Correlation Functions of Higher-Spin Analogues of the 8-Vertex Model

Robert Weston

Florence - Sept 2003

Plan

- The vertex-IRF correspondence
- Fusion
- Corr fns of vertex models in terms of SOS
- The CTM/Kyoto Approach
- $U_{q,p}(\widehat{sl_2})$
- The tail operator
- Comments/questions

With Hitoshi Konno, Takeo Kojima and Marco Rossi

The 8-vertex/IRF correspondence

- In the beginning there was Baxter, and SOS models were one bi-product of his soln of the 8-vertex model [B73].

  Baxter's 'intertwiners' $t_\varepsilon^{a,b}(u)$ and $t_\varepsilon^{*\,a,b}(u)$, $\varepsilon \in \{0,1\}$, $a,b \in \mathbb{Z}$, $a-b \in \{\pm\}$ obey:

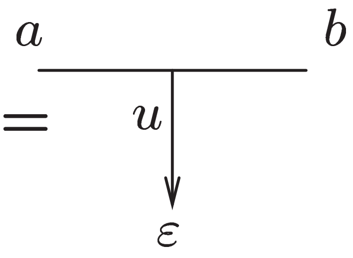
$$\sum_{\tilde\epsilon_1,\tilde\epsilon_2} R(v-u)_{\varepsilon_1,\varepsilon_2}^{\tilde\epsilon_1,\tilde\epsilon_2}\, t_{\tilde\epsilon_1}^{a,b}(v)\, t_{\tilde\epsilon_2}^{b,d}(u)$$

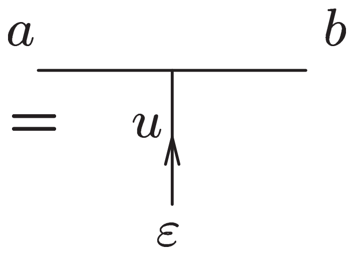
$$= \sum_c t_{\varepsilon_2}^{a,c}(u)\, t_{\varepsilon_1}^{c,d}(v) W\left(\begin{array}{cc|c} a & b & \\ & & v-u \\ c & d & \end{array}\right)$$

$$\sum_{\tilde\epsilon_1,\tilde\epsilon_2} t_{\tilde\epsilon_1}^{*\,d,c}(v)\, t_{\tilde\epsilon_2}^{*\,c,a}(u)\, R(v-u)_{\tilde\epsilon_1,\tilde\epsilon_2}^{\varepsilon_1,\varepsilon_2}$$

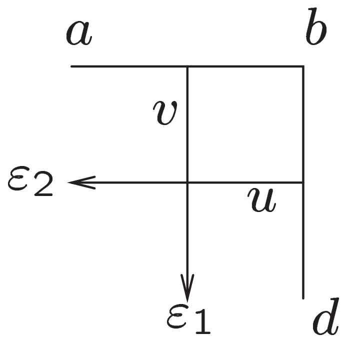
$$= \sum_b W\left(\begin{array}{cc|c} a & b & \\ & & v-u \\ c & d & \end{array}\right) t_{\varepsilon_2}^{*\,d,b}(u)\, t_{\varepsilon_1}^{*\,b,a}(v).$$

