# Symmetry breaking boundary conditions in CFT

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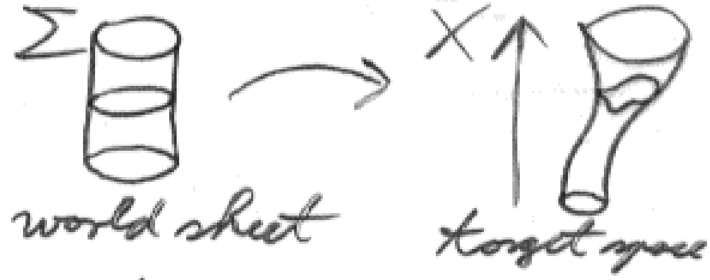
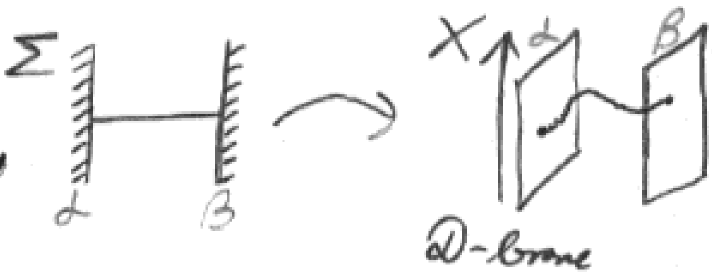
# Applications of BCFT

String theory

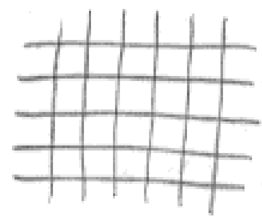



bulk CFT
≙ closed string background

boundary CFT
≙ open strings / D-branes

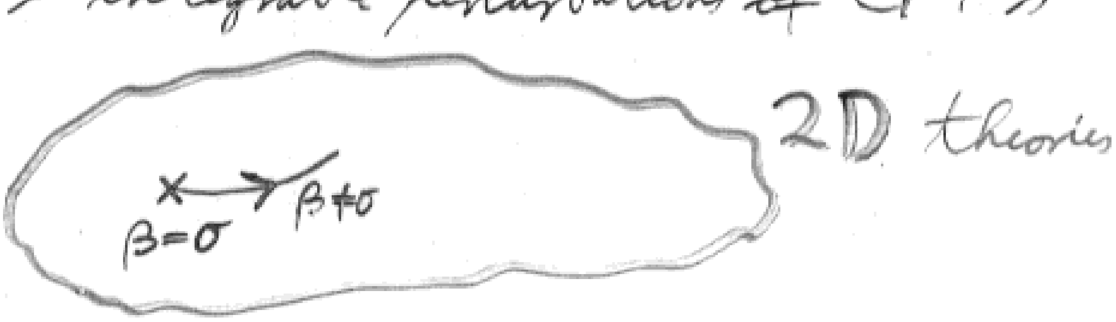



Statistical physics
→ continuum limit of vertex models
Ising, percolation, …



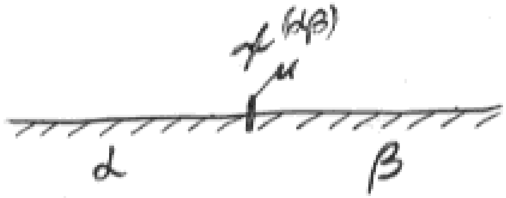
Condensed matter physics
→ quantum wires, Kondo effect, …

Integrable systems
→ integrable perturbations of CFT's



# Boundary conditions

$\boxed{z}$




Symmetry: $\mathcal{A}\otimes\bar{\mathcal{A}}$

$\mathcal{A}\cong\bar{\mathcal{A}}$

In RCFT the boundaries are characterized by boundary states $|\alpha\rangle$ which

i) implement the gluing conditions

$$[T(z)-\bar{T}(\bar{z})]|\alpha\rangle=0$$

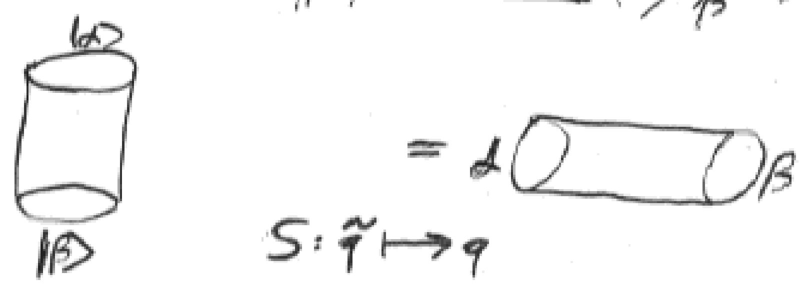
$$[\Omega(W)(z)\mp\bar{W}(\bar{z})]|\alpha\rangle=0$$

