Quantum dimer models on the kagome lattice

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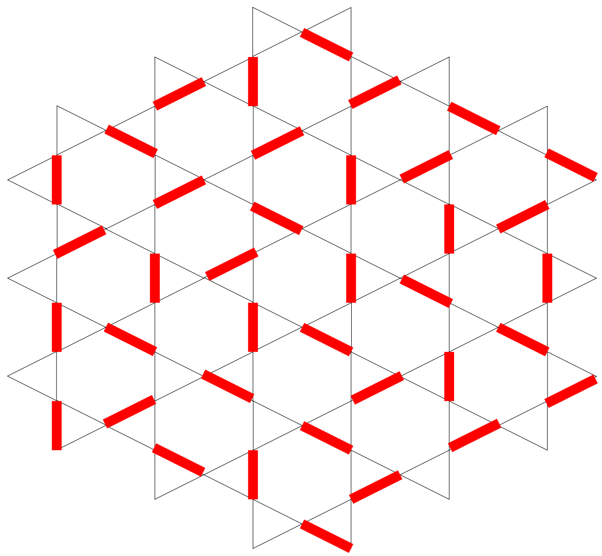
Integrable Models and Applications: from Strings to Condensed Matter

Florence, 15-20 September 2003

Dimer models

e.g. dimer covering of the kagome lattice



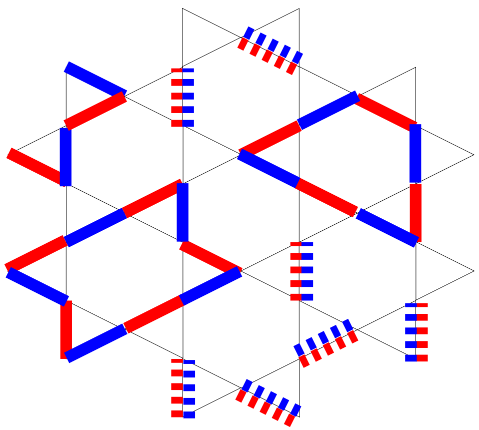
- **classical dimers**

[Kasteleyn[63]]: enumeration of dimer configurations on a planar lattice → Pfaffian → fermion partition functions

[M.Fisher[66]]: solution of the 2d Ising model via dimer representation

- **quantum dimer models:** Hilbert space $\leftrightarrow$ dimer configurations

  - dynamics: dimer moves around closed loops (resonances)



relevance: - RVB physics [Anderson[87]], [Rokhsar, Kivelson[88]]

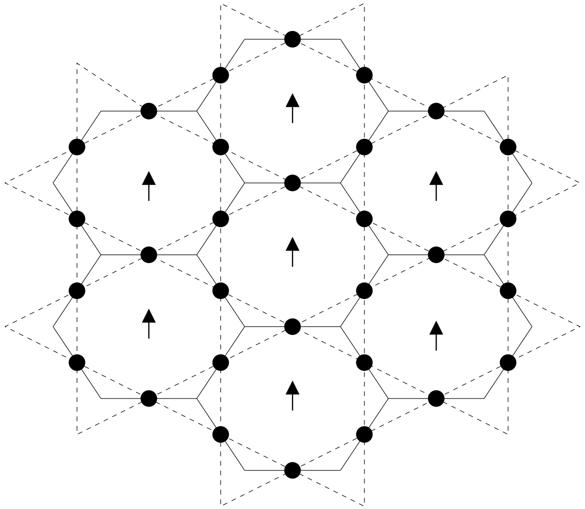
- frustrated quantum Heisenberg antiferromagnets (dimer $\sim$ spin singlet) [RK[88]], [Elser, Zeng[93]]

- quantum computation [Nayak, Shtengel[01]], [Ioffe et al.[02]]