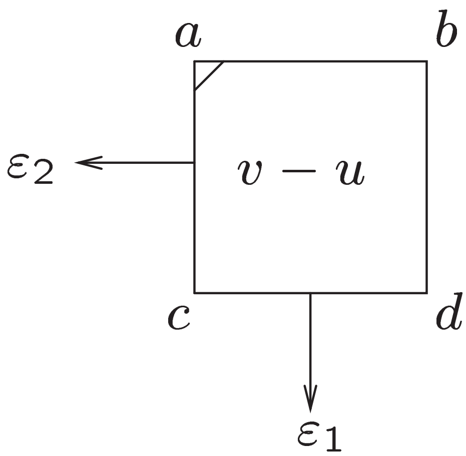
- Pictures: If

$t_{\varepsilon}^{a,b}(u) =$ 

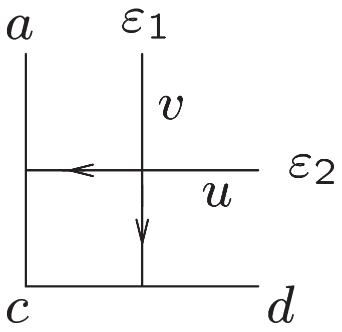
$t_{\varepsilon}^{*\,a,b}(u) =$ 

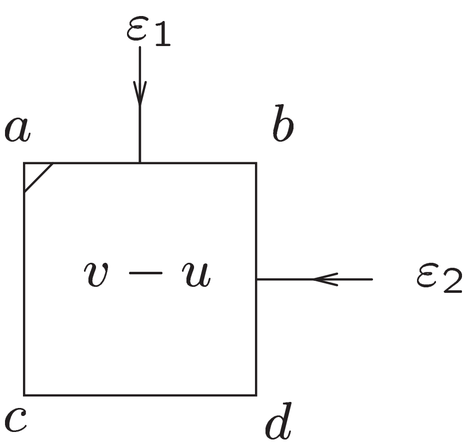
then



$= \sum_{c}$


and

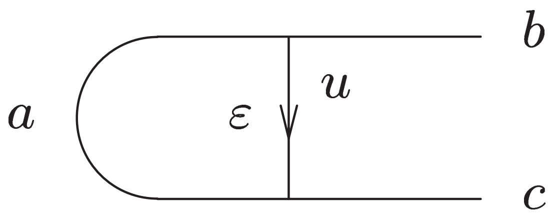


$= \sum_{b}$


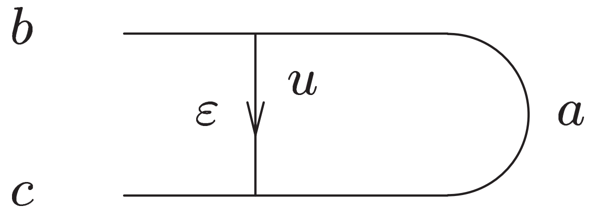
- Another useful property of intertwiner is

$$\sum_{\varepsilon} t^{*\,c,a}_{\varepsilon}(u)\, t^{a,b}_{\varepsilon}(u) = \delta_{c,b}$$

