (locally inertial system). Gravitational forces on a particle, affine connection and metric tensor. [W, ch 3].

Relation between affine connection and metric tensor. The Newtonian limit. [W, ch 3].

Time dilation. [W, ch 3] The principle of general covariance. Tensor analysis [W, cap 4].

Tensor analysis: tensors, connection, covariant derivatives, tensorial equations [W, ch 4].

Covariant derivatives along a curve. Effects of gravitation [W, ch 5].

Curvature and Riemann tensor [W, ch 6]. Algebraic properties of Riemann and Ricci tensors. Bianchi identities.

Exercises. Einstein's equations, cosmological constant. [W, ch 7]

Einstein's equations, gauge symmetry, Lie derivative of the metric, Killing vectors and symmetries, analogy with Maxwell's equations.

The harmonic gauge. Linearized Einstein equations, linearized gauge symmetries, gauge-fixing, the wave equation, gravitational waves.

Gravitational waves and physical polarizations [OR, ch 5].

Solutions with spherical symmetry, metric in standard form [W, ch 8.1]. Schwarzschild solution. Birkhoff's theorem. [OR, ch 8].

Symmetries, isometries, and maximally symmetric spaces. Extension of the Schwarzschild solution in the presence of a cosmological constant. Black holes, infinite red-shift surface, event horizon, classical analogy.

Rindler space. Spacetime diagrams and description of the Schwarzschild black hole. More general black holes. [OR, ch 8].