Bethe ansatz equations and Scattering for the so(m), sp(n) and osp(m|n) open spin chains

joint work with

(D. Arnaudon, J. Avan, N. Crampé, L. Frappat, E. Ragoucy)

• outline

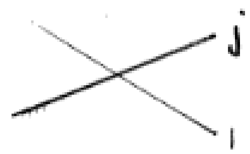
• Introduce the R matrix related to the models and the K-matrix

• Derive the transfer matrix ⇒ eigenvalues and Bethe ansatz equations

• study of the ground state and excitations.

• Finally explicit computation of bulk and boundary S-matrices

- R-matrix acting on $V \otimes V$ solution of the YBE



$= R_{ij}$

$$R_{12}(\lambda_1)\, R_{13}(\lambda_1+\lambda_2)\, R_{23}(\lambda_2) = R_{23}(\lambda_2)\, R_{13}(\lambda_1+\lambda_2)\, R_{12}(\lambda_1)$$

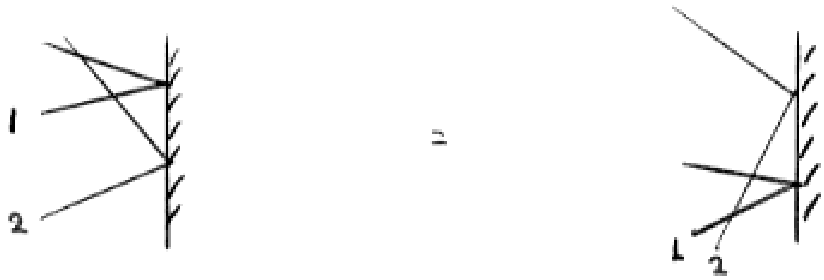
$$\boxed{R(\lambda) = \lambda(\lambda+i\rho)\,\mathbb{1} + (\lambda+i\rho)\,P - \lambda Q} \,.$$

$P^2 = 1$, $Q^2 = \theta_0 Q\,(n-m)$, $PQ = QP = \theta_0 Q$

$\theta_0 = \pm 1$ $(-1 \to Sp,\ +1 \to O)$ $\rho = \theta_0 \dfrac{(n+m-2)}{2}$

- unitarity $R(\lambda)\,R(-\lambda) \propto \mathbb{1}$
- crossing $V_1\, R_{12}^{t_1}(-\lambda-i\rho)\, V_1 = R_{12}(\lambda)$

2. The K-matrix acting on V satisfies the reflection equation (Cherednik)



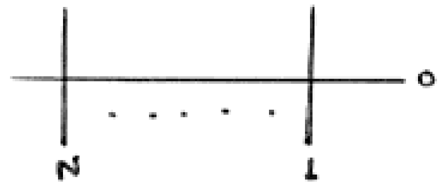
$$\left[\begin{array}{l} R_{12}(\lambda_1-\lambda_2)\ K_1(\lambda_1)\ R_{21}(\lambda_1+\lambda_2)\ K_2(\lambda_2) = \\ K_2(\lambda_2)\ \ R_{12}(\lambda_1+\lambda_2)\ \ K_1(\lambda_1)\ R_{21}(\lambda_1-\lambda_2) \end{array}\right]$$

We focus here only on the diagonal solutions related to $so(m)$, $sp(n)$.

- Transfer Matrix (Sklyanin)

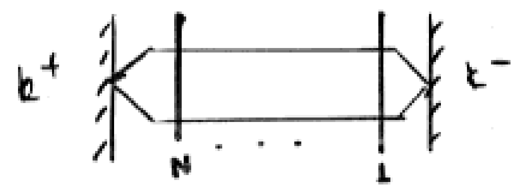
$$T(\lambda) = R_{0N}(\lambda) \cdots R_{01}(\lambda)$$

act $V_0 \otimes \underbrace{V_1 \cdots \otimes V}_{N}$



$$t(\lambda) = \mathrm{tr}_0 \, K_0^+(\lambda) \, T_0(\lambda) \, K_0^-(\lambda) \, T_0^{-1}(-\lambda)$$

$$\left(K^-(\lambda) \equiv K(\lambda) \qquad K^+ = K(-\lambda - i\rho)^t \right)$$



$[t(\lambda), t(\mu)] = 0$
"integrability"

- Aim: Diagonilize $t(\lambda)$ $\Rightarrow$

e.g. eigenvalues of energy $\left(H \sim \frac{d}{d\lambda} t(\lambda) \Big|_{\lambda=0} \right)$

and BAE.