Represented graphically by

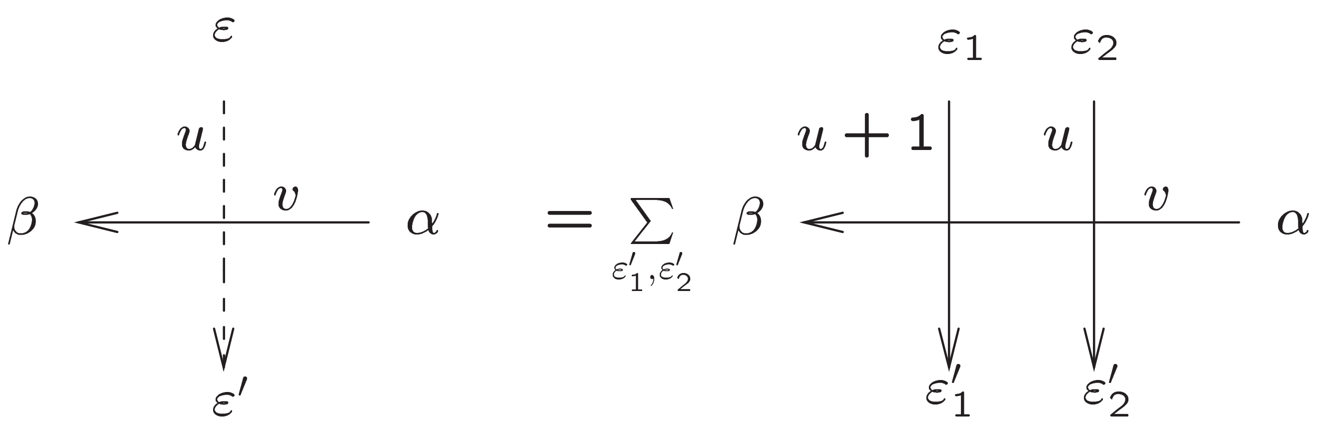
$\sum_{\varepsilon}$  $= \delta_{b,c}$

But

$\sum_{\varepsilon}$  $\neq \delta_{b,c}$

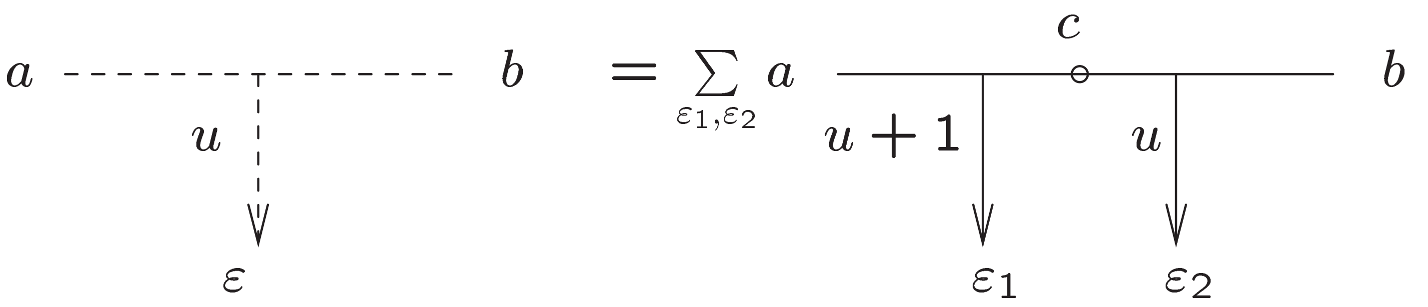
Fusion

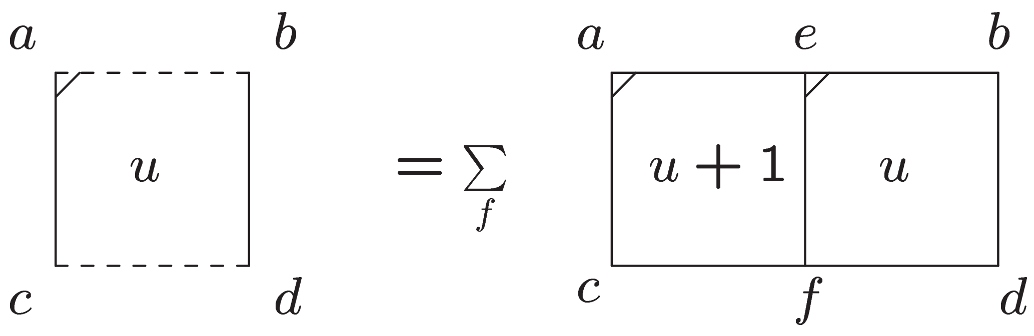
- Fusion of 8-vertex model [KRS 81]:



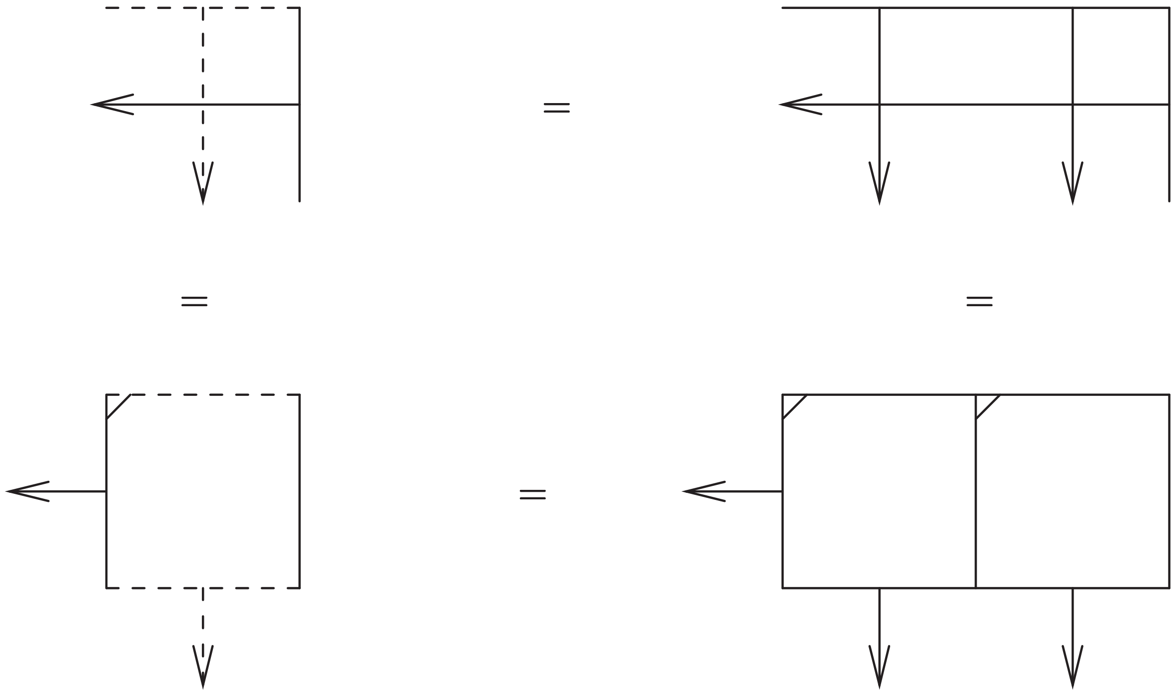
where $\varepsilon = \varepsilon_1 + \varepsilon_2$ and $\varepsilon'_1 + \varepsilon'_2 = \varepsilon'$.

- Fusion SOS and intertwiner [DJMO86]:



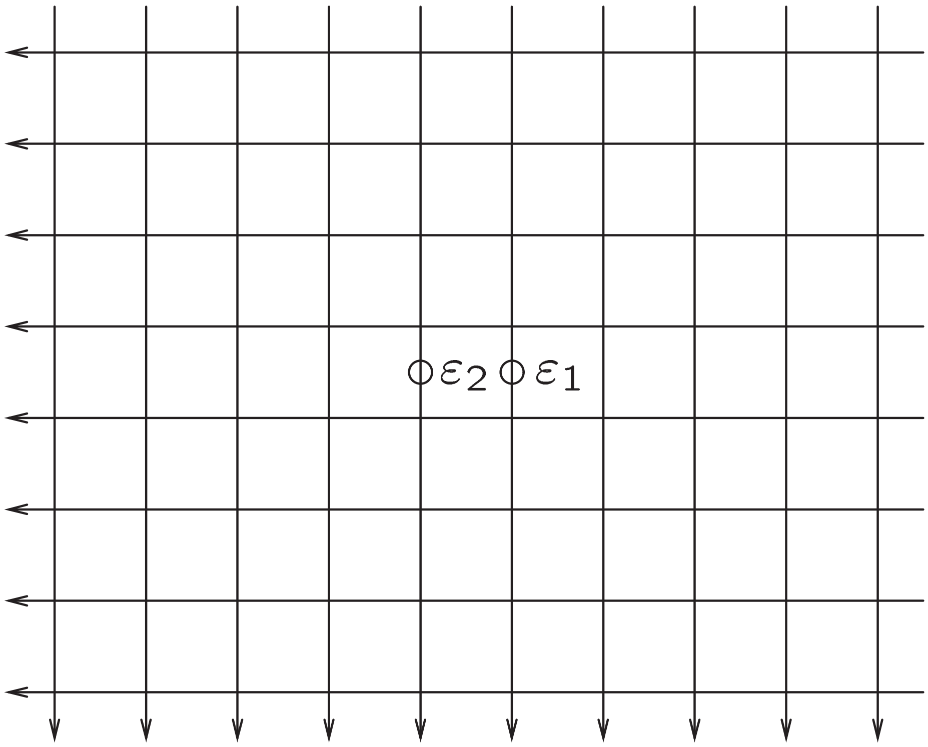


- Constructed to be compatible with with vertex/IRF correspondence, i.e.,



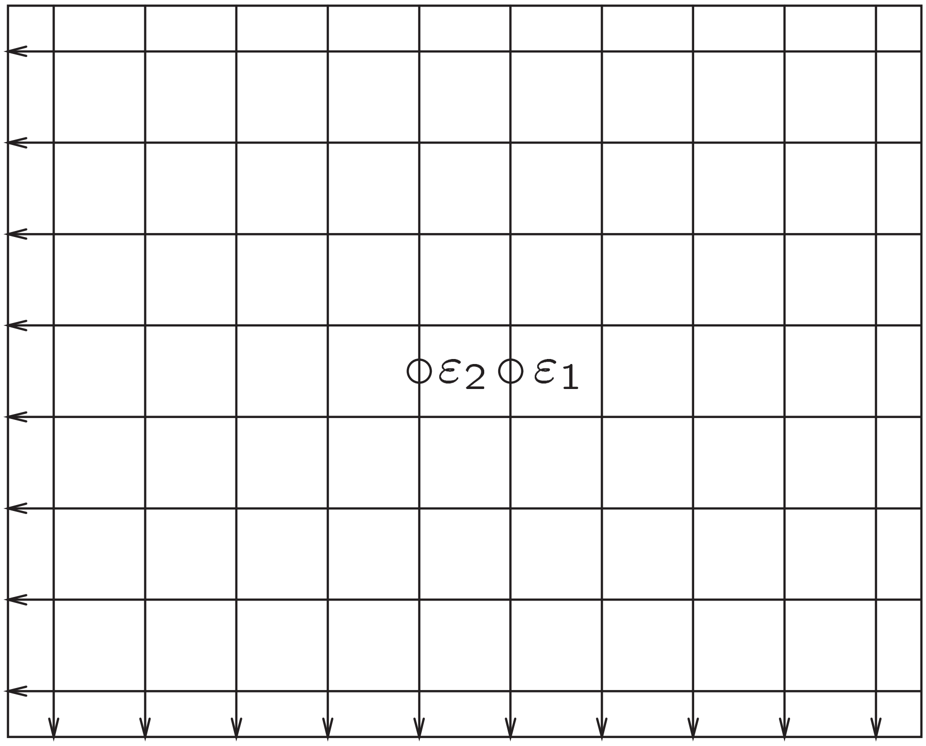
Level-k Fusion 8-Vertex Correlation Functions

These are restricted sums, i.e., sums over all configs with some fixed boundary condition (labelled by $i \in \{0, 1, \cdots, k\}$ for AF model), and with some other spins fixed, e.g.,



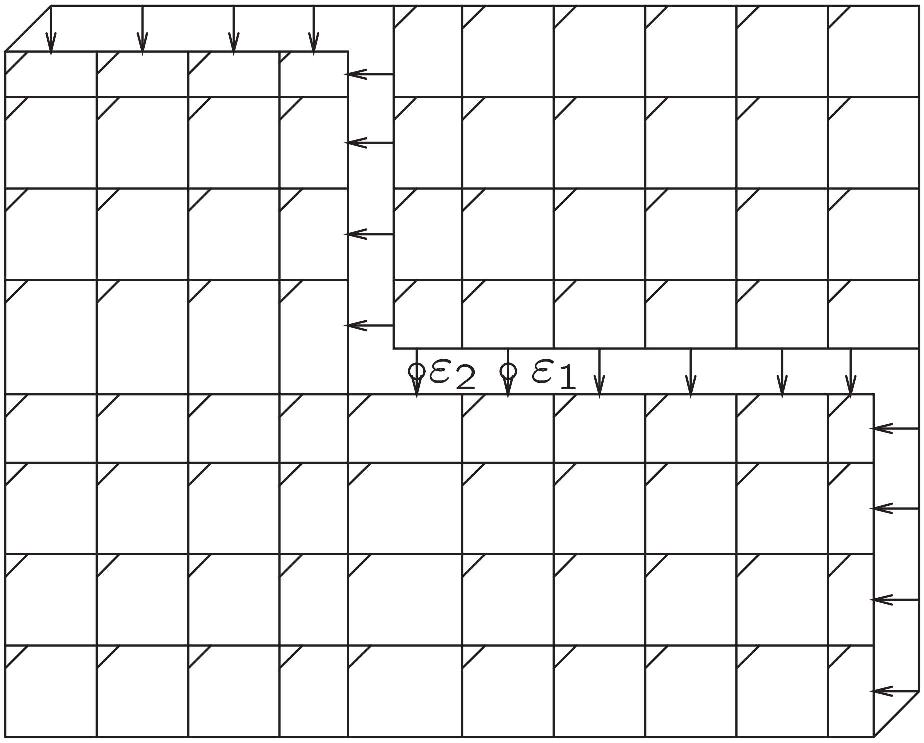
We calculate by connecting with SOS correlation fns (for which a free-field realization may be used) [LaP98].

