Istituto Nazionale di Fisica Nucleare Consiglio Nazionale delle Ricerche University of Trento INFM BOSE EINSTEIN CONDENSATION

# Thermal effects of the Casimir force for surface-atom and surface-surface configurations

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-- **V. Svetovoy** (University of Twente, the Netheralnds) (surface-surface configuration)

Conference QMFPA 2006, Bertinoro (Italy), 4-7 Dic. 2006

# Atom-Atom force

- ***Boyle and Gay-Lussac*** **ideal gas laws** $PV = nRT$ could be explained by the kinetic theory of non-interacting point atoms (Joule, Kroning, Clausius,..), but are **hardly exact**

- ***J.D. van der Waals*** (1873): eq. of state $\left(P + \frac{a}{v^2}\right)(V - b) = nRT$

- ***London*** (1930!): interaction potential between two atoms due to fluctuations of the atomic electric dipole moment **d**

$$V_{VL} \propto -\frac{1}{r^6}$$

$$\langle d_i \rangle = 0, \langle d_i^2 \rangle \neq 0, \vec{d} = \alpha \vec{E}$$

➡ ***dispersion forces*** (it is necessary only that $\alpha \neq 0$, the vacuum is a q.s. with **observable physical consequences!)**

+ ***orientation forces*** (***Keesom***,T, perm. dipoles)
+ ***induction forces*** (***Debye***,q-d) = 3 types vdW forces

- ***Casimir*** and ***Polder*** (1947): inclusion of retardation effect $c \neq \infty$ and at large distance $r >> \lambda_c$

$$V_{CP} \propto -\frac{1}{r^7}$$

# Lifshitz Theory

- The sum of the vdW force between the atoms of the **two plates**, assuming a pairwise potential V=-B/r^n, was experimentally wrong!

- the vdW force is not additive: the force between two atoms depends on the presence of a third atom

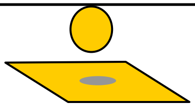
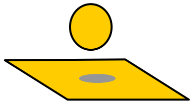
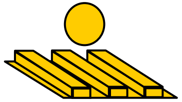
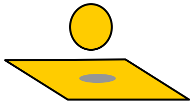
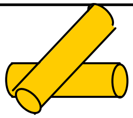
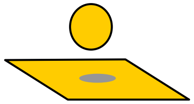
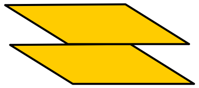
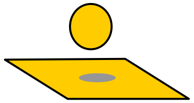
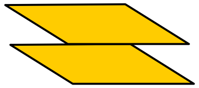
- ***Lifshitz*** (1955), ***Dzyaloshinskii*** and ***Pitaevskii*** (1961) developed a **Macroscopic General Theory of the vdW Forces** motivated by the experimental discrepancy with microscopic-additive theories

**I.E. Dzyaloshinskii, E.M. Lifshitz and L.P. Pitaevskii, Advances in Physics 10, 165 (1961).**

-***Lifshitz*** (Macroscopic Electrodynamics) assumed the dielectrics characterized by **randomly fluctuating sources** as demanded by the **FDT** and solved the Maxwell equations using the Green function method

-***Ginzburg*** (1979): "the calculations are so cumbersome that they were not even reproduced in the relevant Landau and Lifshitz volume where, as a rule, all important calculations are given"

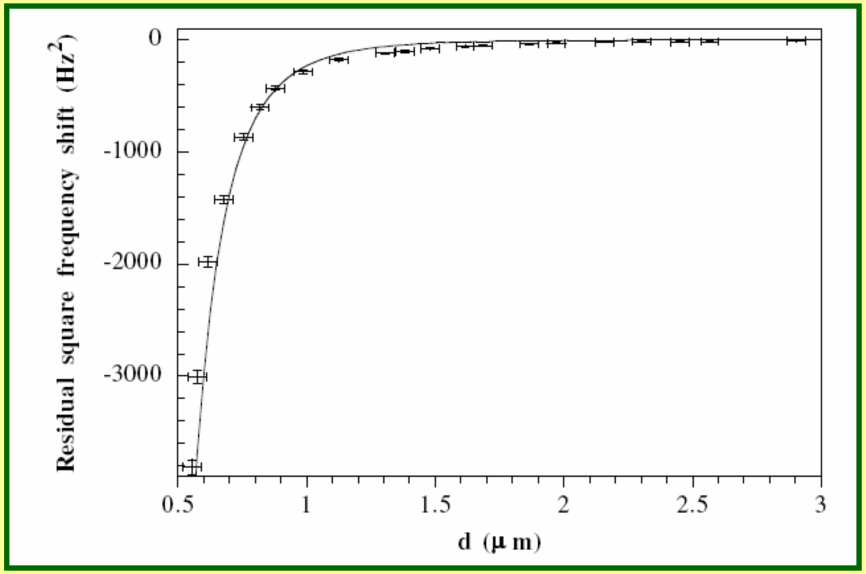
# Recent Measurements of Casimir Force

| Investigators | Year | Geometry | Method | Distance Scale (nm) | Materials | Pressure (mbar) | Temp (K) | Accuracy (%) |
|---|---|---|---|---|---|---|---|---|
| S. K. Lamoreaux | 1997 |  | Torsion pendulum | 600 - 6000 | Au(500nm) | $10^{-4}$ | 300 | 5 |
| U. Mohideen & A. Roy | 1998 |  | AFM | 100 - 900 | Al (300nm) + AuPd (20nm) | 5x$10^{-2}$ | 300 | 2 |
| A. Roy and U. Mohideen | 1999 |  | AFM | 100 - 900 | Al (250nm)+ AuPd (8nm) | 5x$10^{-2}$ | 300 | 2 |
| G. L. Klimthitskaya, A. Roy, U. Mohideen and V. M. Mostepanenko | 1999 |  | AFM | 100 - 900 | Al (300nm) + AuPd (20nm) | 5x$10^{-2}$ | 300 | 1 |
| T. Ederth | 2000 |  | Piezo-tube manipulator | 20 - 100 | 50µm Au wires coated in thiol SAM | 1000 | 300 | 1 |
| H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop & F. Capasso | 2001 |  | MEMS torsion bar capacitance | 90 - 1000 | Au (200nm) + Cr underlayer | 1000 | 300 | 1 |
| G. Bressi, G. Carugno, R. Onofrio & G. Ruoso | 2002 |  | Interfero-metry | 500 - 3000 | Cr (50nm) on Si | $10^{-5}$ | 300 | 15 |
| R. S. Decca, D. Lopez, E. Fischbach & D. E. Krause | 2003 |  | MEMS torsion bar capacitance | 200 - 2000 | Cu/Au | $10^{-4}$ | 300 | 1 |
| NANOCASE | 2005 - |  | AFM, MEMS | 10 - 1000 | Si, Au | $10^{-11}$ | 20 - 1000 | <1 |

# Measurement of Casimir-Polder and Lifshitz force

- Behaviour of **Casimir-Polder force** well **explored** experimentally at **short distances** (mainly forces between metallic bodies)

*Bressi et al. PRL 2002*
*(plate-plate configuration)*



- Behaviour at **larger distances** (few microns) **less explored**. In particular thermal effects of the force not yet measured

- **Cold atoms** are natural **candidates** to explore **thermal** effects of the force at moderately large distances (5-10 microns).

**Surface-atom interaction** has been the object of systematic experimental and theoretical studies in recent years.

***Motivations :***

- Open theoretical and experimental questions
  (ex: role of **e.m. thermal fluctuations, usually masked**)
- Perspectives for applications **(atom chips, ..)**
- New constraints on hypothetical **non-Newtonian forces** at short distances

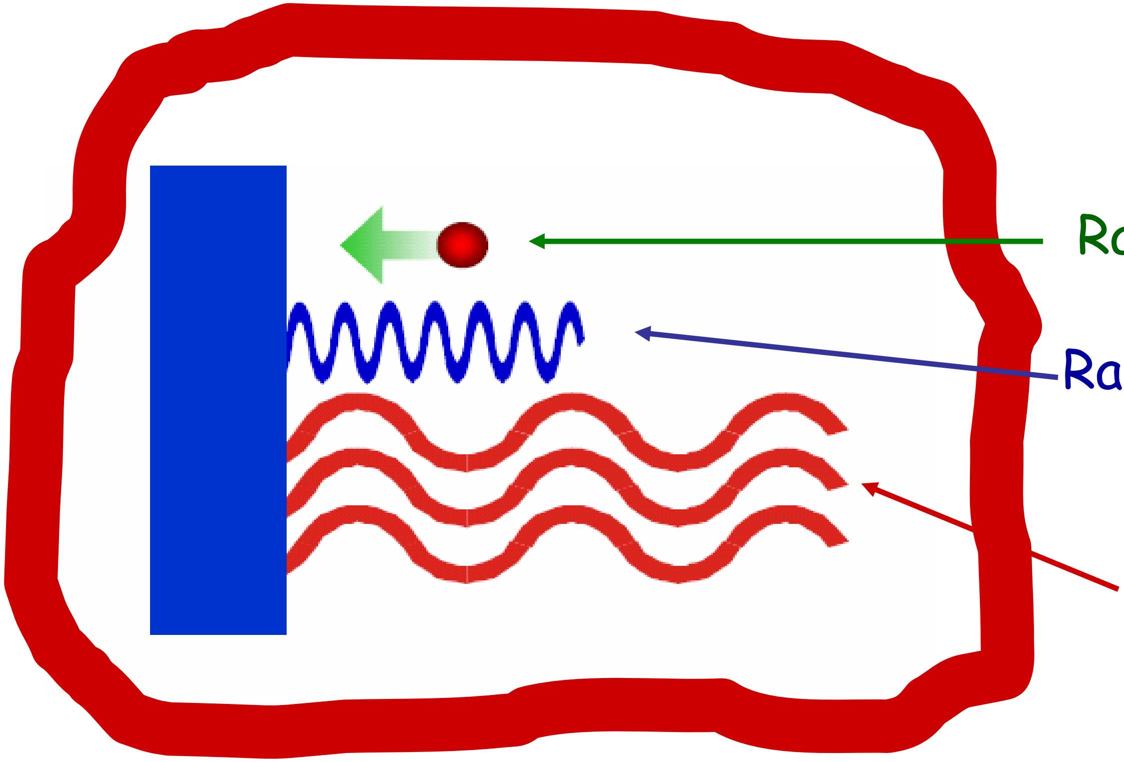
# Experiments with cold atoms about surface-atom interaction

- Shih and Parsegian (1975): deflection of atomic beam (VL)
- Anderson (1988): deflection of atomic beam (VL), Rydberg atoms
- Hinds (1993): deflection of atomic beam (CP)
- Aspect (1997): reflection from atomic mirror
- Shimizu (2001, 2005): reflection from solid surface
- Vuletic (2004): BEC stability near surfaces
- Ketterle (2004): BEC reflection from solid surface
- Cornell (2005): BEC center of mass oscillation (CP)
- Cornell (2006): BEC center of mass oscillation (Thermal)

# Plan of the talk

- Surface-Atom force at thermal equilibrium
- Surface-Atom force out of thermal equilibrium
- BEC oscillations: recent experimental results
- Surface-Surface force out of thermal equilibrium

# Surface-atom force



Radiation from the atom

Radiation from the substrate $\boxed{T_S}$

Radiation from
the environment (cell) $\boxed{T_E}$

$$\vec{F}(\vec{r})=\left\langle d_i^{tot}(t)\vec{\nabla}E_i^{tot}(\vec{r},t)\right\rangle\approx\left\langle d_i^{ind}(t)\vec{\nabla}E_i^{fl}(\vec{r},t)\right\rangle+\left\langle d_i^{fl}(t)\vec{\nabla}E_i^{ind}(\vec{r},t)\right\rangle$$

Force includes **zero-point** (**or vacuum**) fluctuations effects +
**thermal** (**or radiation**) fluctuations effects (**crucial at large distances!**)

## Electric Field

$$\vec{E}[\omega;\vec{r}] = \int_V \overline{G}[\omega;\vec{r},\vec{r}\,'] \bullet \vec{P}[\omega] \quad \mathrm{d}\vec{r}$$

## Fluctuations Dissipation Theorem

$$\left\langle P_i^{fl}[\omega;\vec{r}]P_j^{fl+}[\omega',\vec{r}\,']\right\rangle_S = \frac{\hbar\varepsilon''(\omega)}{2} \coth\left(\frac{\hbar\omega}{2k_B T}\right) \delta(\omega-\omega')\delta(\vec{r}-\vec{r}\,')\delta_{ij}$$

# Result at thermal equilibrium: L-D-P Theory

$$F^{eq}(T,z)=\frac{\hbar}{\pi}\int_0^{\infty}d\omega\coth\left(\frac{\hbar\omega}{2k_BT}\right)\mathrm{Im}\left[\alpha(\omega)\partial_{z_2}G_{ii}[\omega;\vec{r}_1,\vec{r}_2]\Big|_{\vec{r}_1=\vec{r}_2=\vec{r}}\right]$$

$$\coth\left(\frac{\hbar\omega}{2k_BT}\right)=1+\frac{2}{e^{\hbar\omega/k_BT}-1}$$

Vacuum fluctuations : T=0

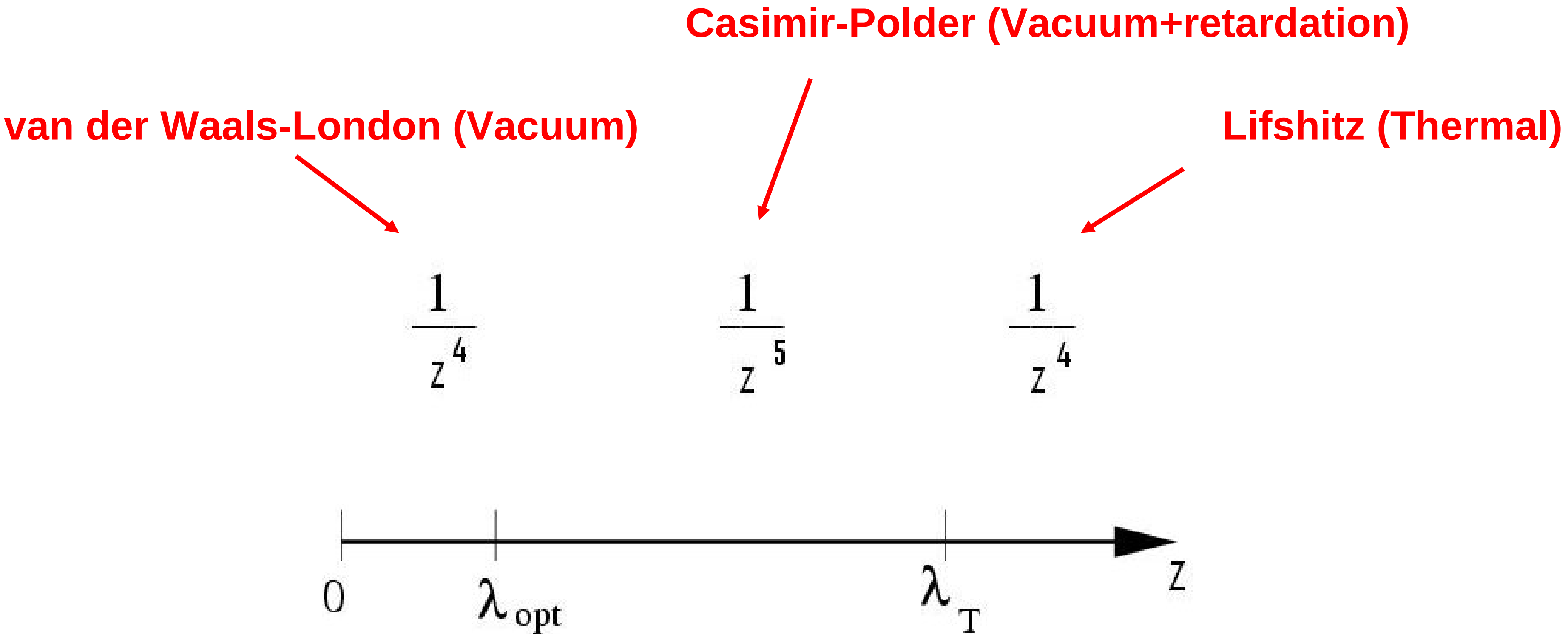
Thermal fluctuations

$$F^{eq}(T,z)=F_0(z)+F_{th}^{eq}(T,z)$$

# Relevant length scales at equilibrium

-**Optical** length $\lambda_{opt}$ fixed by optical properties of the substrate ( typically fractions of microns)