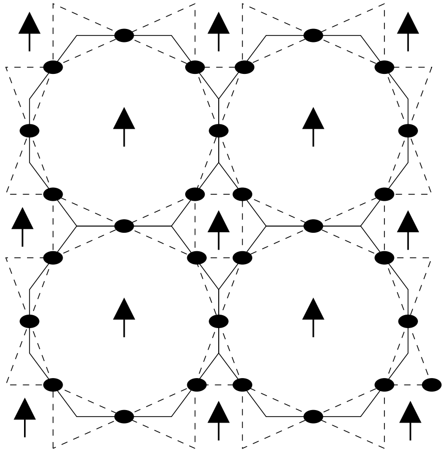
expected physics: various phases including dimer liquids [Moessner, Sondhi[01]], topological excitations, fractionalisation of quantum numbers, (spinon) deconfinement

Kagome lattice: medial lattice construction

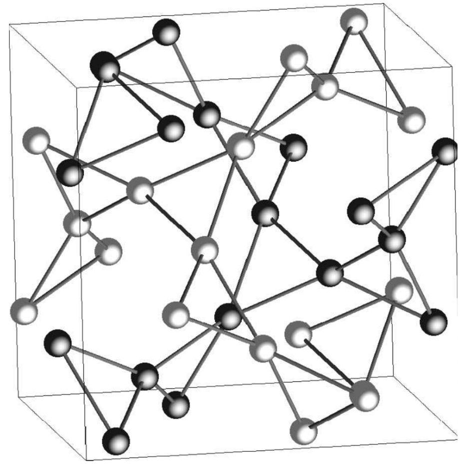
lattice K made by corner sharing triangles = medial lattice of a trivalent lattice H



kagome



squagome



GGG (gyroid)

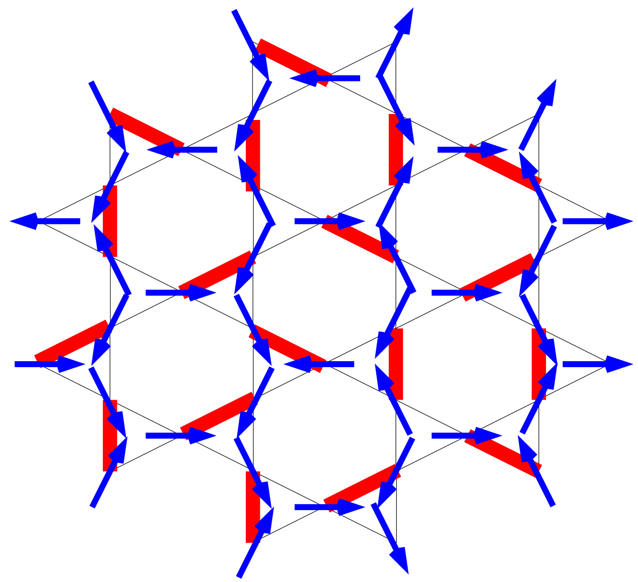
the dimer degrees of freedom can be mapped into

- Ising-like gauge degrees of freedom living on the links of H or into
- Ising spins living on the plaquettes of H.

- **Arrow representation:**

Z_2 gauge degrees of freedom

constraint: $n_{in} = 0 \bmod 2$



- **Ising spins**: - take a reference configuration of arrows and assign to it $\sigma_h^z = 1$ for all the plaquettes h

- any other configuration differs from the reference state on a collection of non-intersecting closed loops $\Rightarrow$ domain walls for σ_h^z

Notes: -the definition of σ_h^z is non-local (need for a reference spin $\sigma_0^z = 1$);

-on closed surfaces $\prod_h \sigma_h^x = 1$;

-topologically nontrivial loops $\rightarrow$ different topological sectors .