- First, we dress the boundary with t and t^*

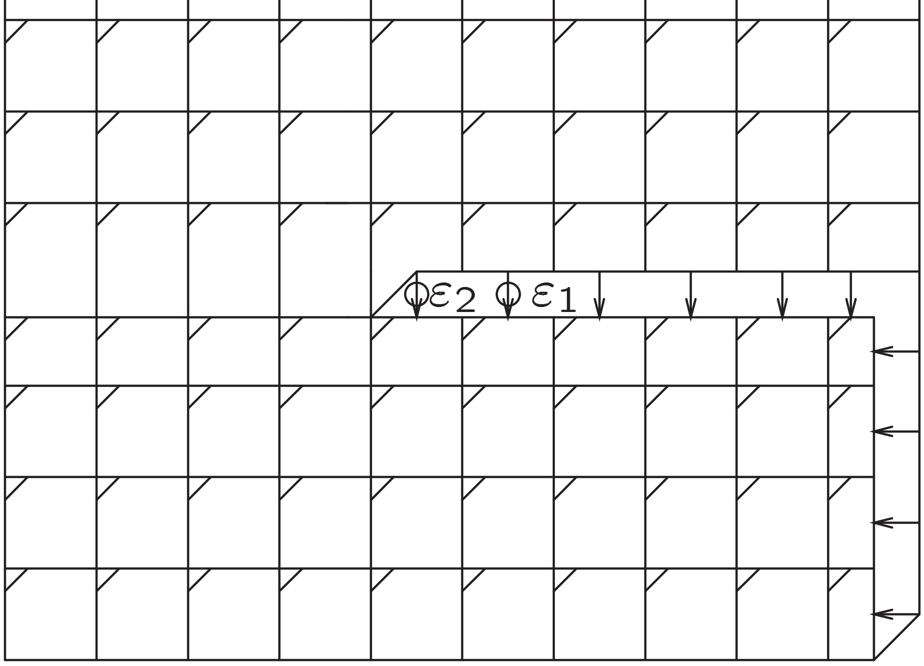


As long as these preserve boundary condition, they will not contribute to correlation fn in ∞ volume limit.