- **Thermal photon** wavelength ( $\lambda_T = \hbar c / k_B T \approx 7.6 \mu m$ at room temperature)



**Casimir-Polder (Vacuum+retardation)**

**van der Waals-London (Vacuum)**

**Lifshitz (Thermal)**

$$\frac{1}{z^4} \qquad \frac{1}{z^5} \qquad \frac{1}{z^4}$$

# Asymptotic behaviour at thermal equilibrium

$$z << \lambda_{opt}$$

$$F_0(z) \rightarrow F_{VL}(z) = -\frac{3\hbar}{4\pi z^4}\int_0^\infty \alpha(i\xi)\frac{\varepsilon(i\xi)-1}{\varepsilon(i\xi)+1}d\xi$$

$$\varepsilon(i\xi) = 1 + \frac{2}{\pi}\int_0^\infty \omega\frac{\varepsilon''(\omega)}{\omega^2+\xi^2}d\omega$$

$$\alpha(i\xi) = \frac{2}{\pi}\int_0^\infty \omega\frac{\alpha''(\omega)}{\omega^2+\xi^2}d\omega$$

$$\lambda_{opt} << z << \lambda_T$$

$$F_0(z) \rightarrow F_{CP}(z) = -\frac{3}{2}\frac{\hbar c\alpha_0}{\pi z^5}\frac{(\varepsilon_0-1)}{(\varepsilon_0+1)}\phi(\varepsilon_0)$$

only static optical properties $(\alpha_0 , \varepsilon_0)$

$$z >> \lambda_T$$

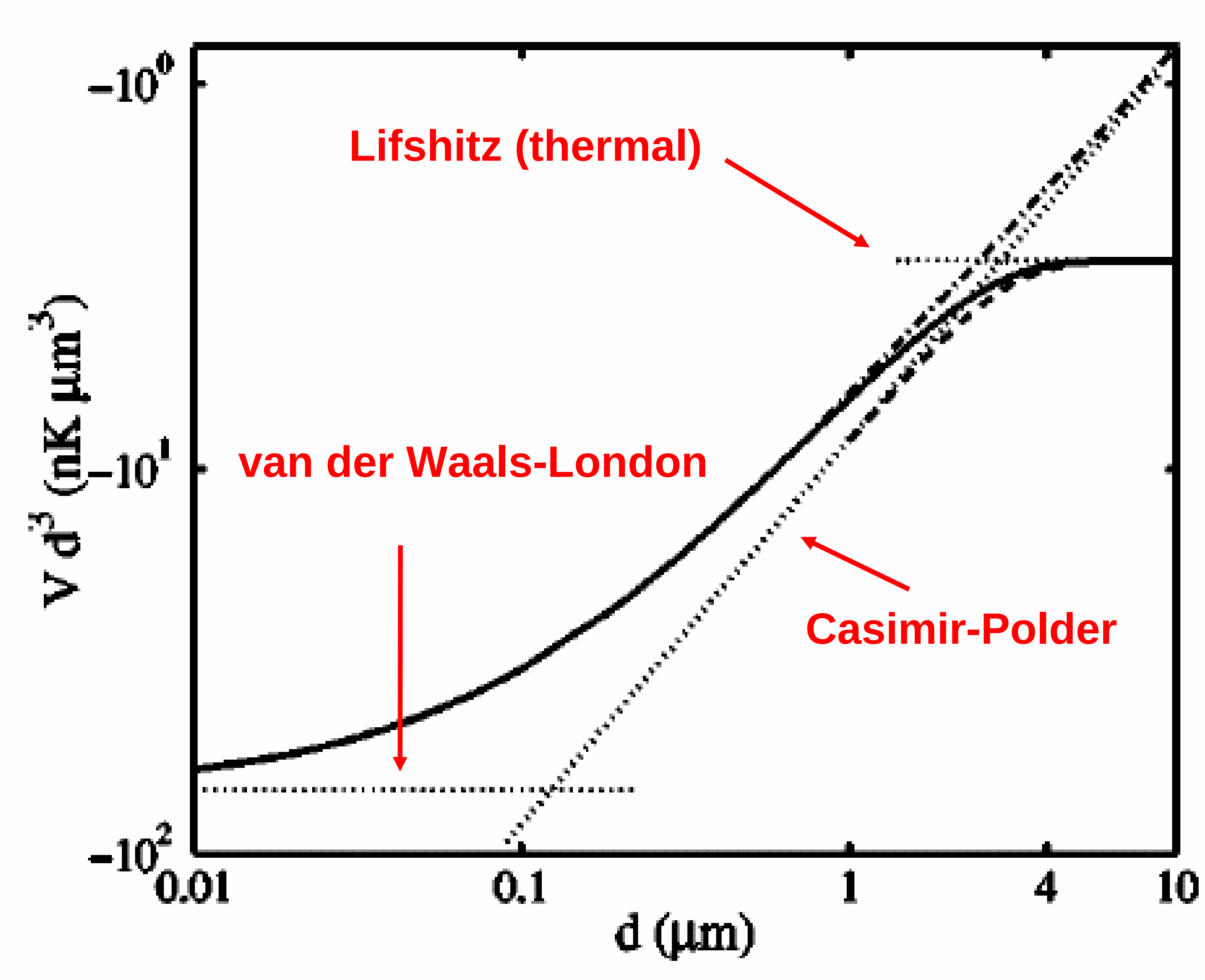
$$F^{eq}(T,z) \rightarrow F_{Lif}(T,z) = -\frac{3}{4}\frac{kT\alpha_0}{z^4}\frac{(\varepsilon_0-1)}{(\varepsilon_0+1)}$$

increases linearly with T

- Only **static optical properties** $(\alpha_0 , \varepsilon_0)$ characterize the asymptotic behaviour of Casimir-Polder and thermal (Lifshitz) forces
- At smaller distances (van der Waals regime) dynamical optical properties $\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$ and $\alpha(\omega) = \alpha'(\omega) + i\alpha''(\omega)$ are needed

# Surface (sapphire) atom (rubidium) interaction at T=300K

[ M. Antezza, L.P. Pitaevskii, S.Stringari, Phys.Rev A70, 053619 (2004)]



- Casimir-Polder force already detected in various experiments
- How to detect thermal effects ?

-Surface-atom force extremely **weak at large distances**
(typically 10E-4 gravity at 4-5 microns)

- **At room temperature thermal effects** prevail only above 5-6 microns and are consequently **difficult to measure**

-***Possible strategies:***

- ***increase T***
  (thermal effect increases linearly with T, but vacuum in the chamber?)

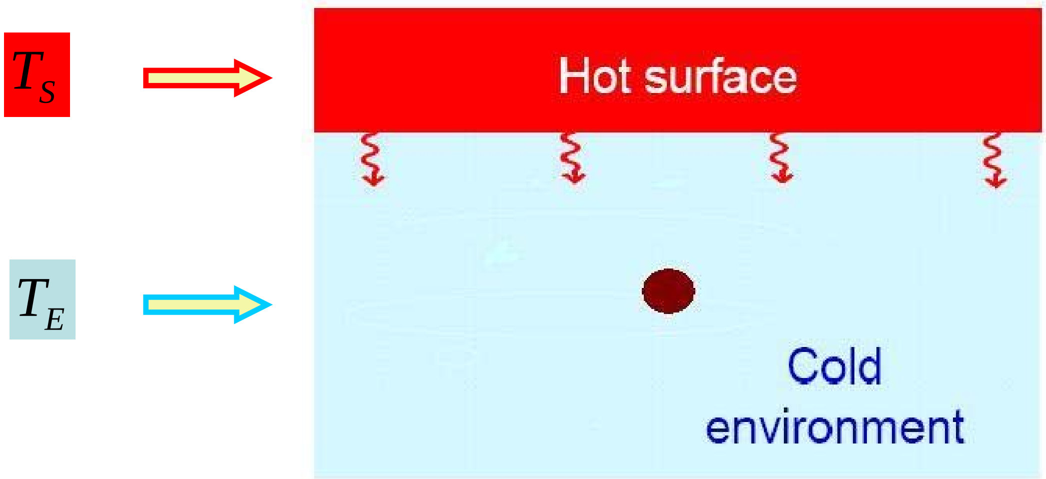
- ***out of thermal equilibrium*** configurations
  (if surface is hotter than environment thermal effect increases **quadratically with surface temperature**)

# Surface-atom force out of thermal equilibrium

- **Thermal effect** in surface-atom force can be **tunable** by varying substrate and environment temperatures.

- What happens if substrate and environment **temperatures are different** ?

- How to **describe environment** radiation and to calculate **field average values**?



**Or viceversa: cold surface and hot environment**

short distances

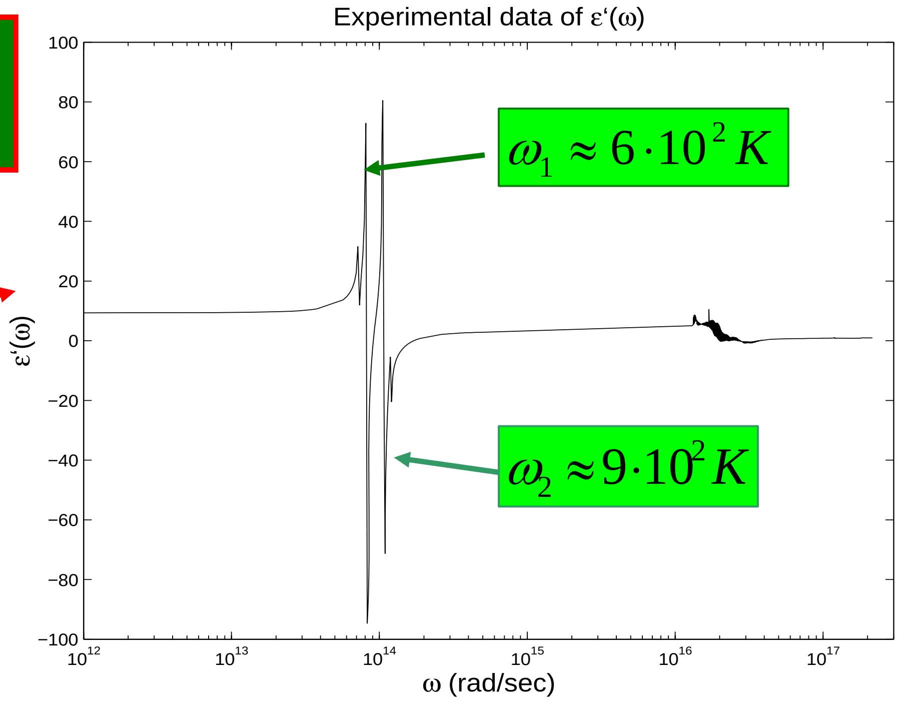
C. Henkel, K. Joulain, J.-P. Mulet and J.-J. Greffet, J. Opt. A **4**,S109 (2002)
M. Antezza, L.P. Pitaevskii and S. Stringari, PRL **95**, 113202 (2005)

medium and large distance behaviour

# Dynamic dielectric and polarizability functions

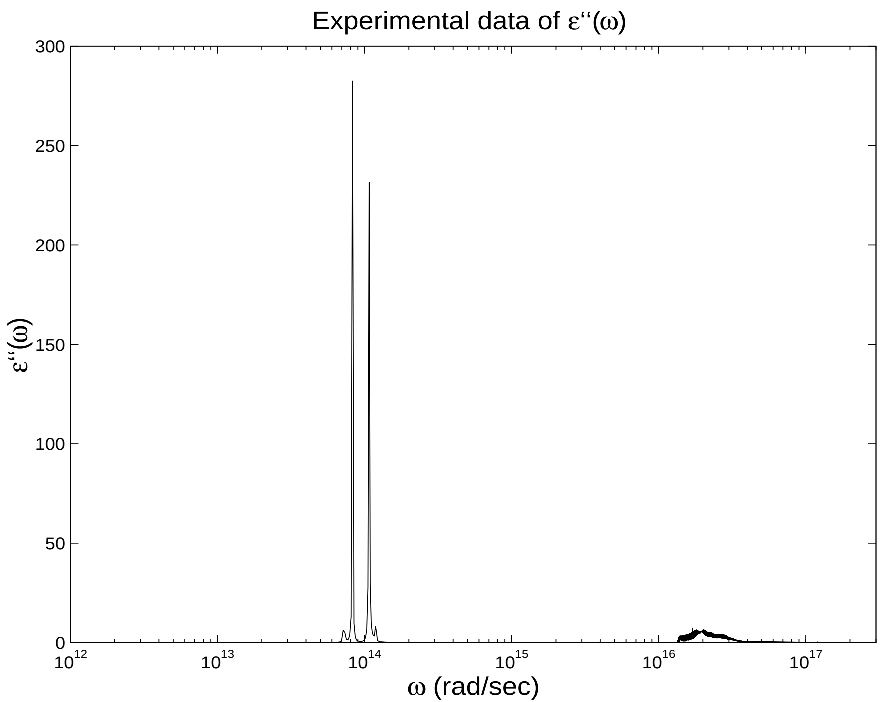
Sapphire ($Al_2O_3$) substrate

$$\varepsilon(\omega)=\varepsilon'(\omega)+i\varepsilon''(\omega)$$



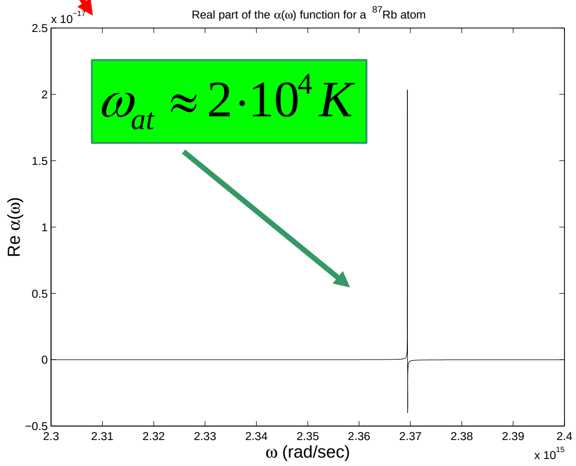
$\omega_1 \approx 6\cdot 10^2 K$

$\omega_2 \approx 9\cdot 10^2 K$



Rubidium ( $^{87}Rb$) atoms

$$\alpha(\omega)=\alpha'(\omega)+i\alpha''(\omega)$$



$\omega_{at} \approx 2\cdot 10^4 K$

# Thermal component of the force

$$F_{th}^{eq}(T,z)$$

$$F_{th}^{eq,ff}(T,z) = \frac{2\hbar}{\pi}\int_0^\infty d\omega\, \frac{\alpha'(\omega)\partial_{z_2}\, \mathrm{Im}\, G_{ii}[\omega;\vec r_1,\vec r_2]\Big|_{\vec r_1=\vec r_2=\vec r}}{e^{\hbar\omega/k_BT}-1}$$

$$F_{th}^{eq,df}(T,z) = \frac{2\hbar}{\pi}\int_0^\infty d\omega\, \frac{\alpha''(\omega)\partial_{z_2}\, \mathrm{Re}\, G_{ii}[\omega;\vec r_1,\vec r_2]\Big|_{\vec r_1=\vec r_2=\vec r}}{e^{\hbar\omega/k_BT}-1}$$

$$\alpha''(\omega) \approx \delta(\omega-\omega_{at})$$

$$k_BT \ll \hbar\omega_{at} \Rightarrow F_{th}^{eq}(T,z) \cong F_{th}^{eq,ff}(T,z)$$

$$k_BT_S, k_BT_E \ll \hbar\omega_{at}$$

**Field fluctuations provide leading term also out of thermal equilibrium**

- **Atom does not contribute to thermal radiation!**

- **Thermal component of the force is determined by Stark effect**

$$\Rightarrow \quad F_{th} = \frac{1}{2}\alpha_0\partial_z < E^2 >_{th}$$

# New asymptotic behaviour out of thermal equilibrium

$$F(T_S,T_E,z) = F^{eq}(T_E,z) + F_{th}(T_S,0,z) - F_{th}(T_E,0,z)$$

**substrate** **environment**

$$F^{neq} = -\frac{\pi}{6}\frac{\alpha_0 k_B^2 (T_S^2 - T_E^2)}{z^3 c\hbar}\frac{\varepsilon_0 + 1}{\sqrt{\varepsilon_0 - 1}}$$