$\updownarrow$

$[R(\lambda=0) = P]$

$K^{\pm} = 1$ for simplicity

- ANALYTICAL BETHE ANSATZ. (Reshetikhin, Mezincescu + Nepomechie

1. Reference state (pseudovacuum)

2 Properties:

i.) Crossing $\quad t(\lambda) = t(-\lambda - i\rho)$

ii) Fusion

$$\left[\tilde{t}(\lambda) = \zeta(2\lambda + 2i\rho)\, t(\lambda)\, t(\lambda + i\rho) - \zeta(\lambda + i\rho)^{(N}\, q(2\lambda + i\rho)\, q(-2\lambda - 3i\rho)\right]$$

$$\zeta(\lambda) = (\lambda + i)(\lambda + i\rho)(\lambda - i)(\lambda - i\rho)$$

$$q(\lambda) = (\theta_0 \lambda + i)(-\lambda + i\rho)$$

iii) Analyticity

iv). Symmetry (asymptotic behavior $t(\lambda \to \pm\infty)$)

$$[t(\lambda), g] = 0 \qquad [R_{12}, U_1 + U_2] = 0$$

1. Reference state:

$$|\Omega+\rangle = \bigotimes_{i=1}^{N} \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_i$$

$$t(\lambda)\, |\Omega+\rangle = \Lambda^{\circ}(\lambda)\, |\Omega+\rangle$$

$$\Lambda^{\circ}(\lambda) = \alpha^{2N}(\lambda)\, g_0(\lambda) + b^{2N}(\lambda) \sum_{i=1}^{n+m-2} g_i(\lambda) + c^{2N}(\lambda)\, g_{n+m-1}(\lambda)$$

- Conjecture $\forall$ eigenvalue

$$\Lambda(\lambda) = \alpha^{2N}(\lambda)\, g_0(\lambda)\, \underline{A_0(\lambda)} + b^{2N} \sum_{i=1}^{n+m-2} g_i(\lambda)\, \underline{A_i(\lambda)} + c^{2N} g_{n+m-1}(\lambda)\, \underline{A_{n+m-1}(\lambda)}$$

$$\alpha(\lambda) = (\lambda+i\rho)(\lambda+i), \quad b(\lambda) = \lambda(\lambda+i\rho), \quad c(\lambda) = (\lambda+i\rho-i)\lambda$$

- A_i "dressing functions" to be derived explicitly.

e.g. $so(2K+1)$

$$A_0(\lambda) = \prod_{j=1}^{M^{(1)}} \frac{\lambda+\lambda_j^{(1)} - i/2}{\lambda+\lambda_j^{(1)}+i/2} \frac{\lambda-\lambda_j^{(1)} - i/2}{\lambda-\lambda_j^{(1)}+i/2}$$

$$A_\ell(\lambda) = \prod_{j=1}^{M^{(\ell)}} \frac{\lambda+\lambda_j^{(\ell)} + \frac{i\ell}{2} + i}{\lambda+\lambda_j^{(\ell)} + \frac{i\ell}{2}} \frac{\lambda - \lambda_j^{(\ell)} + \frac{i\ell}{2} + i}{\lambda-\lambda_j^{(\ell)} + \frac{i\ell}{2}}$$

$$\prod_{j=1}^{M^{(\ell+1)}} \frac{\lambda+\lambda_j^{(\ell+1)} + \frac{i\ell}{2} - i/2}{\lambda+\lambda_j^{(\ell+1)} + \frac{i\ell}{2} + i/2} \frac{\lambda-\lambda_j^{(\ell+1)} + \frac{i\ell}{2} - i/2}{\lambda-\lambda_j^{(\ell+1)} + \frac{i\ell}{2} + i/2}$$