- Then use vertex/IRF correspondence to rewrite sum as



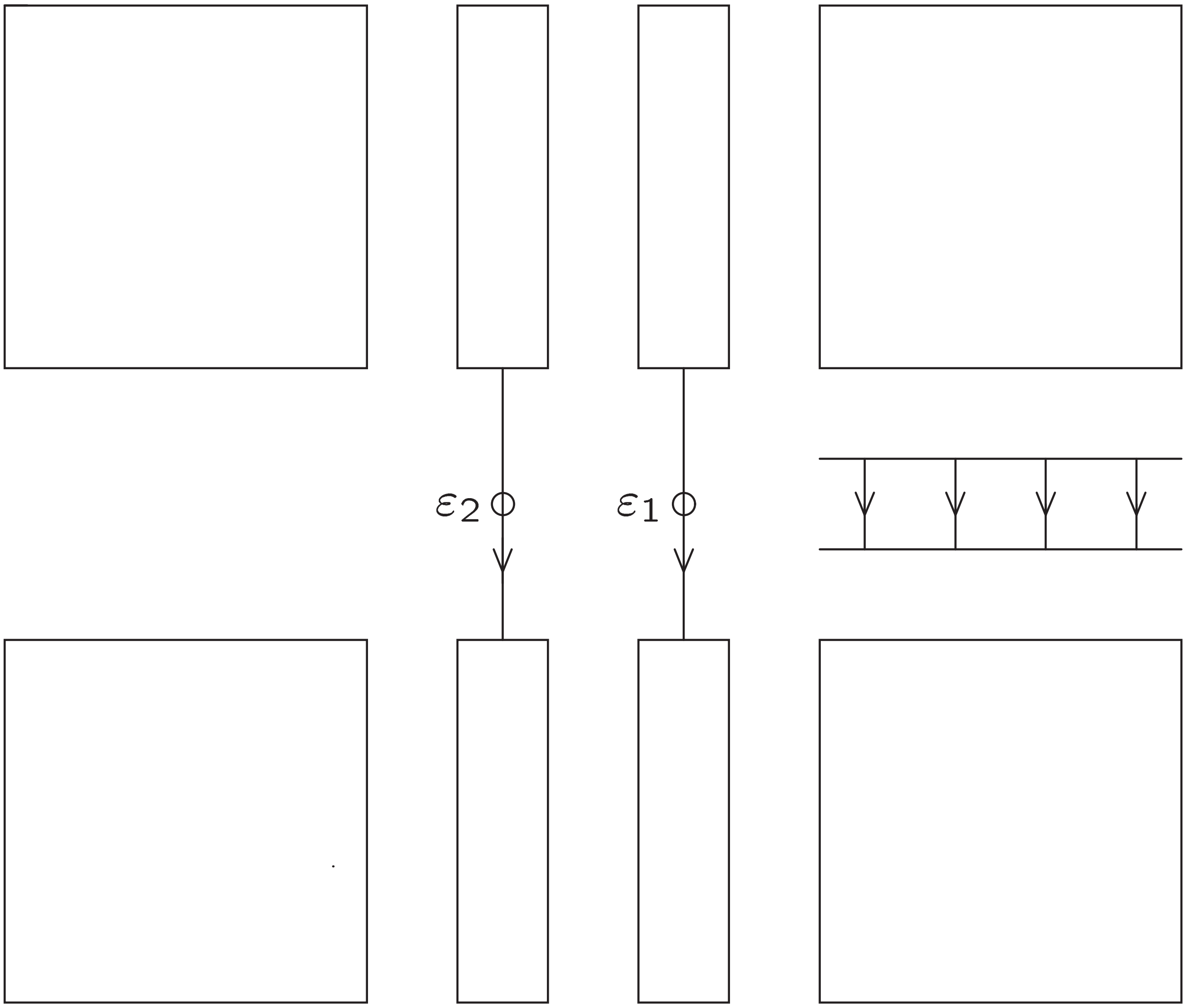
- Finally, we use unitarity to rewrite as



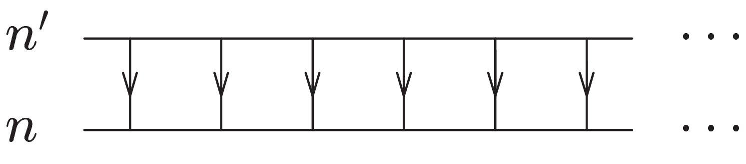
We are stuck with the horizontal tail operator shown

The CTM/Kyoto Approach

Write sum as trace over 9 operators [FJMMN94]. Schematically:



New part over pure SOS models is 'tail operator' $\Lambda_{n',n}(u)$ $(n - n' \in \mathbb{Z})$ defined graphically by:



$U_{q,p}(\widehat{sl_2})$

- $U_{q,p}(\widehat{sl_2})$ is elliptic algebra generated by currents $E(z)$, $F(z)$ and $k(z)$ with certain relations [K98].

- 8 SOS operators and space on which they act can be identified in terms of free-field realization of $U_{q,p}(\widehat{sl_2})$ at level k [LuP96,JKOS99].

- Using this, one can compute trace that occurs in SOS correlation function expression. Get multiple integral expression for corr fns.