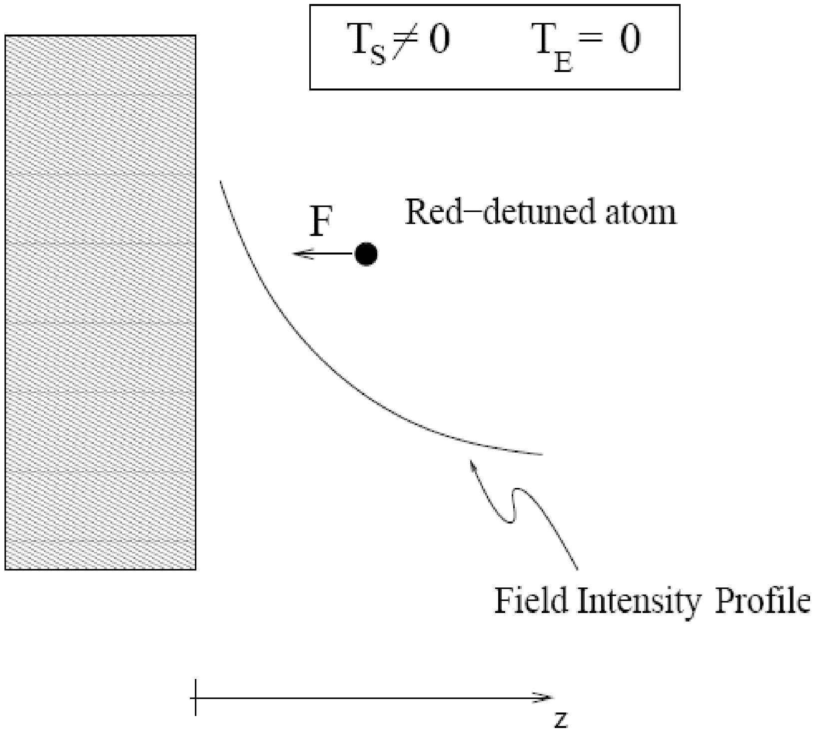
holds at low temperature

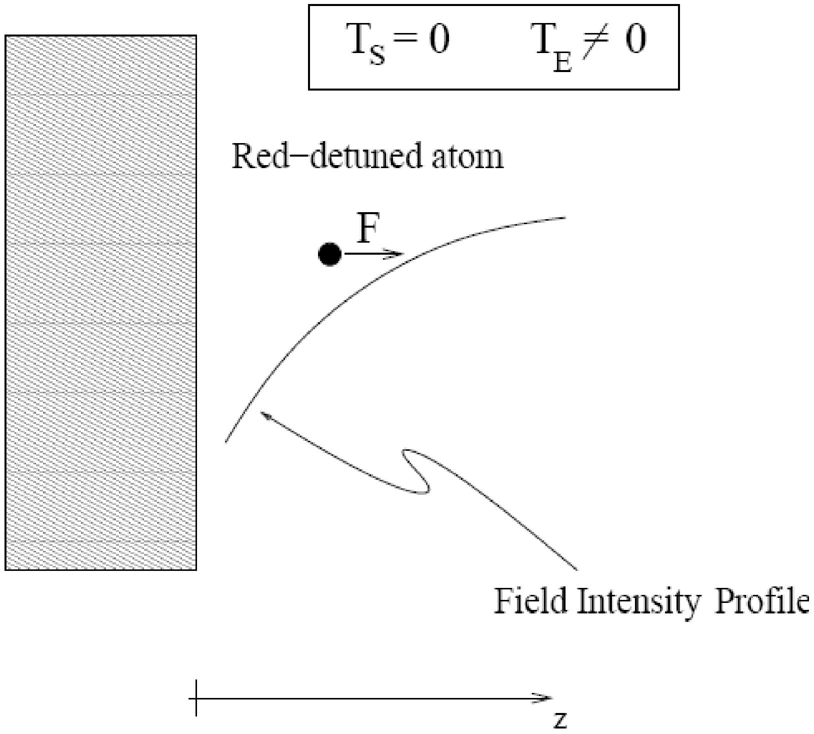
M. Antezza, L.P.Pitaevskii and S.Stringari, Phys. Rev. Lett. **95**,093202 (2005)

- force **decays slower than** at thermal **equilibrium**:

$$F^{eq} = -\frac{3k_B T \alpha_0 (\varepsilon_0 - 1)}{4z^4(\varepsilon_0 + 1)}$$

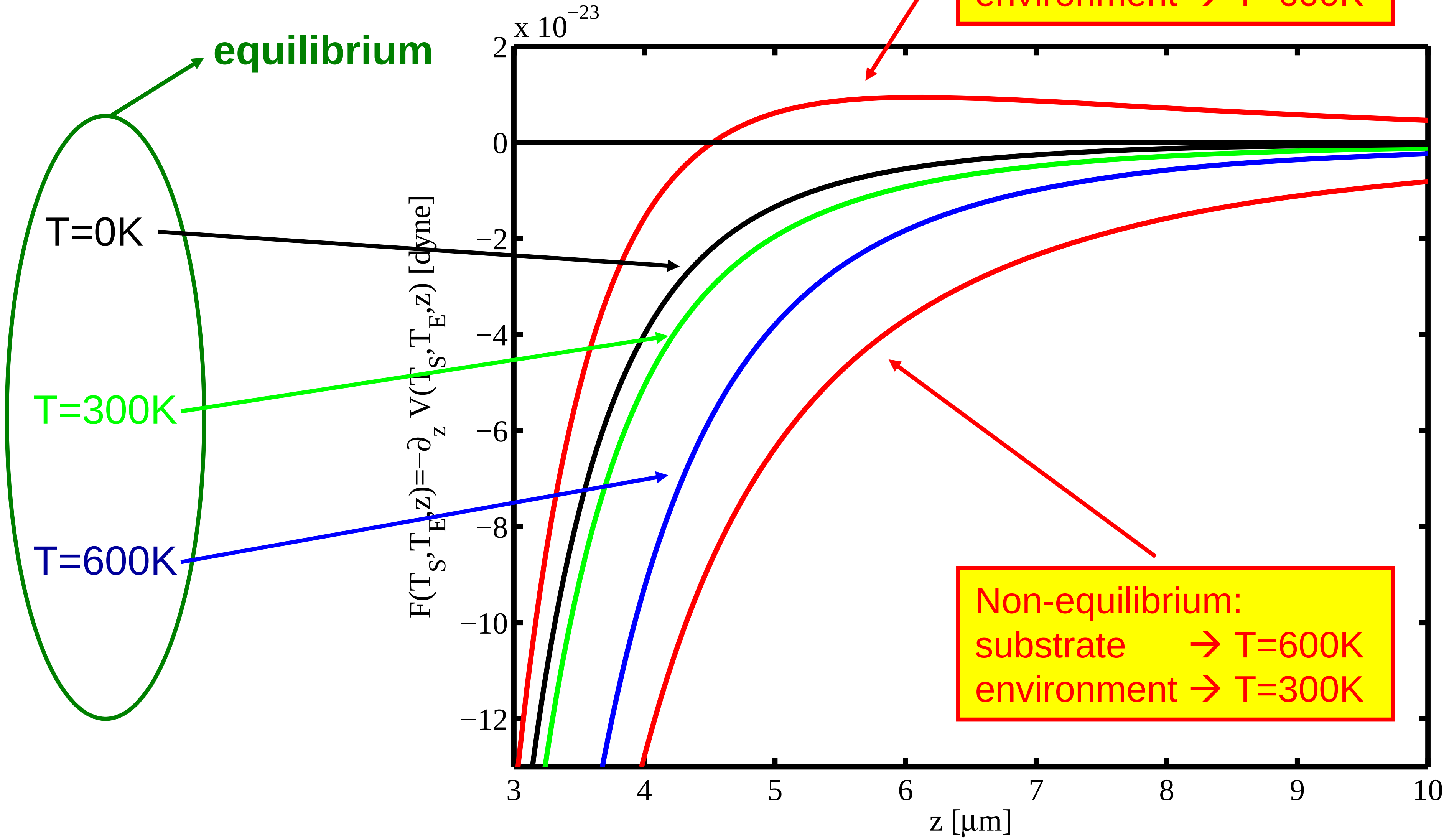
- force depends on **temperature** more **strongly** than at equilibrium
- force can be **attractive** or **repulsive** depending on relative temperatures of substrate and environment
- force has **quantum** nature
- simple extension to **metals** (Drude model $\varepsilon'' = 4\pi\sigma/\omega$ )


$T_S \neq 0$ $T_E = 0$
F
Red−detuned atom
Field Intensity Profile
z


$T_S = 0$ $T_E \neq 0$
Red−detuned atom
F
Field Intensity Profile
z

# Thermal effects on the surface-atom force

- Sapphire substrate
- Rubidium atoms



Non-equilibrium:
substrate → T=300K
environment → T=600K

**equilibrium**

T=0K

T=300K

T=600K

Non-equilibrium:
substrate → T=600K
environment → T=300K

# Measuring the Casimir-Polder force using ultracold atomic gases

Availability of Bose-Einstein condensates and degenerate Fermi gases yields new perspectives in the study of surface-atom forces

## Experiments

- **Collective oscillations** with BEC: **first experiment** at **JILA** (2005)
  (sensitive to the **gradient** of the force)

oscillations

**Bose-Einstein-condensed gases are dilute, ultracold samples characterized by unique properties of coherence and superfluidity. They give rise, among others, to a variety of collective oscillations (S. Stringari (1996))**

- **Bloch oscillations** with ultracold degenerate gases: **MICRA Project** (TN-FI)
  (sensitive to the **force**)

oscillations+interference

**Sensitive measurement of forces at micron scale using Bloch oscillations**
I. Carusotto, L. Pitaevskii, S. Stringari, G. Modugno, and M. Inguscio,
Phys. Rev. Lett. **95**, 093202 (2005)

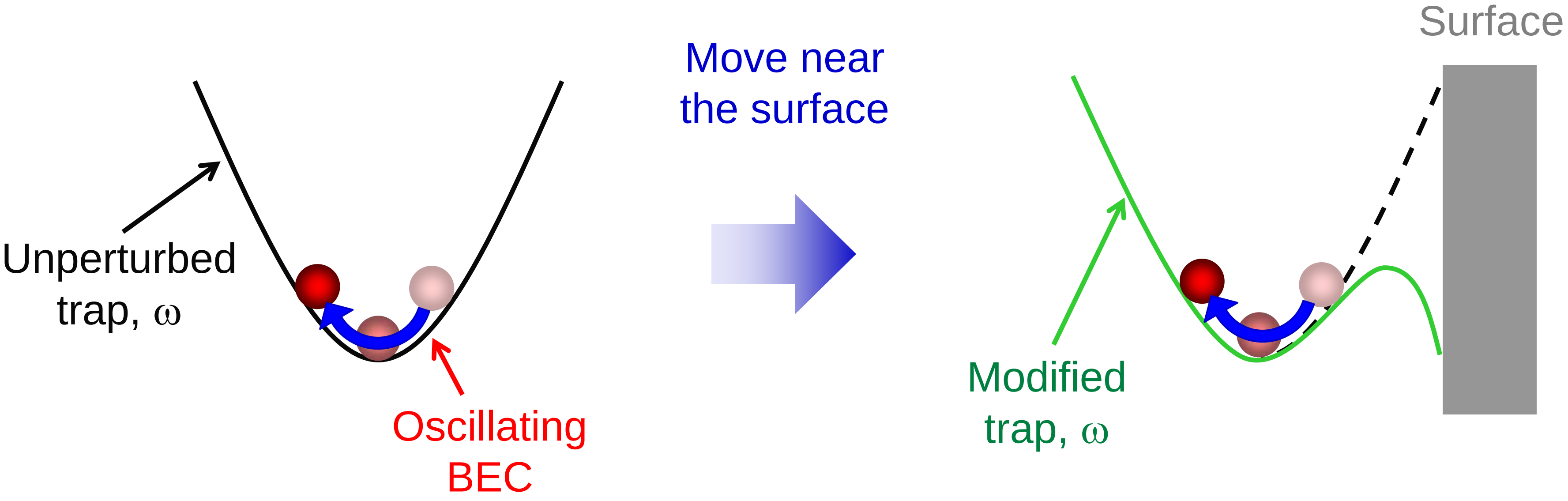
- Macroscopic **BEC** phase **interference** in **double well** potentials
  (sensitive to the **potential**)

interference

# Measuring atom-surface interactions: dipolar oscillations of a BEC

**Use trapped BEC as a mechanical oscillator:**
Measure changes in oscillation frequency

**Attractive force ->** Trap frequency decrease



Unperturbed trap, ω

Oscillating BEC

Move near the surface

Surface

Modified trap, ω

# Frequency shift of collective oscillations of a BEC

**In M. Antezza, L.P. Pitaevskii and S. Stringari, PRA 70, 053619 (2004), the surface-atom force has been calculated and used to predict the frequency shift of the center of mass oscillation of a trapped Bose-Einstein condensate, including:**

- **Effects of finite size of the condensate**
- **Non harmonic effects due to the finite amplitude of the oscillations**
- **Dipole (center of mass) and quadrupole (long living mode) frequency shifts**

In the presence of harmonic potential + surface-atom force frequency of center of mass motion is given by

$$V_{ho}(\vec{r}) = \frac{m}{2}\omega_x^2 x^2 + \frac{m}{2}\omega_y^2 y^2 \frac{m}{2}\omega_z^2 z^2$$

$$\omega_{cm}^2 - \omega_z^2 = \frac{1}{m}\int n_0(\vec{r})\partial_z^2 V_{surf-at}(z)d\vec{r} + \frac{a^2}{8m}\int n_0(\vec{r})\partial_z^4 V_{surf-at}(z)d\vec{r}$$