$$A_K(\lambda) = \prod_{j=1}^{M^{(K)}} \frac{\lambda+\lambda_j^{(K)} + \frac{iK}{2} - i}{\lambda+\lambda_j^{(K)} + \frac{iK}{2}} \frac{\lambda+\lambda_j^{(K)} + \frac{iK}{2} - i}{\lambda-\lambda_j^{(K)} + \frac{iK}{2}} \frac{\lambda+\lambda_j^{(K)} + \frac{iK}{2} + i/2}{\lambda+\lambda_j^{(K)} + \frac{iK}{2} - i/2} \frac{\lambda-\lambda_j^{(K)} + \frac{iK}{2} + i/2}{\lambda-\lambda_j^{(K)} + \frac{iK}{2} - i/2}$$

$$A_\ell(\lambda) = A_{K-\ell-1}(-\lambda - i\rho) \qquad \ell > K$$

$M^{(\ell)}$ related to the diag. generators of the algebra!

Analyticity requirements $\Rightarrow$ BAE

1. $so(2k+1)$: 

(Dynkin diagr.

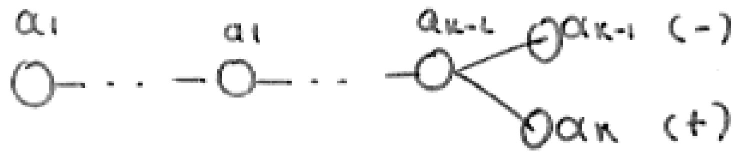
1: $$e_1(\lambda_i^{(1)})^{2N+1} = -\prod_{j=1}^{M^{(1)}} e_2(\lambda_i^{(1)}-\lambda_j^{(1)})\, e_2(\lambda_i^{(1)}+\lambda_j^{(1)}) \times \prod_{j=1}^{M^{(2)}} e_{-1}(\lambda_i^{(1)}-\lambda_j^{(2)})\, e_{-1}(\lambda_i^{(1)}+\lambda_j^{(2)})$$

$l = 2, \ldots, k-1$
$$e_1(\lambda_i^{(l)}) = -\prod_{j=1}^{M^{(l)}} e_2(\lambda_i^{(l)}-\lambda_j^{(l)})\, e_2(\lambda_i^{(l)}+\lambda_j^{(l)}) \prod_{\tau=\pm 1}\prod_{j=1}^{M^{(l+\tau)}} e_{-1}(\lambda_i^{(l)}-\lambda_j^{(l+\tau)})\, e_{-1}(\lambda_i^{(l)}+\lambda_j^{(l+\tau)})$$

k: $$e_{1/2}(\lambda_i^{(k)}) = -\prod_{j=1}^{M^{(k)}} e_1(\lambda_i^{(k)}-\lambda_j^{(k)})\, e_1(\lambda_i^{(k)}+\lambda_j^{(k)}) \prod_{j=1}^{M^{(k-1)}} e_{-1}(\lambda_i^{(k)}-\lambda_j^{(k-1)})\, e_{-1}(\lambda_i^{(k)}+\lambda_j^{(k-1)})$$

where, $$e_x^{-1}(\lambda) = \frac{\lambda - ix/2}{\lambda + \frac{ix}{2}}$$

2. $SO(2K)$



First $K-3$ eqs as in $so(2K+1)$, last 3 modified:

$$K-2:\quad e_1(\lambda_i^{(K-2)}) = -\prod_{j=1}^{M^{(K-2)}} e_2(\lambda_i^{(K-2)}-\lambda_j^{(K-2)})\, e_2(\lambda_i^{(K-2)}+\lambda_j^{(K-2)})$$

$$\prod_{\tau=\pm}\prod_{j=1}^{M^{(\tau)}} e_{-1}(\lambda_i^{(K-2)}-\lambda_j^{(\tau)})\ e_{-1}(\lambda_i^{(K-2)}+\lambda_j^{(\tau)})$$

$$\prod_{j=1}^{M^{(K-3)}} e_{-1}(\lambda_i^{(K-2)}-\lambda_j^{(K-3)})\, e_{-1}(\lambda_i^{(K-2)}+\lambda_j^{(K-3)})$$