for $z=\bar{z}$

$\Omega$ = glueing automorphism

ii) satisfy Cardy's constraint

$$\langle\alpha|\,\tilde{q}^{\frac{1}{2}(L_0+\bar{L}_0-\frac{c}{12})}|\beta\rangle=\sum(n_\mu)_\beta{}^\alpha\,\chi_\mu(q)$$




spectrum of boundary operators

$S: \tilde{q}\mapsto q$

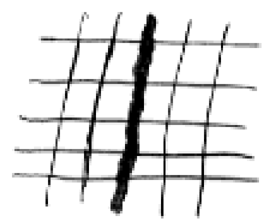
If the system is charge conjugate one always finds the Cardy states with $\Omega = id$

$$|\alpha\rangle=\sum\frac{S_{\alpha\mu}}{\sqrt{S_{0\mu}}}|\mu\rangle\rangle\,,\quad (n_\mu)_\beta{}^\alpha=N_{\mu\beta}{}^{\alpha}$$

# Why symmetry breaking?

- More possible boundary conditions
  (more D-branes, new end-points of RG-flow, ..)
- There exist models which necessitate the introduction of symmetry breaking
  → heterotic CFT's ($\mathcal{A} \not\cong \bar{\mathcal{A}}$)

  In particular: Asymmetric cosets
- In others, symmetry breaking is the only way to obtain non-trivial boundary conditions
  → defect lines with partial transmission / reflection



## Plan:

1. General construction
2. Asymmetric cosets
3. Defect lines

# General strategy

Start with an arbitrary bulk CFT

Symmetry: $\mathcal{A} \otimes \bar{\mathcal{A}}$

Spectrum: $\mathcal{H} = \bigoplus \mathcal{H}_\mu \otimes \bar{\mathcal{H}}_{\bar{\mu}} \, Z^{\mu\bar{\mu}}$

1. Find common conformal subalgebra $\mathcal{A}_{red}$ of $\mathcal{A}$ and $\bar{\mathcal{A}}$.
2. Decompose the theory with respect to the smaller symmetry $\mathcal{A}_{red} \otimes \bar{\mathcal{A}}_{red}$.
3. Define gluing conditions and boundary states with respect to $\mathcal{A}_{red}$.

If the theory remains rational with respect to $\mathcal{A}_{red} \otimes \bar{\mathcal{A}}_{red}$ one is left with the

Spectrum: $\mathcal{H} = \bigoplus Z^{\mu\bar{\mu}} \, b_\mu^{\ a} \, b_{\bar{\mu}}^{\ \bar{a}} \, \mathcal{H}_a \otimes \bar{\mathcal{H}}_{\bar{a}}$

# Concrete Example

Consider a WZNW model with

Symmetry: $\mathcal{A}(G_k) \otimes \overline{\mathcal{A}(G_k)}$

Spectrum: $\mathcal{H} = \bigoplus \mathcal{H}_\mu \otimes \overline{\mathcal{H}}_{\mu^+}$

Natural rational conformal embeddings read

$$A_{red} = \mathcal{A}(G_k/H_{xk}) \otimes \mathcal{A}(H_{xk}) \hookrightarrow \mathcal{A}(G_k)$$

Spectrum: $\mathcal{H} = \bigoplus \mathcal{H}_{(\mu,a)} \otimes \mathcal{H}_a \otimes \overline{\mathcal{H}}_{(\mu,b)^+} \otimes \overline{\mathcal{H}}_{b^+}$

The boundary states and the boundary spectrum for trivial glueing are schematically given by

$$|(\alpha,r)\rangle = \sum \frac{S^G_{\alpha\mu}}{\sqrt{S^G_{0\mu}}} \frac{\overline{S}^H_{ra}}{\overline{S}^H_{0a}} \, |(\mu,a)\rangle\!\rangle \otimes |a\rangle\!\rangle$$

$$Z_{(\alpha,r),(\alpha',r')} = \sum N^{\alpha}_{\alpha'\nu} \, N^{r}_{r'd} \, N^{d}_{bc^+} \, \chi_{(\nu,b)} \chi_c$$