Linear approximation

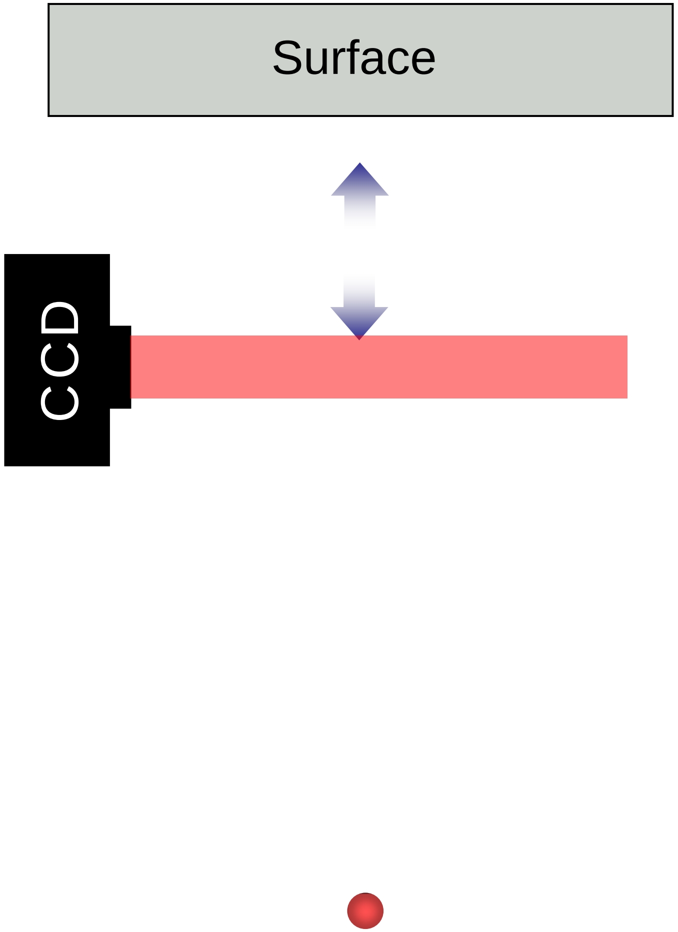
First non-linear correction

a= amplitude of c.m. oscillation

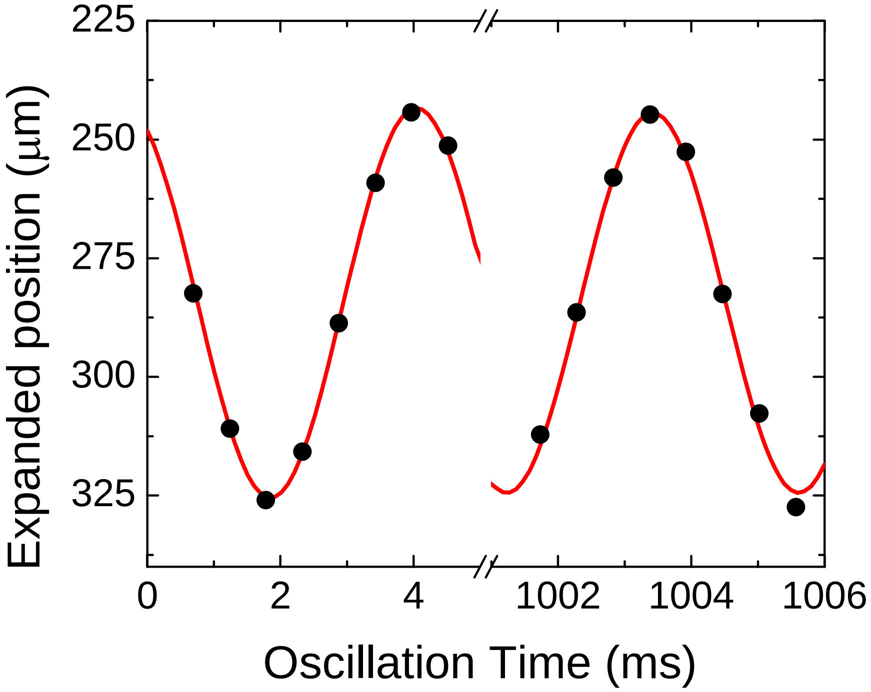
$$Z_{cm} = Z_0 + a\cos(\omega t)$$

$n_0(r) \equiv$ Thomas-Fermi inverted parabola

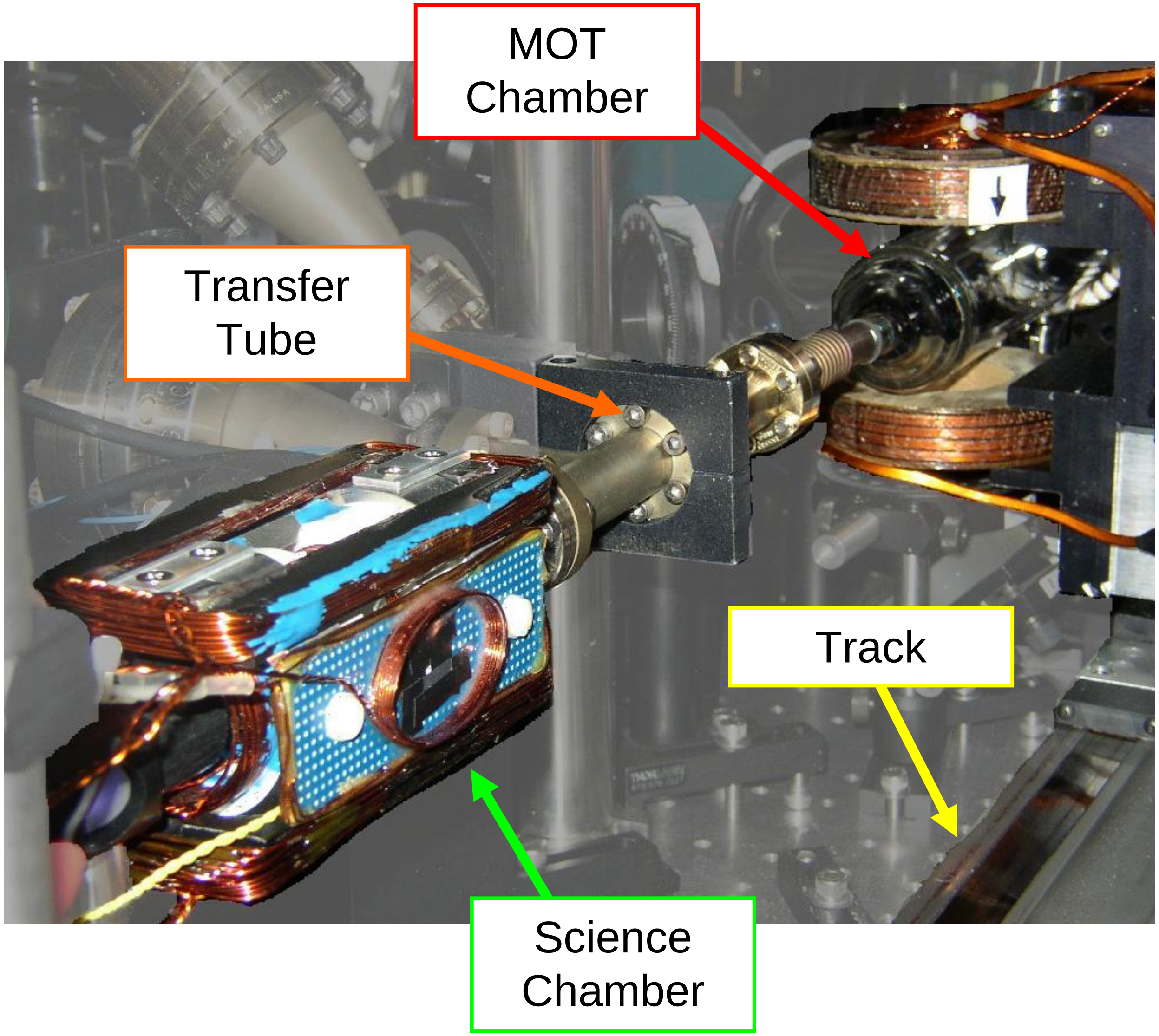
# Measuring atom-surface interactions: dipolar oscillations of a BEC



1) Make BEC far from surface
2) Push BEC a few microns from surface
3) Excite oscillation vertically
4) Switch to anti-trapped state (atoms fall)
5) Image atoms on CCD camera

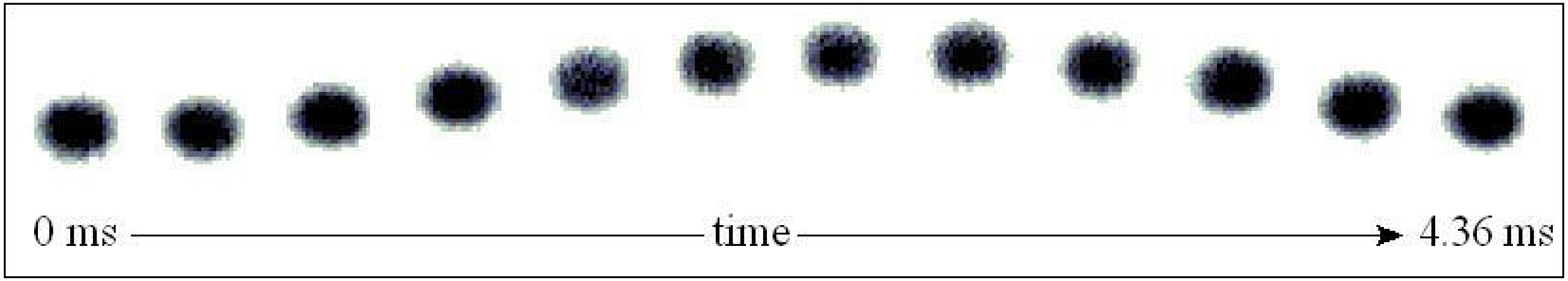


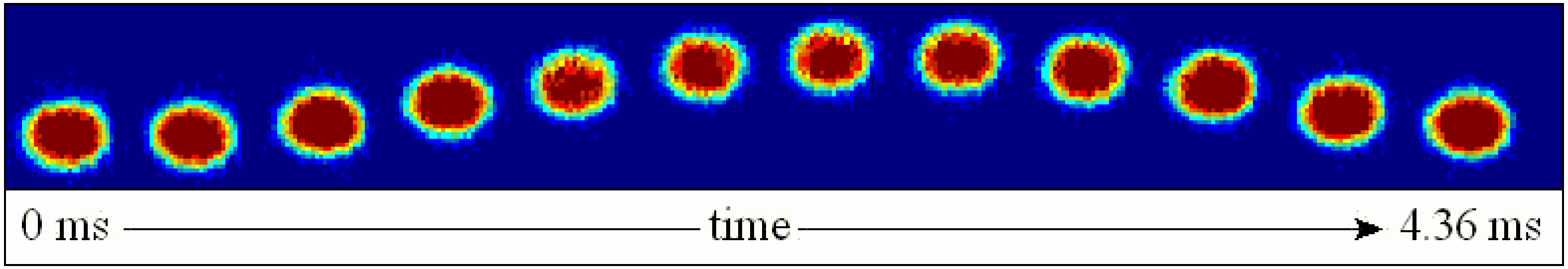
# The experimental apparatus



- Getter-Loaded Rb MOT
- Magnetic coil transfer
- Magnetic trap:
  1. permanent magnets
  2. electromagnets
- Rf evaporation to BEC

* H.J.Lewandowski, *et al*, JLTP **132**, 309 (2003).





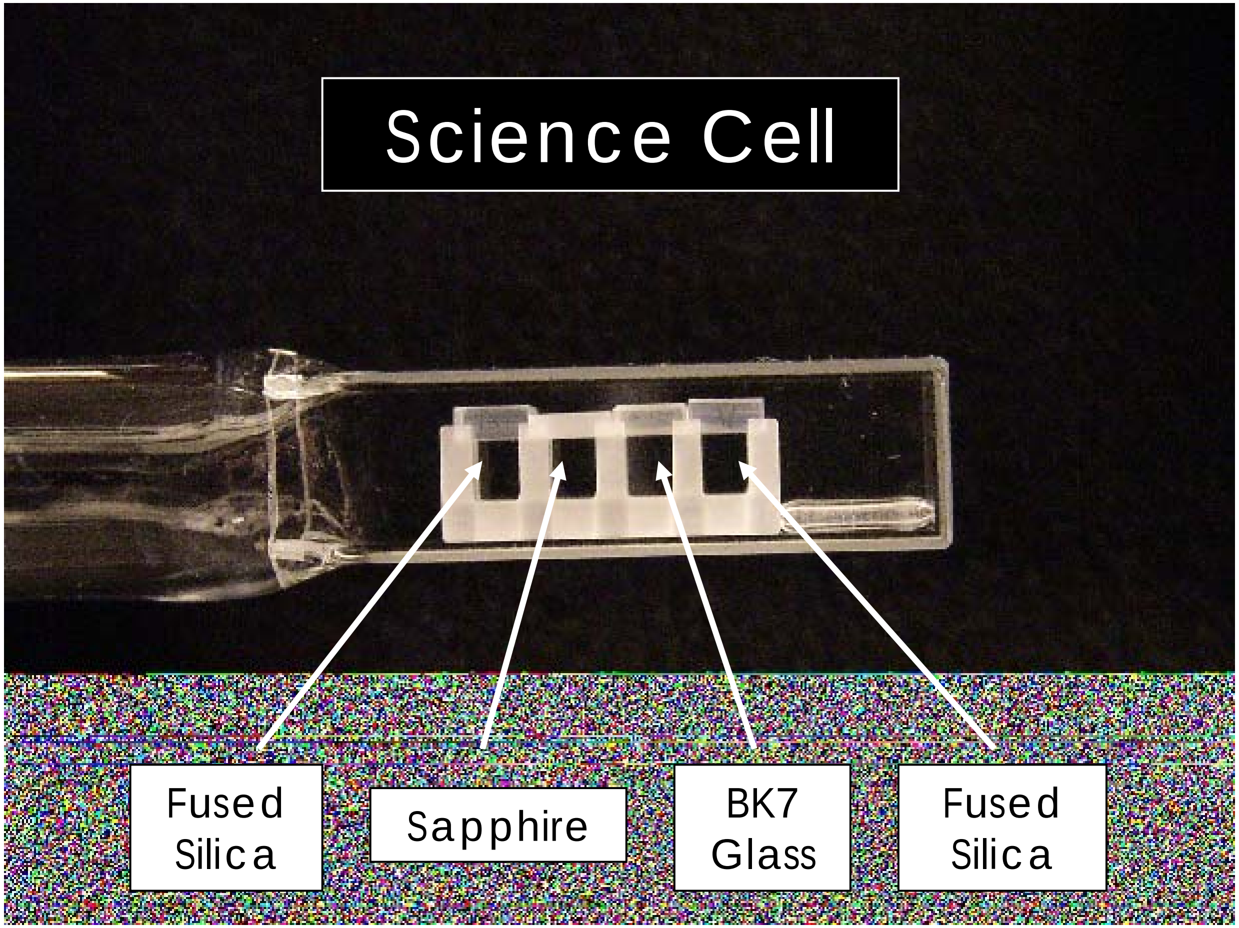
**Dipole mode oscillation:**

Damping time ~**10 seconds**

Frequency resolution ~**10 mHz**

→ *FFS* resolution ~**4 x 10$^{-5}$**

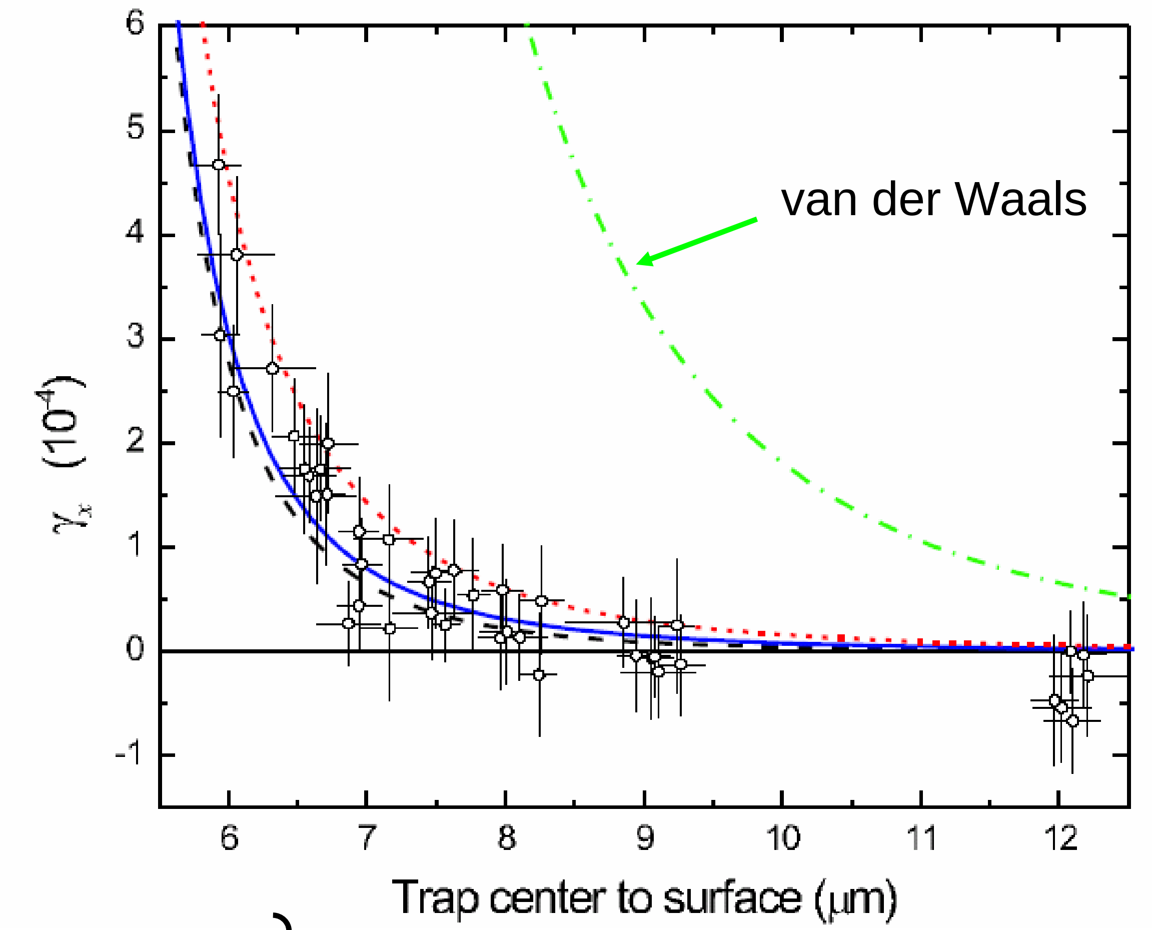
# The experimental apparatus



- Multiple dielectric surfaces! Amorphous glass, crystalline sapphire.
- No conducting objects near atoms!
- Can sustain high temperatures and be compatible with UHV!)