$$(\tau=\pm):\quad e_1(\lambda^{(\tau)}) = -\prod_{j=1}^{M^{(\tau)}} e_2(\lambda_i^{(\tau)}-\lambda_j^{(\tau)})\ e_2(\lambda_i^{(\tau)}+\lambda_j^{(\tau)})$$

$$\prod_{j=1}^{M^{(K-2)}} e_{-1}\ (\lambda_i^{(\tau)}-\lambda_j^{(K-2)})\, e_{-1}(\lambda_i^{(\tau)}+\lambda_j^{(K-2)})$$

("doubled" compared with (Ogievetsky-Reshetikhin-Wiegmann)
BULK.

5. $sp(2k)$

Last 2 equations modified:

$$k-1: \quad e_1(\lambda_i^{(k-1)}) = -\prod_{j=1}^{M^{(k-1)}} e_2(\lambda_i^{(k-1)}-\lambda_j^{(k-1)})\, e_2(\lambda_i^{(k-1)}+\lambda_j^{(k-1)})$$
$$\prod_{j=1}^{M^{(k-2)}} e_{-1}(\lambda_i^{(k-1)}-\lambda_j^{(k-2)})\, e_{-1}(\lambda_i^{(k-1)}+\lambda_j^{(k-2)})$$
$$\prod_{j=1}^{M^{(k)}} e_{-2}(\lambda_i^{(k-1)}-\lambda_j^{(k)})\, e_{-2}(\lambda_i^{(k-1)}+\lambda_j^{(k)})$$

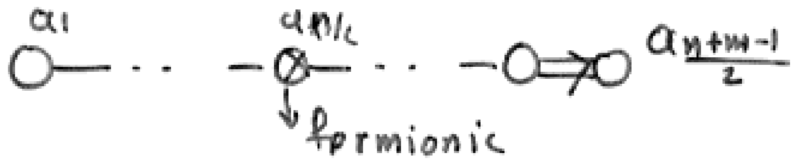
$$k: \quad e_2(\lambda_i^{(k)}) = -\prod_{j=1}^{M^{(k)}} e_4(\lambda_i^{(k)}-\lambda_j^{(k)})\, e_4(\lambda_i^{(k)}+\lambda_j^{(k)})$$
$$\prod_{j=1}^{M^{(k-1)}} e_{-2}(\lambda_i^{(k)}-\lambda_j^{(k-1)})\, e_{-2}(\lambda_i^{(k)}+\lambda_j^{(k-1)})$$

(again "doubled" compared to Ogievetsky-
- Reshetikhin- Wiegmann)

open $SU(n)$ case: (Doikou + Nepomechie)

"SUPER"

4. $osp(m|n)$ m: odd



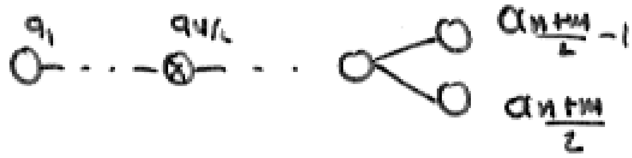
$K = \frac{n+m-1}{2}$

$\forall\ l \neq \frac{n}{2}$ as in $so(2K+1)$

$$l = \frac{n}{2}: \quad 1 = \prod_{j=1}^{M^{(\frac{n}{2}+1)}} e_1\left(\lambda_i^{(\frac{n}{2})} - \lambda_j^{(\frac{n}{2}+1)}\right) e_1\left(\lambda_i^{(\frac{n}{2})} + \lambda_j^{(\frac{n}{2}+1)}\right) \prod_{j=1}^{M^{(\frac{n}{2}-1)}} e_{-1}\left(\lambda_i^{(\frac{n}{2})} - \lambda_j^{(\frac{n}{2}-1)}\right) e_{-1}\left(\lambda_i^{(\frac{n}{2})} + \lambda_j^{(\frac{n}{2}-1)}\right)$$

The last equation as in $so(2K+1)$ $\left(K = \frac{n+m-1}{2}\right)$

5. $osp(m|n)$ m: even



$K = \frac{n+m}{2}$

Similarly everything is as in $so(2K)$ only $l = n/2$ equ. corresponds to fermionic root!