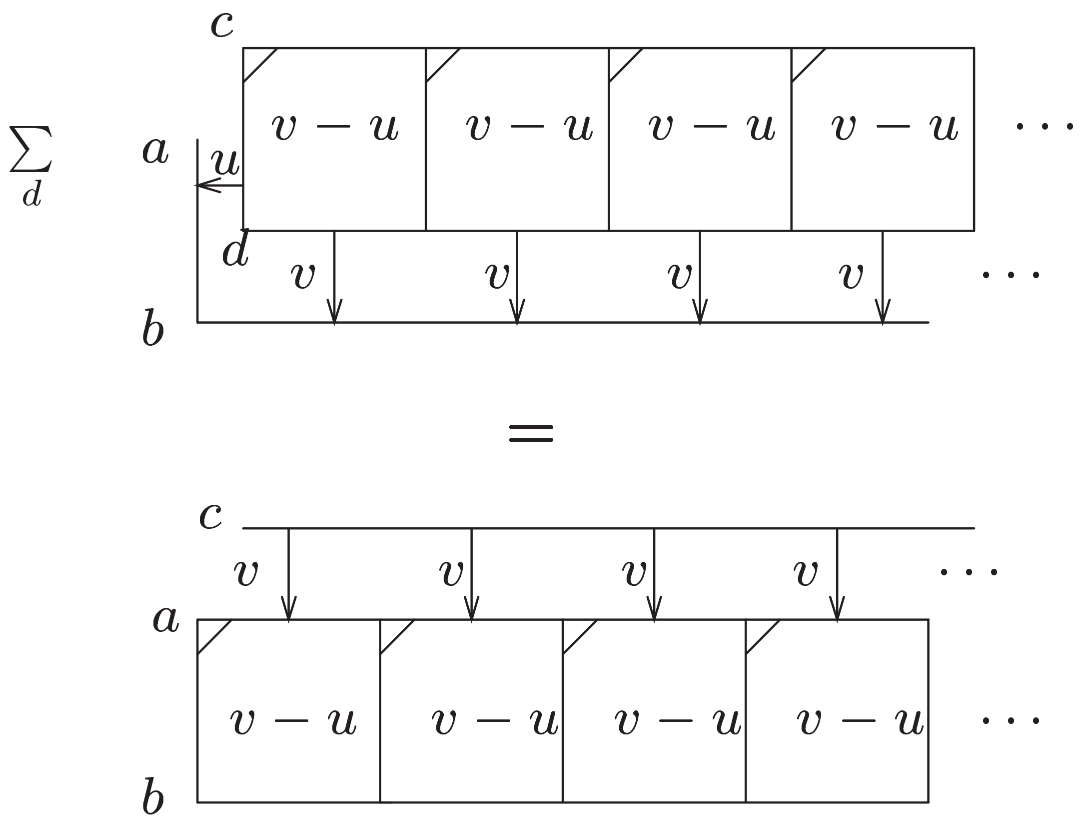
- What about $\Lambda_{n',n}(u)$?

The Tail Operator $\Lambda_{n',n}(u)$

The tail operator obeys 2 requirements

I) Unitarity: $\Lambda_{n,n}(u) = 1$

II)



Solution:

$$\Lambda_{n',n}(u) \sim F^{+}(-u-2)^{|n-n'|}$$

Hence correlation fns of vertex models (for $k=2$).

Comments/Questions

- Currently working with general formula to get spontaneous polarisation

- What is algebraic origin of $t_{\varepsilon}^{a,b}(u)$ and $t_{\varepsilon}^{*a,b}(u)$ [BBB96]?

- Why is tail operator of this form?

References

[B73] R. Baxter: Annals of P, 76, 25-47 (1973)

[BBB96] O. Babelon, D. Bernard, E. Billey, PLB 375, 89 (1996)

[DJMO86] E. Date, M. Jimbo, T. Miwa, M. Okado: LMP 12, 209-215 (1986)

[FJMMN94] O. Foda, M. Jimbo, T. Miwa, K. Miki, A. Nakayashiki: JMP 35,13-46 (1994)

[JKOS99] M. Jimbo, H. Konno, S. Odake, J. Shiraishi: CMP 199, 605-647 (1999)

[K98] H. Konno: CMP 195, 373-403 (1998)

[KRS81] P.P. Kulish, N. Yu. Reshetikhin, E.K. Sklyanin LMP 5, 393 (1981)

[LaP98] M. Laskevich, Y. Pugai: NP B516, 623-651 (1998)

[LuP96] S. Lukyanov, Y. Pugai: NP B473, 631-658 (1996)