# Measurement of Casimir-Polder (+Lifshitz?) force with Bose-Einstein condensates

**Exp: D.M. Harber, J.M. Obrecht, J.M. McGuirk ,and E.A. Cornell, PRA <u>72</u>, 033610 (2005)**



- **Fused Silica substrate**
- **Rubidium atoms**
- **Experiment at room temperature**

**Thermal effects not yet evident some months ago!**

**Frequency shifts strongly enhanced by non equilibrium effects !?!**

---- 500 K
---- 300 K
---- 0 K

Theory: Antezza, Pitaevskii, Stringari, Phys. Rev A **70**, 053619 (2004)

D.M. Harber, M. Obrecht, J.M. McGuirk, E.A. Cornell , Phys. Rev. A **72**, 033610 (2005)

Physical Review Focus

Focus Archive PNU Index Image Index Focus Search

**8 September 2005**

## Just Another Tool

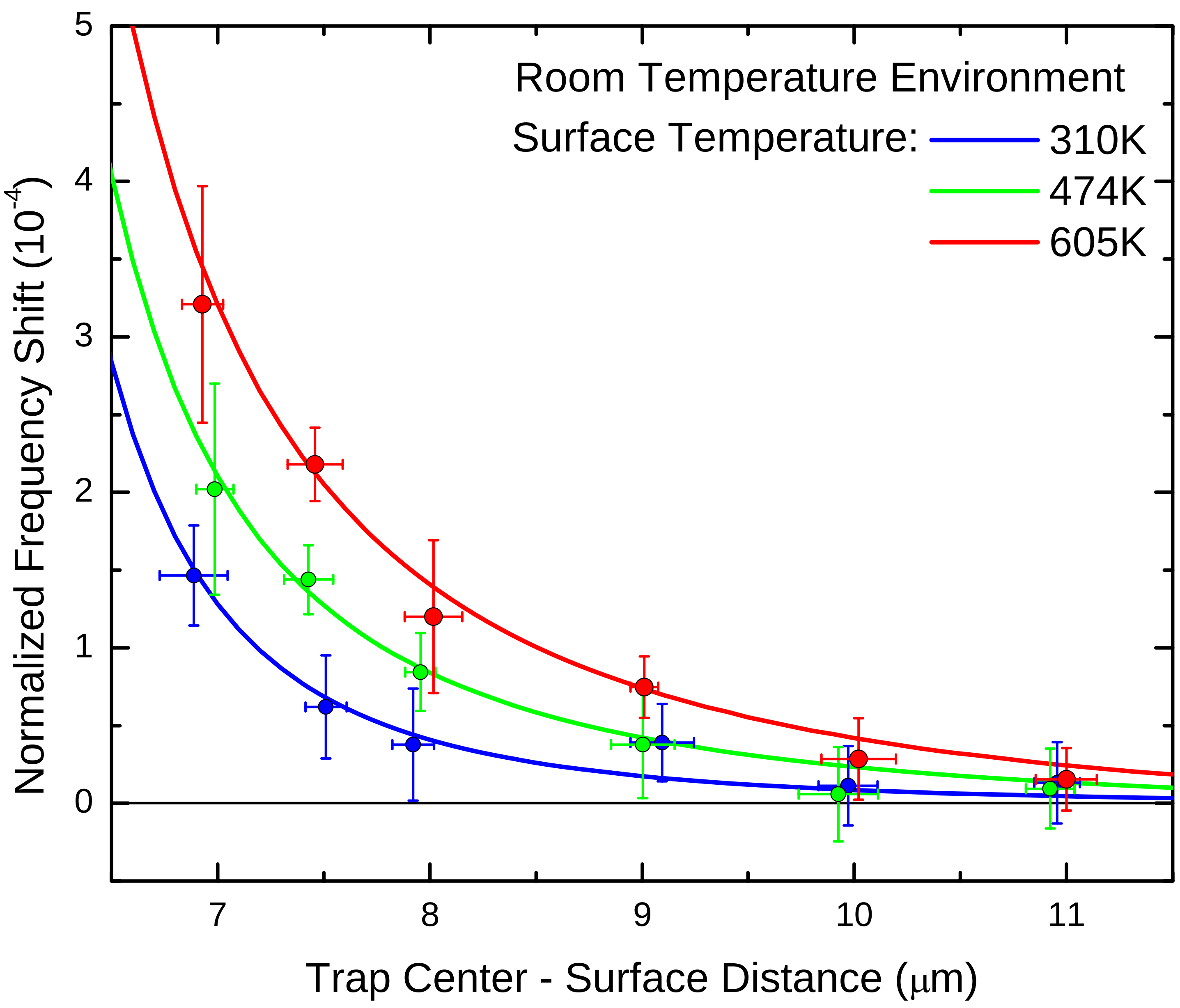
………….

………….

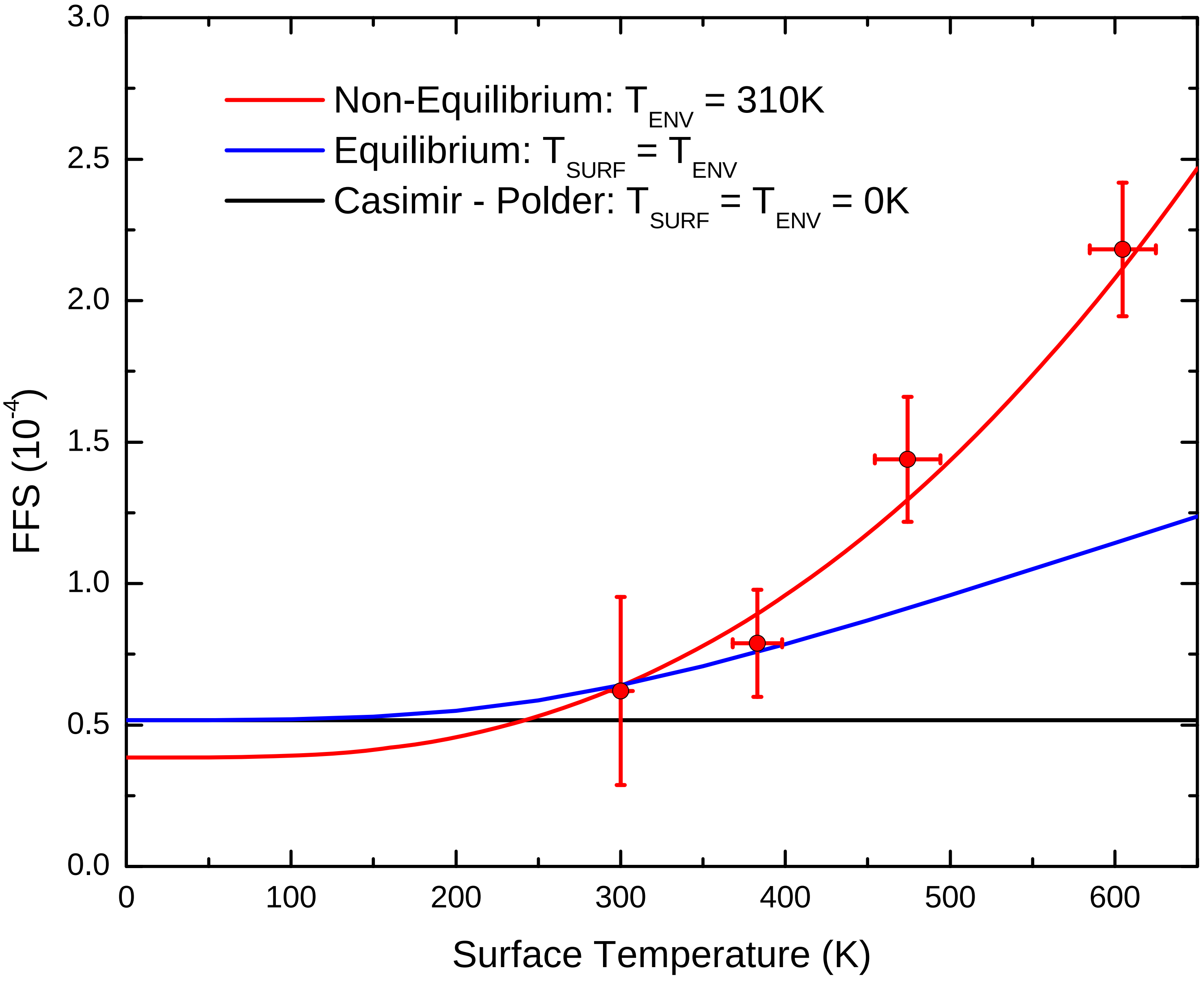
………….

Steve Lamoreaux of Los Alamos National Lab in New Mexico, who first cleanly measured the Casimir force, calls the new experiment a **"marvel of modern science."** He adds that **"this is one of the first direct applications [of BEC] to a measurement where the condensate itself wasn't the focus."**

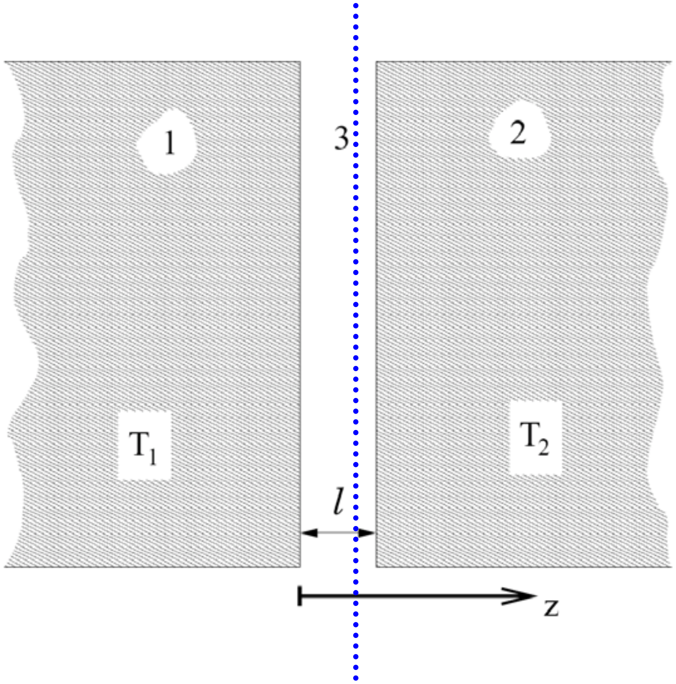
# Recent Experimental results from JILA



J.M. Obrecht, R.J. Wild, M. Antezza, L.P. Pitaevskii, S. Stringari, and E.A. Cornell, submitted (2006), arXiv:physics/0608074


Non-Equilibrium: $T_{ENV}$ = 310K
Equilibrium: $T_{SURF}$ = $T_{ENV}$
Casimir - Polder: $T_{SURF}$ = $T_{ENV}$ = 0K
FFS ($10^{-4}$)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
100
200
300
400
500
600
Surface Temperature (K)

# Surface-surface pressure out of thermal equilibrium



$$P^{neq}(T,l) = \langle T_{zz}(\vec{r},t) \rangle$$

- Role of thermal fluctuation?
- is it possible to recover the surface-atom force?
- what for asymptotics?

- Dorofeyev, J. Phys. A: Math. Gen. 31, 4369 (1998) – identical materials
- Dorofeyev et al., Phys. Rev. E 65, 026610 (2002) – different materials

⇩

$$P_{th}^{neq}(T_1, T_2, l) = \frac{P_{th}^{eq}(T_1, l)}{2} + \frac{P_{th}^{eq}(T_2, l)}{2}$$

# Our Results

$$P_{th}^{neq}(T_1,T_2,l) = P_{th}^{neq}(T_1,0,l) + P_{th}^{neq}(0,T_2,l)$$

$$P_{th}^{neq}(T,0,l) = \frac{P_{th}^{eq}(T,l)}{2} + \Delta P_{th}(T,l)$$

$$P_{th}^{neq}(0,T,l) = \frac{P_{th}^{eq}(T,l)}{2} - \Delta P_{th}(T,l)$$

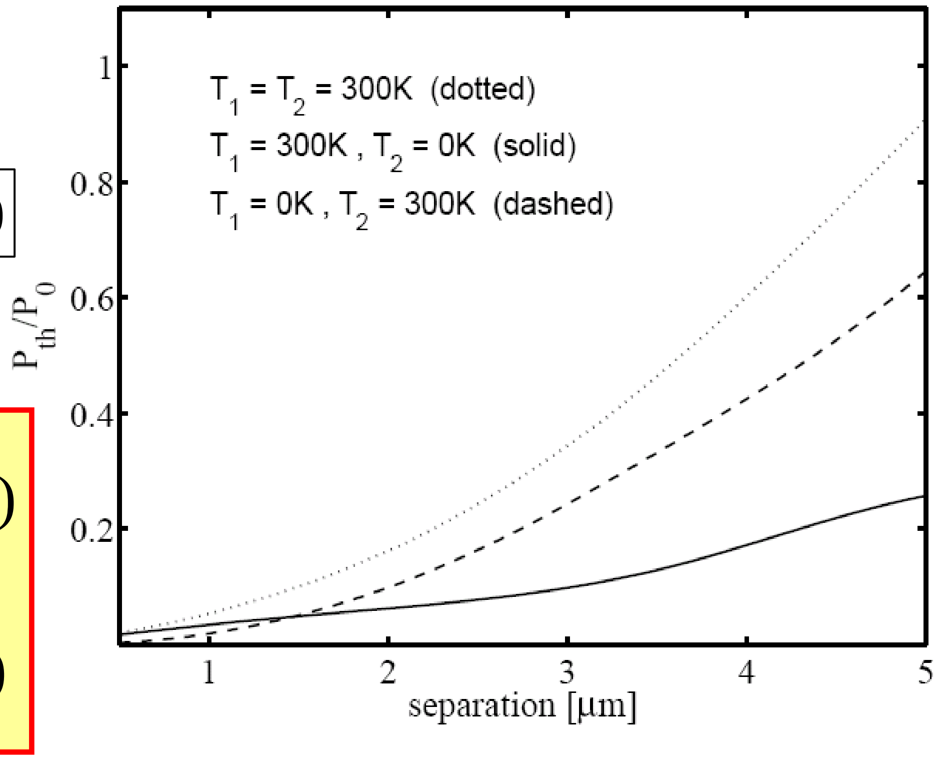


FIG. 1: Relative contribution of the thermal component with respect to the zero-temperature component of the pressure between two different materials: Fused Silica ($SiO_2$, body 1) and Silicon (Si, body 2).

$$\Delta P_{\text{th}}^{\text{PW}}(T,l) = -\frac{\hbar}{4\pi^2}\int_0^\infty \mathrm{d}\omega \frac{1}{e^{\hbar\omega/k_BT}-1}\int_0^k \mathrm{d}Q\, Q\, q_z \sum_{\mu=s,p}\left(|r_2^\mu|^2-|r_1^\mu|^2\right)\left(\frac{1}{|D_\mu|^2}-\frac{1}{1-|r_1^\mu\, r_2^\mu|^2}\right),$$

$$\Delta P_{\text{th}}^{\text{EW}}(T,l) = \frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\omega \frac{1}{e^{\hbar\omega/k_BT}-1}\int_k^\infty \mathrm{d}Q\, Q\, \mathrm{Im}q_z\, e^{-2l\mathrm{Im}q_z}\sum_{\mu=s,p}\frac{\mathrm{Im}\,(r_1^\mu)\,\mathrm{Re}\,(r_2^\mu)-\mathrm{Im}\,(r_2^\mu)\,\mathrm{Re}\,(r_1^\mu)}{|D_\mu|^2},$$