- Special Cases:

- osp(2|n) 

$l = 2, \dots, \frac{n}{2}$ as usual, $l = 1$: fermionic

$l = \frac{n}{2}$ as the last equ. of sp(2K) $\left(K = \frac{n}{2} + 1 \right)$

- osp(1|n) 

first $\frac{n}{2} - 1$ equs as usual,

$$\left(l = \frac{n}{2} = K \right) : \quad e_1(\lambda_i^{(K)})\, e_{-1/2}(\lambda_i^{(K)}) = \prod_{j=1}^{M^{(K)}} e_2(\lambda_i^{(K)} - \lambda_j^{(K)})\, e_2(\lambda_i^{(K)} + \lambda_j^{(K)})$$
$$\times\, e_{-1}(\lambda_i^{(K)} - \lambda_j^{(K)})\, e_{-1}(\lambda_i^{(K)} + \lambda_j^{(K)}) \prod_{j=1}^{M^{(K-1)}} e_{-1}(\lambda_i^{(K)} - \lambda_j^{(K-1)})\, e_{-1}(\lambda_i^{(K)} + \lambda_j^{(K-1)})$$

* osp(1|2) $\longrightarrow$ su(3) spin chain "soliton non-preserving" BC (Doikou)

$$\left[e_1(\lambda)^{2N+1} e_{-1/2}(\lambda) = \prod_{j=1}^{M} e_2(\lambda_i - \lambda_j)\; e_2(\lambda_i + \lambda_j) \times e_{-1}(\lambda_i - \lambda_j)\; e_{-1}(\lambda_i + \lambda_j) \right]$$

Agree with Martins + Ramos bulk osp(1|n)

osp(2|n), osp(m|2). "doubled"

Bulk su(m|n) : (Saleur)

D1. $K(\lambda) = \mathrm{diag}(\underbrace{\alpha, \ldots \alpha}_{K}, \underbrace{b, \ldots, b}_{K})$

$so(2K)$, $sp(2K)$ only

$\alpha = -\lambda + i\xi$, $\quad b = \lambda + i\xi$: ξ: free parameter

D2. $K(\lambda) = \mathrm{diag}(\alpha, b, \ldots, b, \gamma)$ (no for $sp(n)$)

$$\alpha = \frac{-\lambda + i\xi_1}{\lambda + i\xi_1}, \quad b = 1, \quad \gamma = \frac{-\lambda + i\xi_n}{\lambda + i\xi_n}$$

$$\left[\xi_1 + \xi_n = p - 1\right]$$

D3. $K(\lambda) = (\underbrace{\alpha, \ldots \alpha}_{p}, \underbrace{b, \ldots b}_{n-2p}, \alpha, \ldots \alpha)$

$\alpha = -\lambda + i\xi$, $\quad b = \lambda + i\xi$ $\quad \xi = \frac{n}{4} - p$ "fixed"

(McKay & short)

D4. $K(\lambda) = (\alpha, b, \gamma, \delta)$ $\quad so(4)$

$\alpha = (-\lambda + i\xi_+)(-\lambda + i\xi_-)$ $\quad b = (\lambda + i\xi_-)(-\lambda + i\xi_+)$

$\gamma = (-\lambda + i\xi_-)(\lambda + i\xi_+)$ $\quad \delta = (\lambda + i\xi_-)(\lambda + i\xi_+)$

(*): K^{+}: $D_1, D_2, D_3\ D_4$, $\quad K^{(+)} = 1$

- Ground state and excitations

[1.] $so(2k+1)$: $k-1$ first "seas" filled with real strings.
k^{th} sea filled with 2-string ($\lambda^0 \pm i/4$)

[2.] $so(2k)$: k "seas" filled with real strings

[3.] $sp(2k)$: $k-1$ "seas" filled with 2-string ($\lambda^0 \pm i/2$)
k "sea" filled with real strings

[4.] $osp(1|n)$ $k = \frac{n}{2}$ filled real λ's.