Solvable dimer model

The symplest dynamics preserving the gauge constraint: reverse the arrows around a plaquette h:

$$H = -\sum_h \sigma_h^x$$

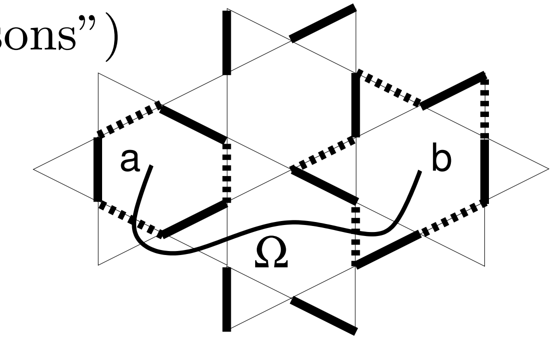
- **Ground state:** $\sigma_h^x = 1$

-RK state [Rokhsar, Kivelson[88]]: sum over all the dimer coverings with equal amplitudes; topologically degenerated

- dimer liquid: the correlation of dimers or of holes ("spinons") are strictly zero beyond few lattice spacings

- **Excitations:** pairs of Ising vortices ("visons")

$\sigma_a^x = \sigma_b^x = -1$: configurations are counted with the parity of the number of dimers crossing the string Ω



Adding a potential term:

$$H = -\Gamma \sum_h \sigma_h^x - J \sum_{<a,b>} \sigma_a^z \sigma_b^z$$

$2D$ Ising model in transverse field $\sim 3D$ Ising model
(dual to Ising gauge theory).

$\mathbf{J = 0} \quad \Rightarrow \quad <\sigma^z>= 0$: disordered or deconfined phase (dimer liquid)

$\mathbf{\Gamma = 0} \quad \Rightarrow \quad <\sigma^z>\neq 0$: ferromagnetic or confined phase (dimers "solidify" to the reference state)

Spin 1/2 Heisenberg model on the kagome lattice

- Numerical results [Waldtman et al[98]]:

- no magnetic long-range order

- possibly a small spin gap

- huge number of low-energy singlet states $\sim 1.15^N$

- Dimer approximation [Rokhsar, Kivelson[88]], [Elser, Zeng[93]]

$$\text{dimer} = \text{singlet} = \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

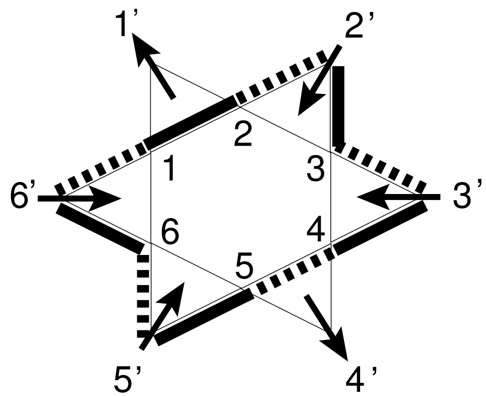
- projection of the Heisenberg Hamiltonian on the dimer space $\Rightarrow$ dimer moves around closed loops with signs

Dimer model with entropy

-retain only the dimer moves around one plaquette with their signs

$$\mathbf{H} = -\sum_{\mathbf{h}} \mu_{\mathbf{h}}$$

$$\mu_{\mathbf{h}} = (-\mathbf{1})^{\mathbf{n}_{\mathrm{out}}/\mathbf{2}}\ \sigma^{\mathbf{x}}_{\mathbf{h}}$$



- the moves on adjacent plaquettes do not commute anymore $\Rightarrow$ frustration

$$\begin{aligned} \{\mu_h, \mu_{h'}\} &= 0\,, \qquad \text{if } h, h' \text{ neighbors} \\ [\mu_h, \mu_{h'}] &= 0\,, \qquad \text{otherwise} \end{aligned}$$

$$\mu_h^2 = 1.$$

Symmetry

Another set of operators, $\tilde{\mu}$, obeying the same algebra as μ and commuting with the Hamiltonian:

$$[\tilde{\mu}_h, \mu_{h'}] = 0 , \qquad \text{for any } h, h' .$$

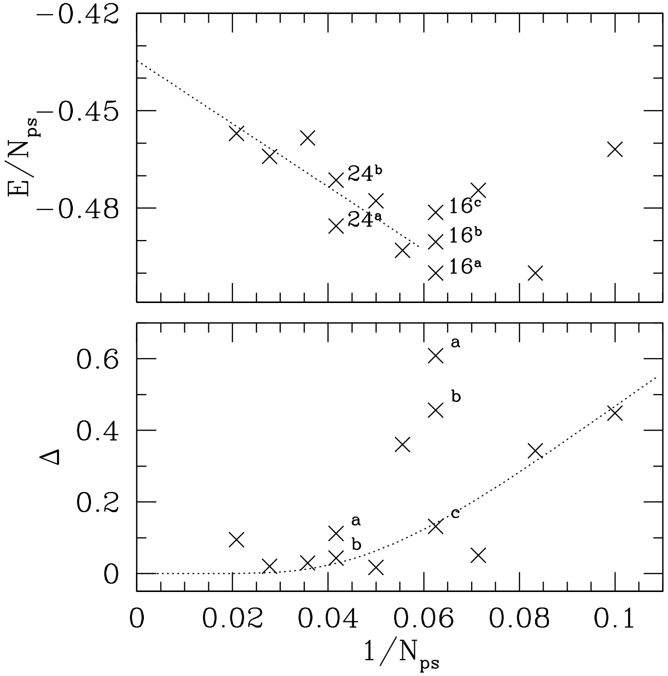
- the center of the algebra $\tilde{\mu}$: $\sim N_{\text{plaquettes}}/2 = N/6$ elements
$\Rightarrow$ degeneracy of the whole spectrum $\sim 2^{N/6} \simeq 1.12^N$

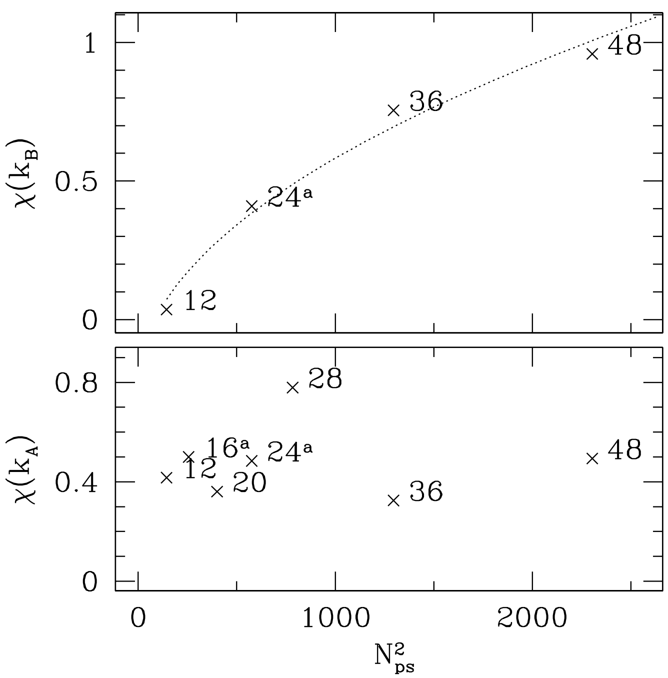
- ground states: no correlations between dimers beyond few lattice spacings $\longrightarrow$ dimer liquid

- reduce the dimension of the representation of μ $\Rightarrow$ treat numerically much larger systems, up to $N = 144$

Numerical results

- critical phase ?: - the gap may extrapolate to zero
  - no cristalline order in μ.





- insertion of static holes: beyond few lattice spacings, the holes do not interact (deconfinement?)

Conclusions

• lattices made of corner sharing triangles $\rightarrow$ exactly solvable models of quantum dimers

• Heisenberg antiferromagnets on these lattices all share the same properties (lack of magnetic ordering, proliferation of low-energy states) $\Rightarrow$ dimer models are good starting approximation ?

- exact solution of the associated dimer models?
- understand the origin of the degeneracy in the spin language?