M. Antezza, L.P. Pitaevskii, S. Stringari, V.B. Svetovoy, Phys. Rev. Lett. **97** , 223203 (2006)

# Asymptotic behaviours for the surface-surface pressure

Equilibrium

$$
\begin{aligned}
P_{\rm th}^{\rm eq,PW}(T,l) &= \frac{k_B T \zeta(3)}{4\pi l^3}, \\
P_{\rm th}^{\rm eq,EW}(T,l) &= -\frac{k_B T \zeta(3)}{4\pi l^3} + \\
& \frac{k_B T}{16\pi l^3} \int_0^\infty \mathrm{d}x \; x^2 \left[ \frac{\varepsilon_{10}+1}{\varepsilon_{10}-1} \frac{\varepsilon_{20}+1}{\varepsilon_{20}-1} \; e^x - 1 \right]^{-1},
\end{aligned}
$$

$$
l \gg \lambda_T \max_{m=1,2} \left[ \frac{\varepsilon_{m0}}{\sqrt{\varepsilon_{m0}-1}} \right].
$$

$$
l \gg \lambda_T
$$

Non Equilibrium

$$
\begin{aligned}
P_{\rm th}^{\rm neq,PW}(T,0,l) &\to \frac{k_B T}{l^3} \frac{\zeta(3)}{16\pi} \left[ 2 - \frac{\sqrt{\varepsilon_{10}-1} - \sqrt{\varepsilon_{20}-1}}{\sqrt{\varepsilon_{10}-1} + \sqrt{\varepsilon_{20}-1}} - \frac{\varepsilon_{20}\sqrt{\varepsilon_{10}-1} - \varepsilon_{10}\sqrt{\varepsilon_{20}-1}}{\varepsilon_{20}\sqrt{\varepsilon_{10}-1} + \varepsilon_{10}\sqrt{\varepsilon_{20}-1}} \right], \\
P_{\rm th}^{\rm neq,EW}(T,0,l) &\to \frac{k_B T}{l^3} \frac{1}{8\pi^2} \int_0^\infty \mathrm{d}t \int_0^\infty \mathrm{d}x \, \frac{x^2 \; e^{-x}}{t} \sum_{\mu=s,p} \frac{\mathrm{Im}\left[r_1^\mu(t)\right] \mathrm{Re}\left[r_2^\mu(t)\right]}{|1 - r_1^\mu(t) r_2^\mu(t) \; e^{-x}|^2},
\end{aligned}
$$

# Asymptotic behaviours for the surface-rarefied body pressure

At equilibrium : for $\ell >> \lambda_T$

$$P_{th}^{eq}(T,\ell) = \frac{k_B T}{16 \ \pi \ \ell^3} \frac{\varepsilon_{10} - 1}{\varepsilon_{10} + 1} \left(\varepsilon_{20} - 1\right)$$

$$\varepsilon_{20} - 1 = 4 \, \pi \, n \, \alpha$$

$$\lambda_T = \frac{\hbar c}{k_B T}$$

## Out of equilibrium: two different limiting procedures

first $\ell \rightarrow \infty$ with fixed $\varepsilon_{20}$

and then $\left( \varepsilon_{20} - 1\right) \rightarrow 0$ :

$$P_{th}^{neq}(T,0,\ell) = \frac{k_B T \quad C}{\ell^3} \frac{\varepsilon_{10} - 1}{\sqrt{\varepsilon_{10} - 1}} \sqrt{\varepsilon_{20} - 1}$$

holding at $\ell >> \dfrac{\lambda_T}{\sqrt{\varepsilon_{20} - 1}}$

first $\left(\varepsilon_{20} - 1\right) \rightarrow 0$ with fixed $\ell$

and then $\ell \rightarrow \infty$ : (*n.b.* PW and EW are equal!)

$$P_{th}^{neq}(T,0,\ell) = \frac{(k_B T)^2}{24 \ \ell^2 \ \hbar c} \frac{\varepsilon_{10} - 1}{\sqrt{\varepsilon_{10} - 1}} \left(\varepsilon_{20} - 1\right)$$

holding at $\lambda_T >> \ell >> \dfrac{\lambda_T}{\sqrt{\varepsilon_{20} - 1}}$

# The cross-over

$$P_{\rm th}^{\rm neq}(T,0,l) = \frac{k_B T\, C}{l^3} \frac{\varepsilon_{10}+1}{\sqrt{\varepsilon_{10}-1}} \sqrt{\varepsilon_{20}-1}\, f(x),$$

$$x = l\sqrt{\varepsilon_{20}-1}/\lambda_T$$

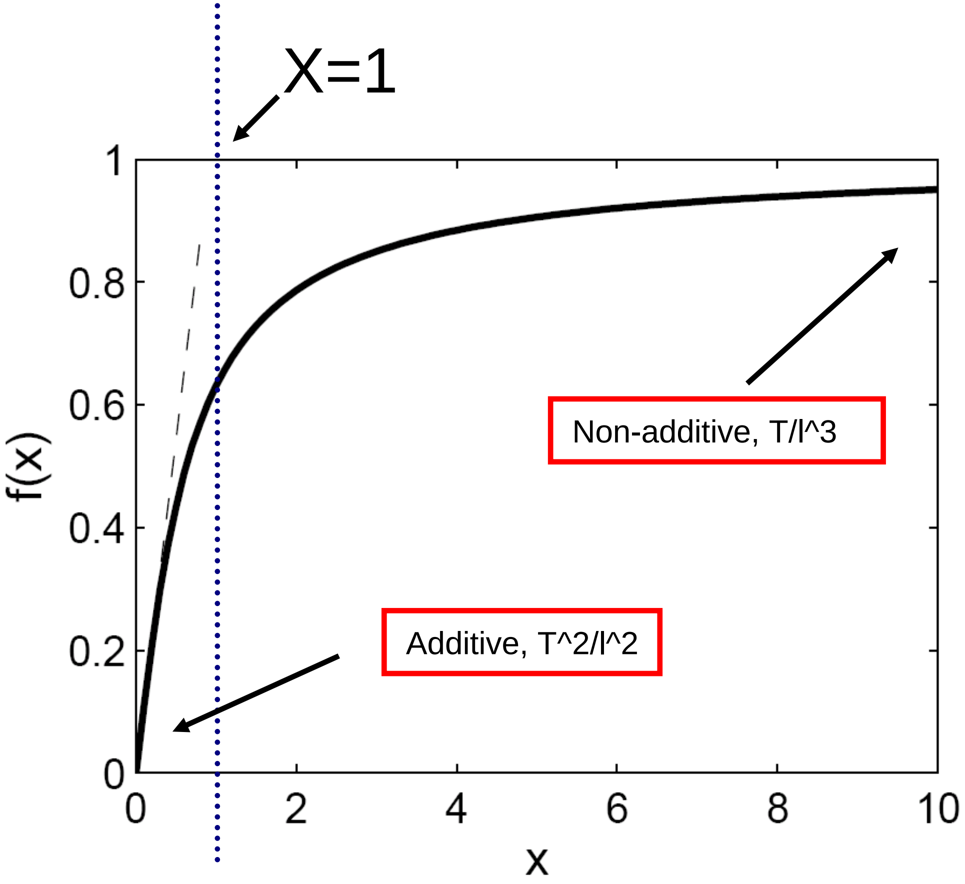


FIG. 2: Dimensionless function $f(x)$.

# What for the surface-atom force?

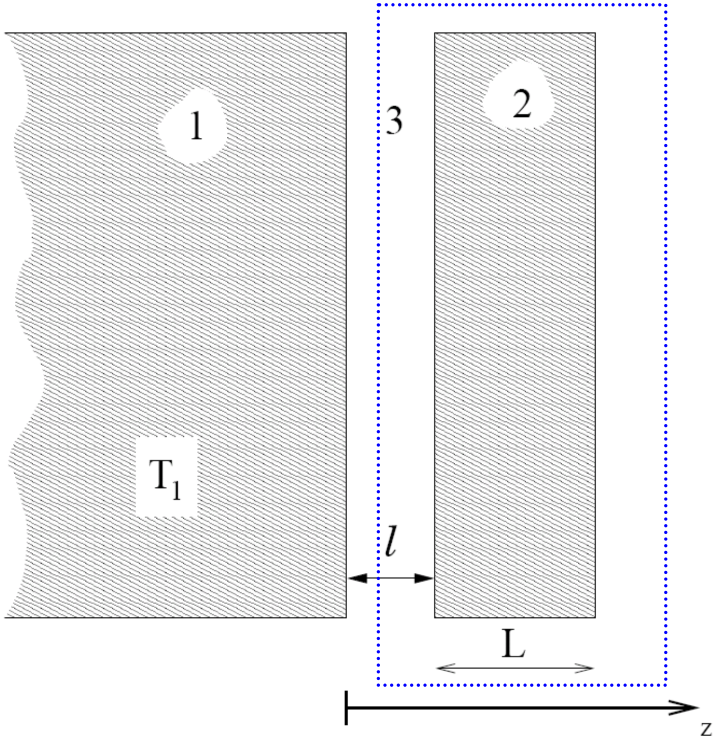
From the surface-rarefied body:

$$P_{th}^{neq}(T,0,\ell) = \frac{(k_B T)^2}{24\ \ell^2\ \hbar c}\ \frac{\varepsilon_{10}-1}{\sqrt{\varepsilon_{10}-1}}\ (\varepsilon_{20}-1)$$

But the surface-atom force is:

$$F_{th}^{neq}(T,0,\ell) = \frac{(k_B T)^2}{24\ \ell^3\ \hbar c}\ \frac{\varepsilon_{10}-1}{\sqrt{\varepsilon_{10}-1}}\ (\varepsilon_{20}-1)$$

What is the problem??



If the gas occupies a finite slab L and does not absorb the thermal radiation:

$$L << \lambda_T^2 / \ell \varepsilon_2''$$

the inclusion of the remote surface results in a PW contribution of the order

$$\propto (\varepsilon_{20}-1)^3$$

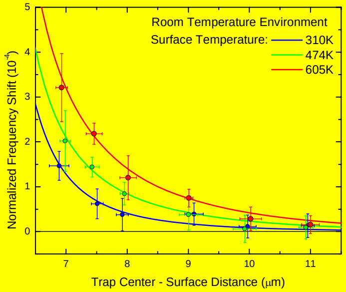
and hence should be omitted!

# Conclusions

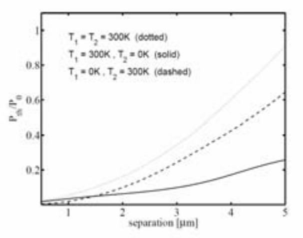
- Surface-atom force out of thermal equilibrium exhibits **new asymptotic (large distance) behaviour** and can provide **a new way to measure thermal effects**

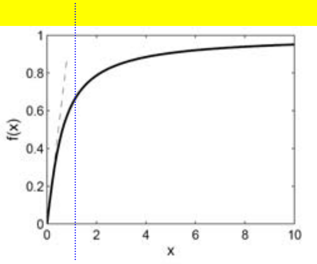
$$F(T_S, T_E, z) \to \frac{(T_S^2 - T_E^2)}{z^3}$$

-**Center of mass oscillation** of a trapped Bose-Einstein condensate provides a powerful **mechanical tool** to detect surface-atom force at large distances, and **agrees** with theoretical **predictions for Casimir-Polder** force
**(first measurement of any thermal effect)** (Trento-Boulder collaboration)



- Study of the **surface-surface force** out of thermal equilibrium and **asymptotic non-additivity**