Holes in the "seas" $\longrightarrow$ particle-like excitations.

Thermodynamic limit $(N \to \infty)$
(take log and derivative of BAE)

state with 2 holes 1^{st} sea:

$$\left[\sigma^{(1)}(\lambda) = 2\epsilon^{(1)}(\lambda) + \frac{1}{N}\varphi_0(\lambda) + \frac{1}{N}\varphi_1(\lambda,\xi)\right]$$

$\hat{\varphi}_{0,1}$ known explicitly , $\epsilon^{(1)}(\lambda)$: energy of the hole

- Scattering

Quantization condition (Korepin, Andrei + Destri)

$$(e^{2iNp}S - 1)\,|\tilde{\lambda}_1,\tilde{\lambda}_2\rangle = 0$$

$$S = e^{i\Phi} \cong \alpha^- \alpha^+$$

Integrate density:

but: $\int_0^{\tilde{\lambda}_1} \sigma^{(1)}(\lambda)\, d\lambda$ = integ. , $\frac{1}{2\pi}\frac{dp^{(1)}}{d\lambda} = e^{(1)}(\lambda)$

$$\sigma^{(1)} \to \quad 2\pi j = 2\pi N p^{(1)}(\lambda) + \frac{1}{2\pi}\int_0^{\tilde{\lambda}_1} d\lambda \, (\varphi_0(\lambda) + \varphi_1(\lambda))$$

Q.C. $\quad 2\pi j i = 2\pi i N p^{(1)}(\lambda) + i\varphi \qquad (\alpha^+\alpha^- \equiv e^{i\varphi})$

$$\therefore \quad \left[\quad \alpha^+\alpha^- = \exp\left[2\pi \int_0^{\tilde{\lambda}_1} (\varphi_0 + \varphi_1)\, d\lambda\right]\right]$$

$$\alpha^- = K_0(\lambda)\, K_1(\lambda, \xi)$$

$$\alpha^+ = K_0(\lambda)$$

Derive $K_{0,1}$ explicitly.

$$K_0(\lambda) = \exp\left[-\frac{1}{2}\int_{-\infty}^{\infty} \frac{d\omega}{\omega}\, e^{-i\omega\lambda}\, \hat{\varphi}_0(\omega)\right]$$

$$K_1(\lambda) = \exp\left[-\int_{-\infty}^{\infty} \frac{d\omega}{\omega}\, e^{-i\omega\lambda}\, \hat{\varphi}_1(\omega)\right]$$

Scattering in terms of Γ-functions:

$$\frac{1}{2}\int_0^\infty \frac{d\omega}{\omega}\;\frac{e^{-\mu\omega/2}}{\operatorname{ch}\omega/2} = \ln \frac{\Gamma\left(\frac{\mu+1}{4}\right)}{\Gamma\left(\frac{\mu+3}{4}\right)}$$

1. so(n)

1a. Bulk Scattering (from terms $(\tilde\lambda_1-\tilde\lambda_2)$)

terms $(\tilde\lambda_1-\tilde\lambda_2)$ to

$$S_0 = \frac{\tan\pi\left(\frac{i\lambda-1}{n-2}\right)}{\tan\pi\left(\frac{i\lambda+1}{n-2}\right)}\;\frac{\Gamma\left(\frac{i\lambda}{n-2}\right)\Gamma\left(\frac{-i\lambda}{n-2}+\frac{1}{2}\right)\Gamma\left(-\frac{i\lambda+1}{n-2}\right)\Gamma\left(\frac{i\lambda+1}{n-2}+\frac{1}{2}\right)}{\Gamma\left(\frac{-i\lambda}{n-2}\right)\Gamma\left(\frac{i\lambda}{n-2}+\frac{1}{2}\right)\Gamma\left(\frac{i\lambda+1}{n-2}\right)\Gamma\left(\frac{-i\lambda+1}{n-2}+\frac{1}{2}\right)}$$

$$S = \frac{S_0}{(i\lambda+1)(i\lambda+\rho)}\left(i\lambda(i\lambda+\rho)\mathbb{1} + (i\lambda+\rho)P - i\lambda Q\right)$$