# Asymmetric Cosets

Aim: New string backgrounds / CFT models

Group $G \longrightarrow$ WZNW model

Invariance: $g \longmapsto g_L(z)\, g\, g_R(\bar{z})$

field $\Sigma \longrightarrow G$

Symmetry: $\mathcal{A}(G_k) \otimes \overline{\mathcal{A}(G_k)}$

2 copies of Kac-Moody algebras

$\Big\downarrow$ gauging of subgroup $H$

Coset $G/H$ $\qquad g \sim \varepsilon_L(h)\, g\, \varepsilon_R(h^{-1}),\ h \in H$

two embeddings $H \hookrightarrow G$

Symmetry: $\mathcal{A}(G_k/H_{x_1 k})_L \otimes \overline{\mathcal{A}(G_k/H_{x_2 k})_R}$

2 possibly different coset chiral alg.

Modular/conformal invariance requires $x_1 = x_2$!

# GMM models

$$
\begin{tikzcd}
G = G_1 & \times & G_2 \\
& H \arrow[ul, bend left, "\varepsilon_1"] \arrow[ur, bend right, "\varepsilon_2"'] &
\end{tikzcd}
$$

gauging common subgroup

Symmetry:

$$\underbrace{\mathcal{A}(G_1/H) \otimes \mathcal{A}(G_2)}_{\text{holomorphic}} \otimes \underbrace{\overline{\mathcal{A}(G_1) \otimes \mathcal{A}(G_2/H)}}_{\text{antiholomorphic}}$$

$\Rightarrow$ heterotic CFT

Symmetry reduction:

$$\mathcal{A}_{red} = \mathcal{A}(G_1/H) \otimes \mathcal{A}(G_2/H) \otimes \mathcal{A}(H)$$

$$|(\rho_1, \rho_2, \tau)\rangle \underset{\text{roughly}}{=} \sum \frac{S^{G_1}_{\rho_1 \mu}}{\sqrt{S^{G_1}_{0\mu}}} \frac{S^{G_2}_{\rho_2 \nu}}{\sqrt{S^{G_2}_{0\nu}}} \frac{S^{H}_{\tau a}}{(S^{H}_{0a})^{\frac{3}{2}}} |(\mu,a),(\nu,a),a\rangle\rangle$$

# Defects as boundaries

Every defect (or junction) can be mapped to a boundary using the folding trick


a) Decoupled systems (full reflection):


$\bar{J}_1 = \Omega_1(J_1)$, $\bar{J}_2 = \Omega_2(J_2)$

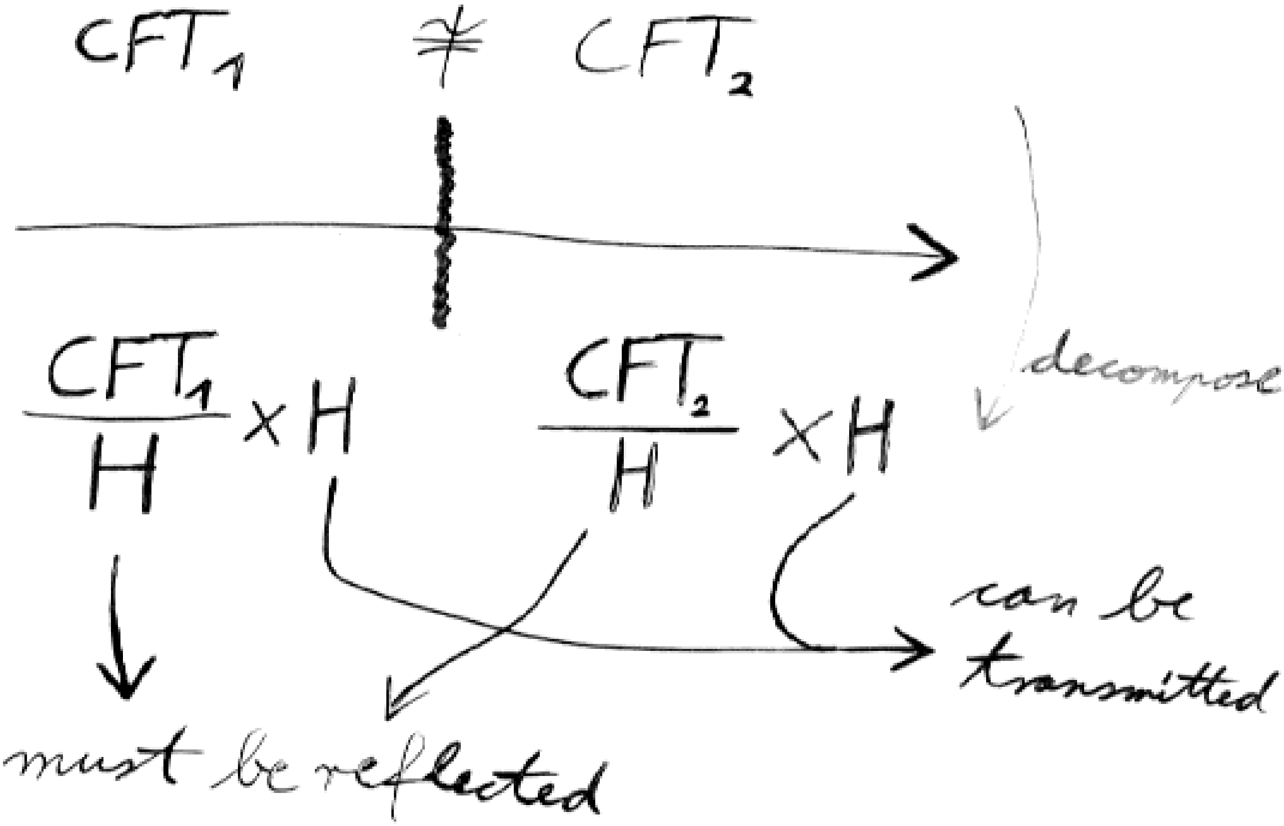
b) Full transmission:


$\bar{J}_1 = \Omega_1(J_2)$, $\bar{J}_2 = \Omega_2(J_1)$

c) Non-trivial defects:

Want to describe partial transmission / reflection

# Non-trivial defects




Aim: Construct boundary states in theory with chiral algebra

$$\mathcal{A} = \frac{\mathrm{CFT}_1}{H} \otimes \frac{\mathrm{CFT}_2}{H} \otimes H \otimes H$$

[An alternative with less obvious interpretation is

$$\mathcal{A} = \frac{\mathrm{CFT}_1 \otimes \mathrm{CFT}_2}{H} \otimes H$$
]

# Non-trivial defects III

In the case of two charge conjugate WZNW models one has

$$\mathcal{H} = \bigoplus \mathcal{H}^{G_1}_{\mu} \otimes \mathcal{H}^{G_2}_{\nu} \otimes \bar{\mathcal{H}}^{G_1}_{\mu^+} \otimes \bar{\mathcal{H}}^{G_2}_{\nu^+}$$

$$= \bigoplus \mathcal{H}^{G_1/H}_{(\mu,a)} \otimes \mathcal{H}^{G_2/H}_{(\nu,b)} \otimes \mathcal{H}^{H}_{a} \otimes \mathcal{H}^{H}_{b}$$

$$\otimes \bar{\mathcal{H}}^{G_1/H}_{(\mu,c)^+} \otimes \bar{\mathcal{H}}^{G_2/H}_{(\nu,d)^+} \otimes \bar{\mathcal{H}}^{H}_{c} \otimes \bar{\mathcal{H}}^{H}_{d}$$

The boundary states roughly read

$$|(\rho_1,\rho_2,r)\rangle = \sum \frac{S^{G_1}_{\rho_1\mu}}{\sqrt{S^{G_1}_{0\mu}}} \frac{S^{G_2}_{\rho_2\nu}}{\sqrt{S^{G_2}_{0\nu}}} \frac{S^{H}_{ra}}{(S^{H}_{0a})^2} \cdot |(\mu,a),(\nu,a),a,a\rangle\!\rangle$$

The spectrum is then given by

$$Z_{(\rho_1,\rho_2,r),(\rho_1',\rho_2',r')} = \sum N_{\rho_1'\sigma}^{\rho_1} N_{\rho_2'\varrho}^{\rho_2} N_{de}^{f} N_{fc^+}^{g} N_{gb^+}^{h} N_{hr'}^{r}$$

$$\chi^{G_1/H}_{(\mu,\sigma)} \chi^{G_2/H}_{(\nu,\varrho)} \chi^{H}_{d} \chi^{H}_{e}$$

# Conclusions

- Symmetry breaking boundary conditions arise naturally in heterotic CFT's (e.g. asymmetric cosets) and the treatment of defect lines

- Explicit constructions are known for WZNW and (certain types of asymmetric) coset theories. They are based on

$$A(G) \longrightarrow A(G/H) \otimes A(H)$$

$$A(G/H) \longrightarrow A(G/U) \otimes A(U/H)$$

$$G \supset U \supset H$$

Open problems:

- Physical properties, eg. Casimir energies, correlation functions, ...

- Generalization to non-oriented surfaces