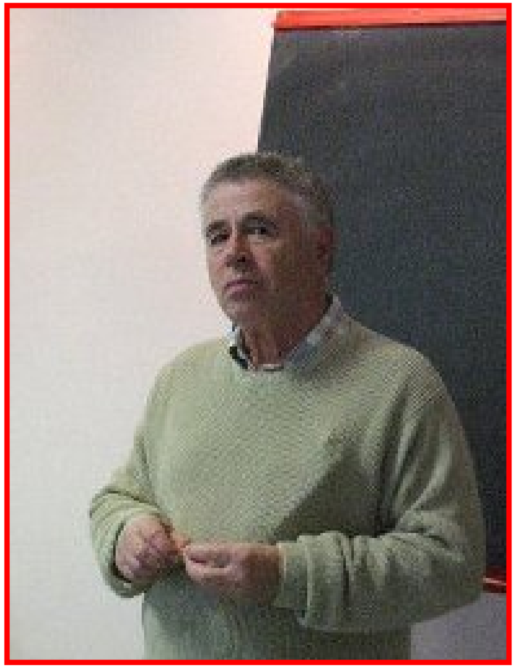




# The Team

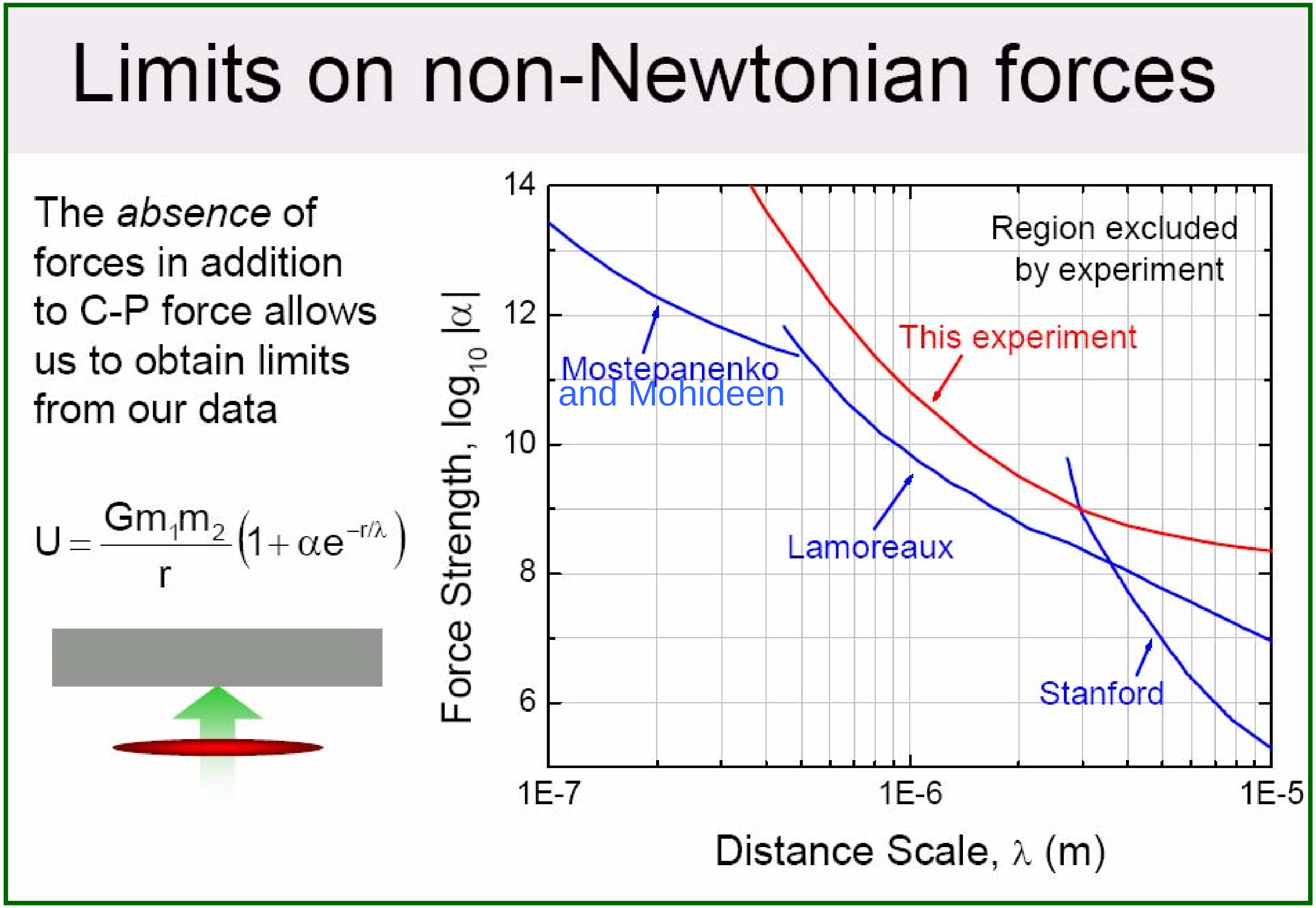
M. Antezza

Prof. Lev P. Pitaevskii



Prof. Sandro Stringari

# Perspective for measurement of non Newtonian forces at the micron scale

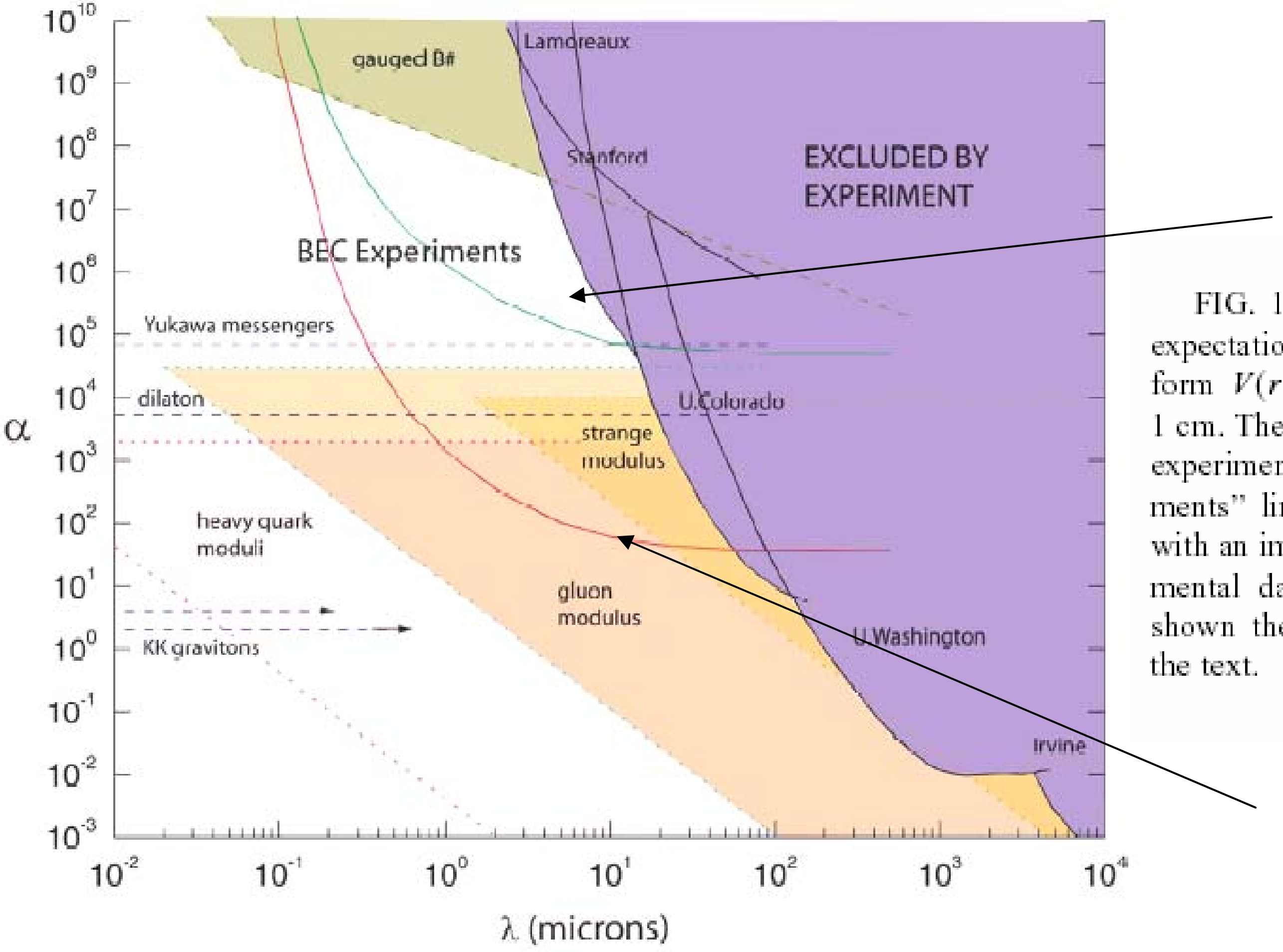


## Limits on non-Newtonian forces

The *absence* of forces in addition to C-P force allows us to obtain limits from our data

$$U = \frac{Gm_1m_2}{r}\left(1+\alpha e^{-r/\lambda}\right)$$

# Non-newtonian forces



$10^{-7}$ g

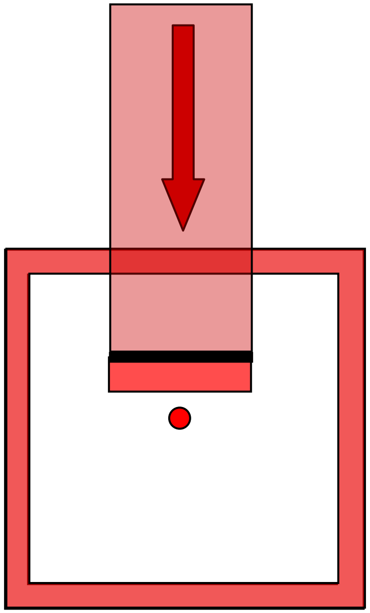
$10^{-10}$ g

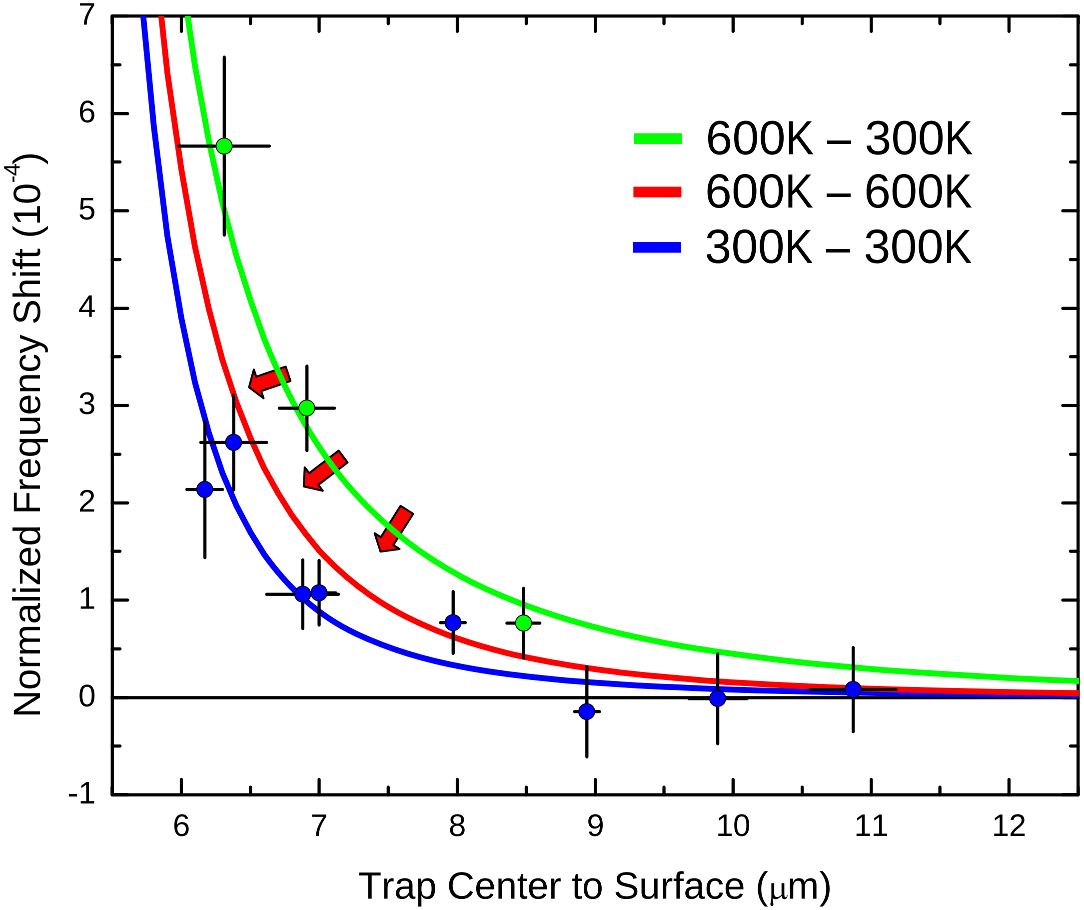
FIG. 1. Experimental bounds and theoretical expectations on new forces from potentials of the form $V(r) = -G_N(m_1 m_2/r)(1+\alpha e^{-r/\lambda})$ below 1 cm. The projected reach of the first-round BEC experiments is shown as the upper "BEC Experiments" line. The lower line indicates the reach with an improved sensitivity of $10^{-7}$ Hz. Experimental data are from Refs. [8–10,14,15]. The shown theoretical expectations are discussed in the text.

S. Dimopulos and A. A. Geraci, Phys. Rev. D 68, 124021 (2003)

# Systematic study of spurious forces

Heat surface AND environment:





Show that by HEATING the environment, the strength of the force DECREASES!

- Experiment on collective oscillation probes **gradient of the force**
- Due to finite size of condensate and amplitude of oscillation **experiment does not probe the effects locally** (average sensitive to shorter distances where thermal effects are weaker).

## Bloch oscillations: new strategy for high precision measurements

**Sensitive measurement of forces at micron scale using Bloch oscillations**
I. Carusotto, L. Pitaevskii, S. Stringari, G. Modugno, and M. Inguscio,
Phys. Rev. Lett. **95**, 093202 (2005)

- **Center of mass** oscillation
  - measures **gradient** of the force
  - **mechanical** approach (oscillation in coordinate space)

- **Bloch** oscillation
  - measures directly the **force**
  - **interferometric** approach (oscillation in momentum space)

# MICRA

## Misure Interferometriche di forze a Corto Range con reticoli Atomici

Obiettivo del progetto:

- Studio della **forza superficie-atomo** su scala micrometrica mediante tecniche **interferometriche** divenute recentemente disponibili in fisica dei **gas atomici degeneri** **intrappolati**.

- La componente dominante della forza nel range di distanze di 5-10 micron e' data dal termine di **Casimir-Polder**, nonché dalle fluttuazioni **termiche** del campo elettromagnetico.

- La verifica e il controllo sperimentale di queste forze sono condizione necessaria per migliorare i limiti di conoscenza su **forze non-newtoniane.**

# MICRA: struttura del progetto

Responsabile nazionale: M. Inguscio

Gruppi di ricerca che partecipano al progetto:
- **Firenze** (progettazione e realizzazione dell’esperimento), responsabile locale: G. Modugno
- **Trento** (teoria della forza superficie-atomo e dinamica dei gas ultrafreddi), responsabile locale: S. Stringari
- Collaborazione con E. Cornell (JILA, Boulder USA)

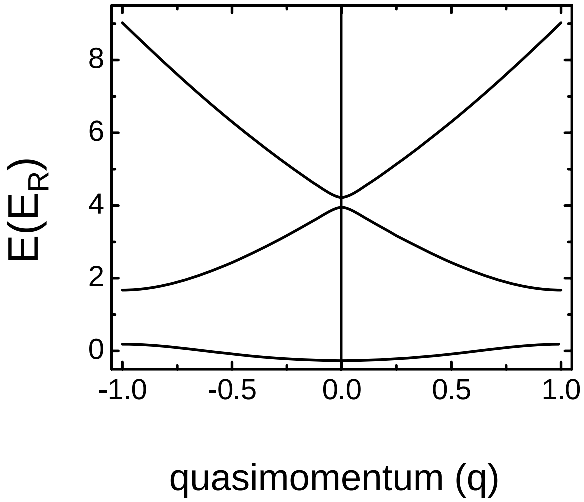
Durata del progetto: 3 anni

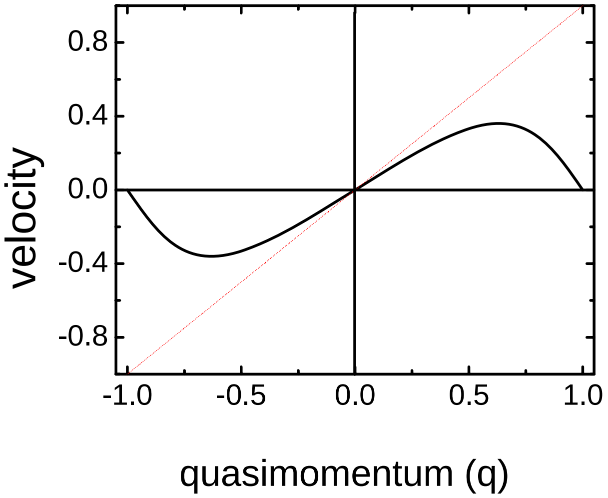
Gruppo sperimentale di **Firenze:**
L. Fallani, C. Fort, M. Inguscio, F. Minardi, G. Modugno, G. Roati, M. Zaccanti
Gruppo teorico di **Trento:**
M. Antezza, I. Carusotto, L. Pitaevskii, S. Stringari

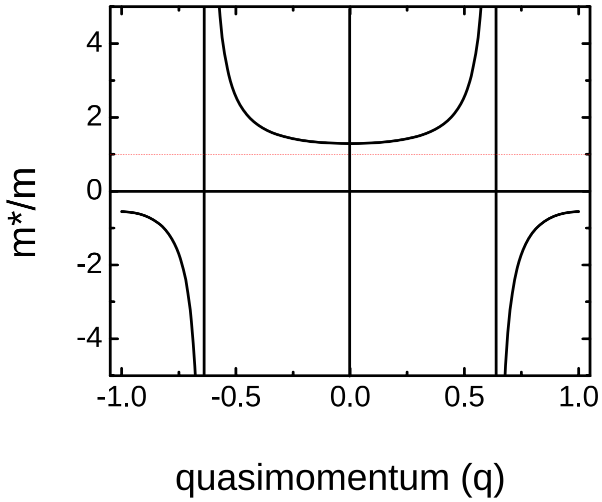
R&D: SQUAT-Super (LNS-Catania + Firenze) (superfici e interazioni)

# Atoms & Periodic Potential

Particles in a periodic potential : band structure







Constant external force => Bloch oscillations

$$\dot{\mathbf{r}} = \mathbf{v}(\mathbf{q}) = \frac{1}{\hbar}\frac{\partial E(\mathbf{q})}{\partial \mathbf{q}}$$

$$\dot{\mathbf{q}} = \frac{1}{\hbar}\mathbf{F}(\mathbf{r},t)$$

## Optical lattice + gravity

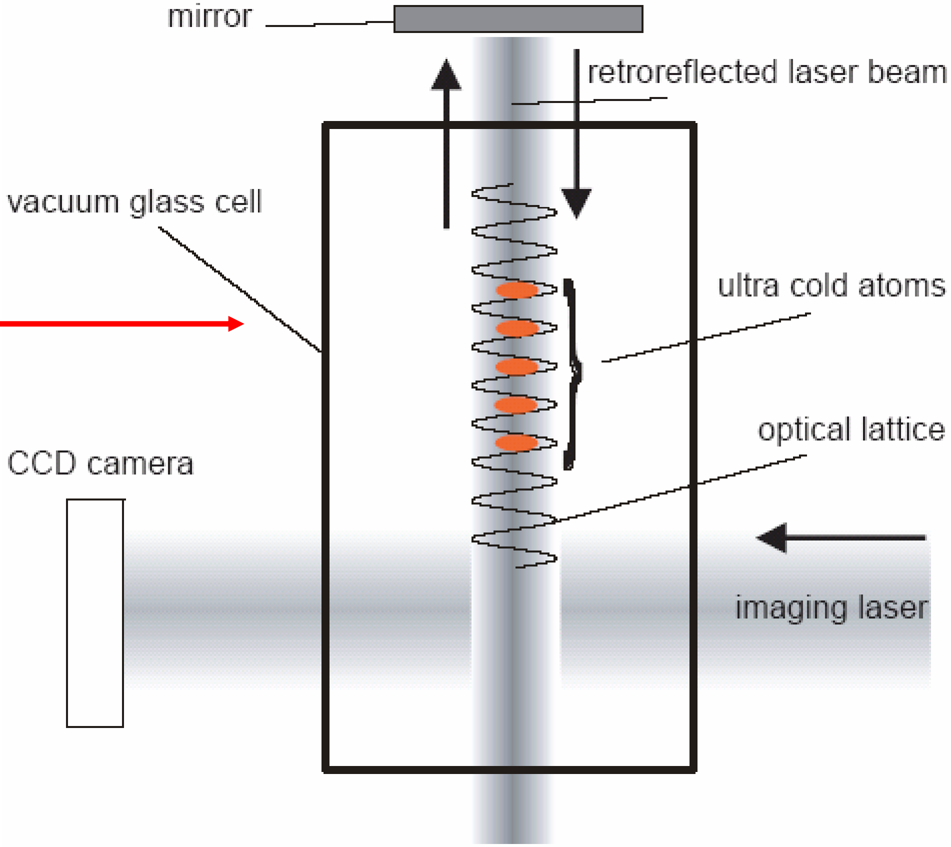
$$H = \frac{P^2}{2m} + s\frac{E_R}{2}\left[1 - \cos(4\pi x/\lambda)\right]\boxed{-\,mgz}$$