Agree with Ogievetsky – Wiegmann – Reshetikhin

1b so(n) boundary Scattering

$$K_0(\lambda) = Y_0(\lambda)\,\frac{\Gamma\left(\frac{i\lambda}{n-2}\right)\Gamma\left(\frac{-i\lambda}{n-2}+\frac{3}{4}\right)\Gamma\left(\frac{i\lambda+1/2}{n-2}+\frac{3}{4}\right)\Gamma\left(\frac{-i\lambda+\frac{1}{2}}{n-2}+\frac{1}{2}\right)}{\Gamma\left(\frac{-i\lambda}{n-2}\right)\Gamma\left(\frac{i\lambda}{n-2}+\frac{3}{4}\right)\Gamma\left(\frac{-i\lambda+1/2}{n-2}+\frac{3}{4}\right)\Gamma\left(\frac{i\lambda+1/2}{n-2}+\frac{1}{2}\right)}$$

$$Y_0(\lambda) = \frac{\sin\pi\left(\frac{i\lambda+1/2}{n-2}-\frac{1}{4}\right)\sin\pi\left(\frac{i\lambda-1/2}{n-2}+\frac{1}{2}\right)\sin\pi\left(\frac{i\lambda}{n-2}+\frac{1}{4}\right)}{\sin\pi\left(\frac{i\lambda-1/2}{n-2}+\frac{1}{4}\right)\sin\pi\left(\frac{i\lambda+1/2}{n-2}-1/2\right)\sin\pi\left(\frac{i\lambda}{n-2}-\frac{1}{4}\right)}$$

D1. $$k(\lambda) = (\underbrace{\alpha \cdots \alpha}_{k}, \underbrace{\beta \cdots \beta}_{k})$$

$$\alpha = K_1(\lambda,\xi') = \frac{\Gamma\left(\frac{i\lambda+\xi'}{n-2}+\frac{1}{2}\right)\Gamma\left(\frac{-i\lambda+\xi'}{n-2}+1\right)}{\Gamma\left(\frac{-i\lambda+\xi'}{n-2}+\frac{1}{2}\right)\Gamma\left(\frac{i\lambda+\xi'}{n-2}+1\right)}$$

$$\frac{\beta}{\alpha} = \frac{\lambda+i\xi'}{-\lambda+i\xi'} \qquad \xi' = \xi - 1/2 \text{ (renormalized).}$$

D2. $K(\lambda) = (\alpha, \beta, \ldots \beta, \gamma)$

$$K_1(\lambda,\xi') = \frac{\tan\pi\left(\frac{i\lambda-\xi_n'}{n-2}\right)}{\tan\pi\left(\frac{i\lambda+\xi_n'}{n-2}\right)} \frac{\Gamma\left(\frac{i\lambda+\xi_1'}{n-2}+\frac{1}{2}\right)\Gamma\left(\frac{-i\lambda+\xi_n'}{n-2}+1\right)}{\Gamma\left(\frac{-i\lambda+\xi_1'}{n-2}+\frac{1}{2}\right)\Gamma\left(\frac{i\lambda+\xi_1'}{n-2}+1\right)}$$

$$\times \frac{\Gamma\left(\frac{i\lambda+\xi_n'}{n-2}+1/2\right)\Gamma\left(\frac{-i\lambda+\xi_n'}{n-2}\right)}{\Gamma\left(\frac{-i\lambda+\xi_n'}{n-2}+\frac{1}{2}\right)\Gamma\left(\frac{i\lambda+\xi_n'}{n-2}\right)}$$

$$\frac{\gamma}{\alpha} = \frac{\lambda+i\xi_1'}{-\lambda+i\xi_1'} \quad \frac{-\lambda+i\xi_n'}{\lambda+i\xi_n'} \qquad (\xi_1'+\xi_n' = p-1)$$

$\xi_1' = \xi_1 - 1/2$

$\xi_n' = \xi_n + 1/2$

$$\frac{\beta}{\alpha} = \frac{\lambda+i\xi_1'}{-\lambda+i\xi_1'}$$

D3. $K(\lambda) = (\underbrace{\alpha \ldots \alpha}_{p}, \underbrace{\beta, \ldots \beta}_{n-2p}, \underbrace{\alpha, \ldots \alpha}_{p})$

$$K_1(\lambda,\xi') = \frac{\Gamma\left(\frac{i\lambda+\xi'}{n-2}+\frac{1}{2}\right)\Gamma\left(\frac{-i\lambda+\xi'}{n-2}+1\right)\Gamma\left(\frac{-i\lambda+1/2}{n-2}+\frac{3}{4}\right)}{\Gamma\left(\frac{-i\lambda+\xi'}{n-2}+\frac{1}{2}\right)\Gamma\left(\frac{i\lambda+\xi'}{n-2}+1\right)\Gamma\left(\frac{i\lambda+1/2}{n-2}+3/4\right)} \times$$

$$\Gamma\left(\frac{i\lambda+1/2}{n-2}+1/4\right) \Big/ \Gamma\left(\frac{-i\lambda+1/2}{n-2}+1/4\right)$$

$$\frac{\beta}{\alpha} = \frac{\lambda+i\xi'}{-\lambda+i\xi'} \qquad \xi' = \frac{n}{4} - p \quad (\text{fixed})$$

(MacKay + Short)

a)• osp(1|n) bulk

$$S_0(\lambda) = \frac{\tan\pi\left(\frac{i\lambda-1}{n+1}\right)}{\tan\pi\left(\frac{i\lambda+1}{n+1}\right)} \; \frac{\Gamma(i\lambda/n+1)\,\Gamma\left(\frac{-i\lambda}{n+1}+\frac{1}{2}\right)\Gamma\left(\frac{-i\lambda+1}{n+1}\right)\Gamma\left(\frac{i\lambda+1}{n+1}+\frac{1}{2}\right)}{\Gamma\left(\frac{-i\lambda}{n+1}\right)\Gamma\left(\frac{i\lambda}{n+1}+\frac{1}{2}\right)\Gamma\left(\frac{i\lambda+1}{n+1}\right)\Gamma\left(\frac{-i\lambda+1}{n+1}+\frac{1}{2}\right)}$$

$$S'(\lambda) = \frac{S_0(\lambda)}{(i\lambda+p)(i\lambda+1)} \left[i\lambda(i\lambda+p) + (i\lambda+p)P - i\lambda Q \right]$$

b) boundary scattering:

$$K_0(\lambda) = Y_0(\lambda) \frac{\Gamma\left(\frac{i\lambda}{n+1}\right)\Gamma\left(\frac{-i\lambda}{n+1}+\frac{3}{4}\right)\Gamma\left(\frac{i\lambda+1/2}{n+1}+\frac{3}{4}\right)\Gamma\left(\frac{-i\lambda+1/2}{n+1}+\frac{1}{2}\right)}{\Gamma\left(\frac{-i\lambda}{n+1}\right)\Gamma\left(\frac{i\lambda}{n+1}+\frac{3}{4}\right)\Gamma\left(\frac{-i\lambda+1/2}{n+1}+\frac{3}{4}\right)\Gamma\left(\frac{i\lambda+1/2}{n+1}+\frac{1}{2}\right)}$$

$$Y_0(\lambda) = \frac{\sin\pi\left(\frac{i\lambda+1/2}{n+1}-\frac{1}{4}\right)\sin\pi\left(\frac{i\lambda-1/2}{n+1}+1/2\right)}{\sin\pi\left(\frac{i\lambda-1/2}{n+1}+\frac{1}{4}\right)\sin\pi\left(\frac{i\lambda+1/2}{n+1}\mp\frac{1}{2}\right)} \times$$

$$\times \frac{\tan\pi\left(\frac{i\lambda+1/4}{n+1}+\frac{1}{4}\right)}{\tan\pi\left(\frac{i\lambda-1/4}{n+1}\mp\frac{1}{4}\right)}.$$

- Conclusion

- Generalize computations for osp(m|n)
  (osp(2|2) starting point)

- Thermodynamic analysis for super spin chains
  with boundaries : conformal properties : (c)
  boundary properties : (g)

- "Soliton non preserving" BC and super chains?