Analogy with a normal conductor with an applied voltage

# Bloch oscillations in atomic gases

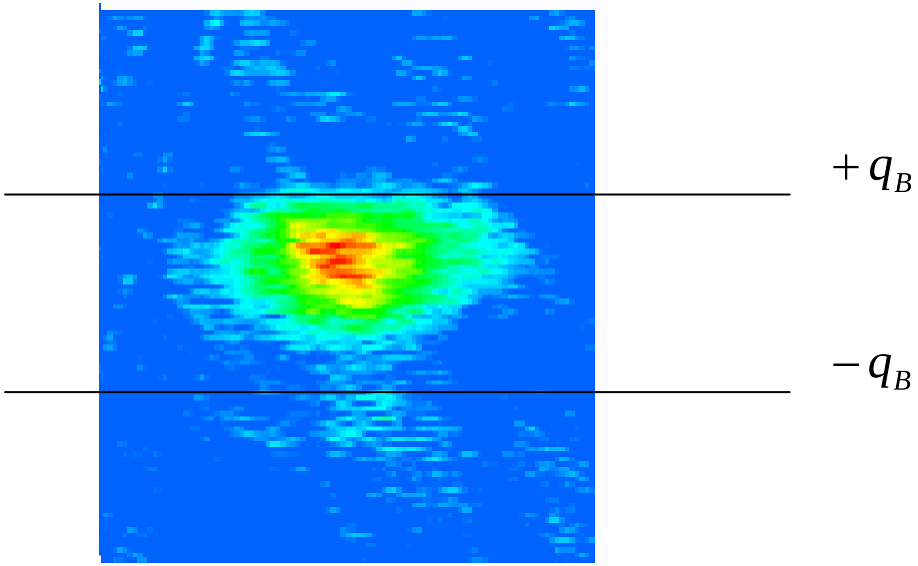
- atomic gas initially feels **3D harmonic** trap+ **gravity** + **periodic** confinement
- at t=0 one switches off harmonic trap System feels periodic potential + gravity and starts oscillating (Bloch oscillation).
- After given evolution time the periodic potential is switched off. Atomic gas falls down, expands and is hence imaged.
- For ideal gas imaged profiles are proportional to initial momentum distribution

$$\omega_{Bloch} = mg\lambda / 2\hbar$$

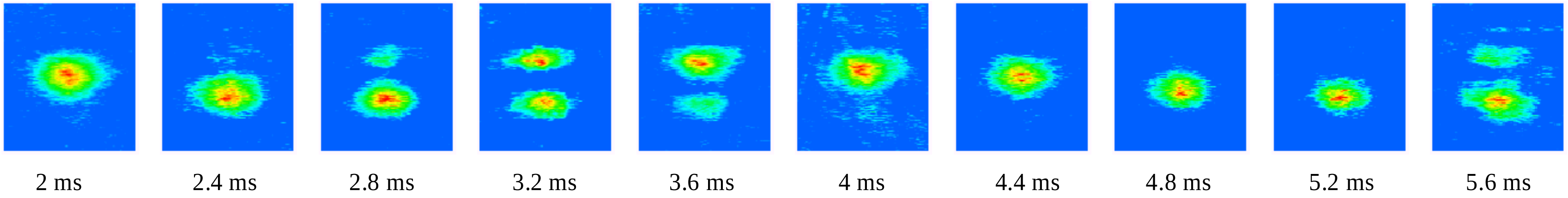


Atoms filling different wells evolve with different phase due to gravity ! (interferometric tool)

# Momentum distribution of a Fermi gas performing Bloch oscillations



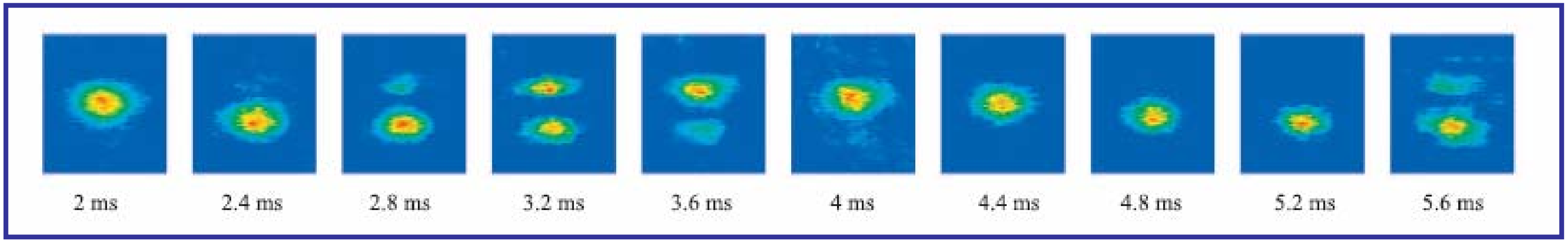
Time-resolved Bloch oscillations of non-interacting fermions

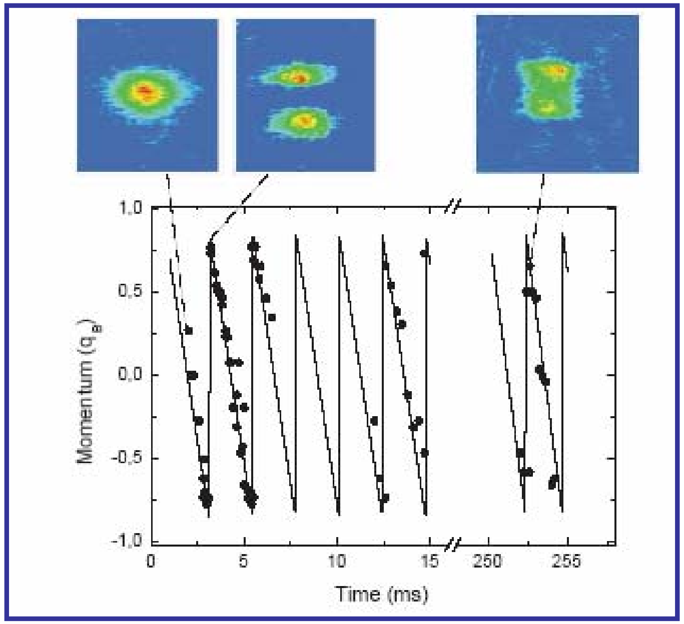


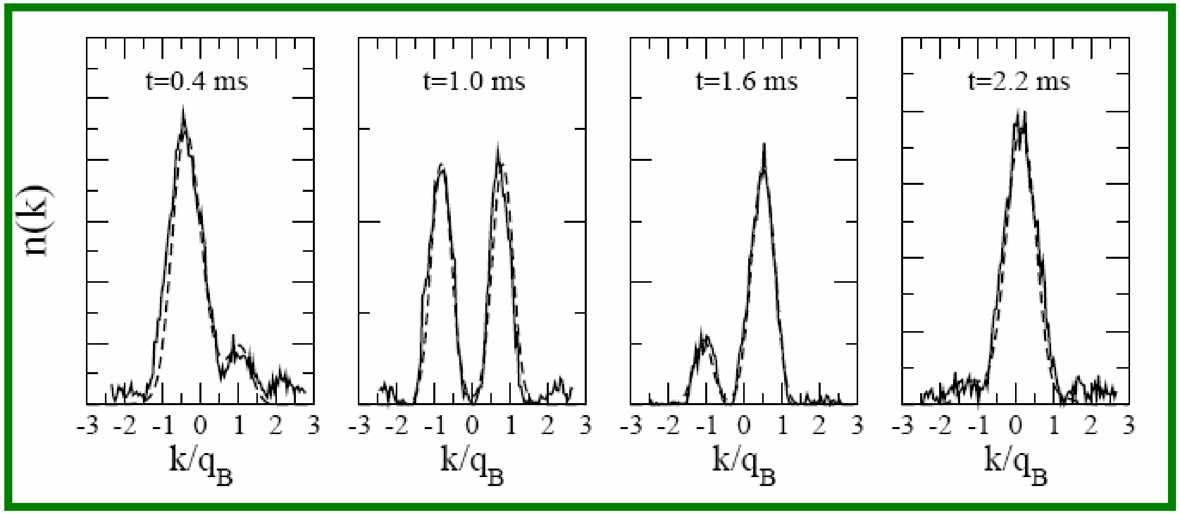
measurement: $T_B = 2.32789(22)\text{ms} \quad \Rightarrow \quad \Delta g / g = 10^{-4}$

# Time-resolved Bloch oscillations of non-interacting fermions

Images of spin polarized Fermi gas performing Bloch oscillation
(Florence 2005, Roati et al. PRL 92, 230402 (2004) )







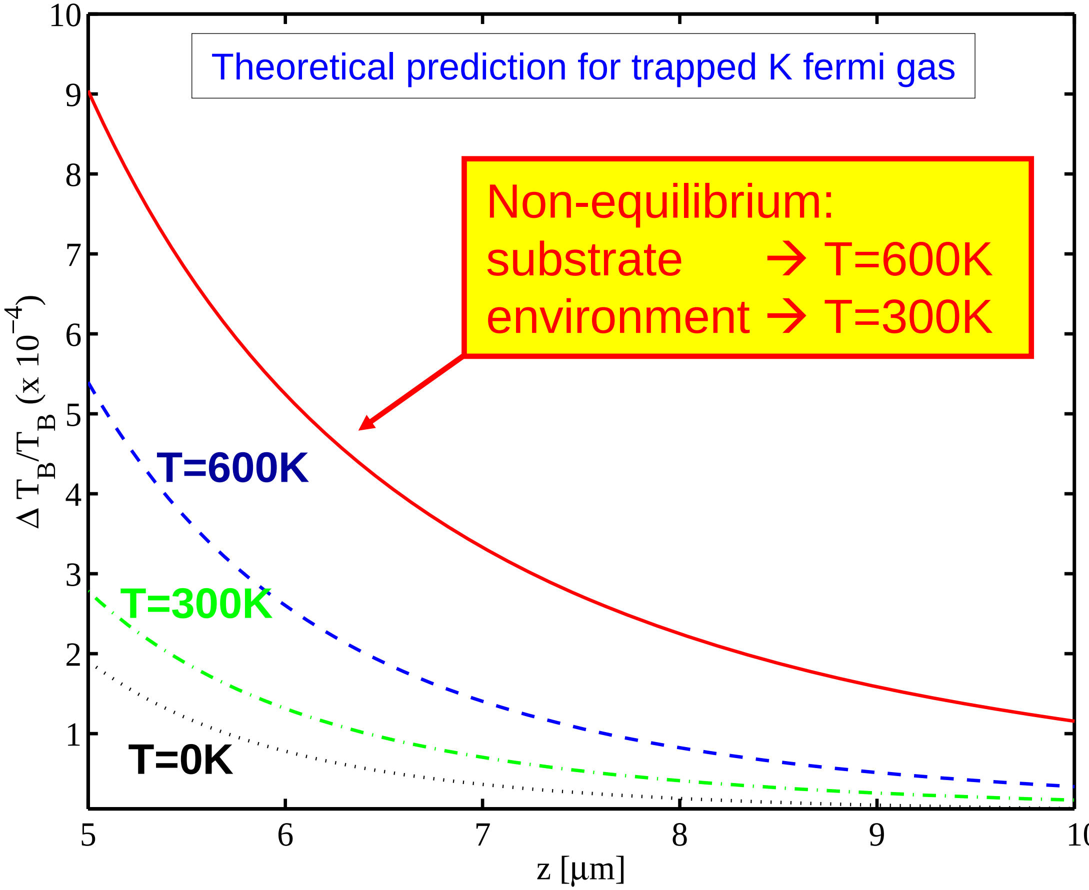
**Excellent th. vs exp. agreement: Carusotto et al., Phys. Rev. Lett. 95, 093202 (2005)**

# Surface-atom force effects on the Bloch frequency

Bloch oscillations of a trapped gas in an optical lattice in presence of gravity and surface-atom interactions: change in the ext. force → change in the Bloch frequency

$$\hbar \dot{q} = F = mg + F_{CP}$$

$$F_g / F_{CP} \approx 10^4$$



Theoretical prediction for trapped K fermi gas

Non-equilibrium:
substrate → T=600K
environment → T=300K

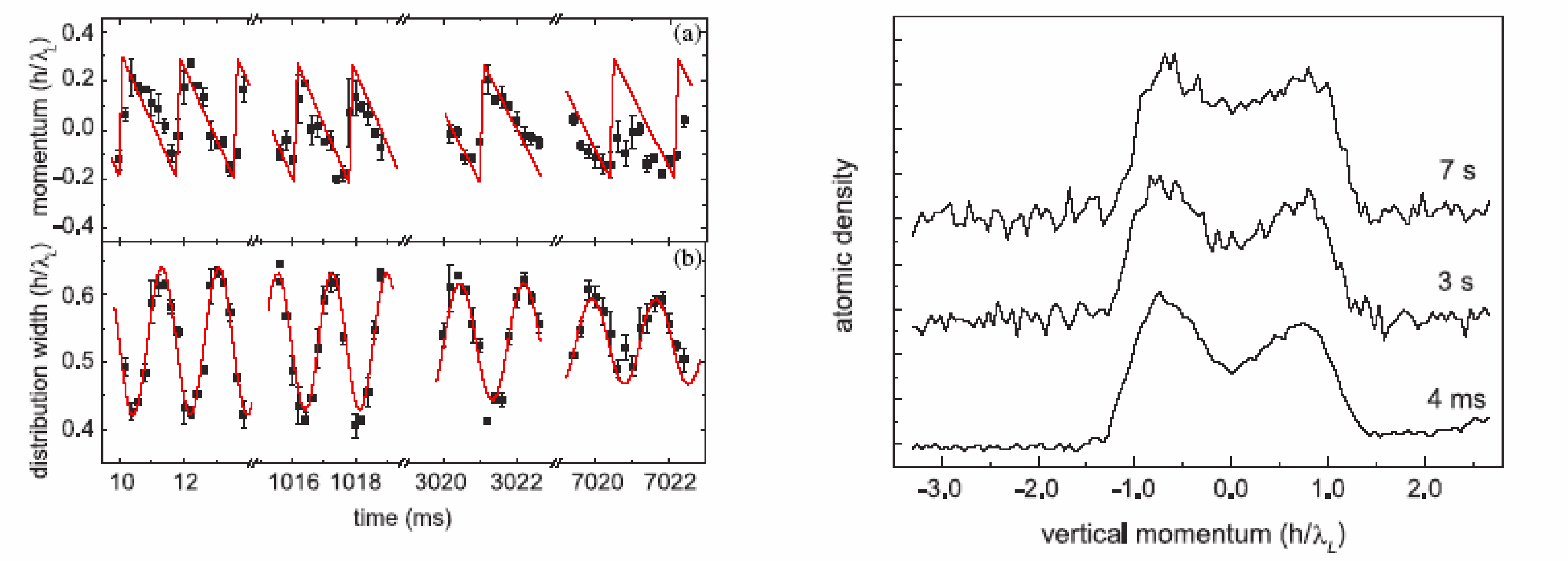
Sensitivity required: $\Delta T_B/T_B = 10^{-4}$ -$10^{-5}$

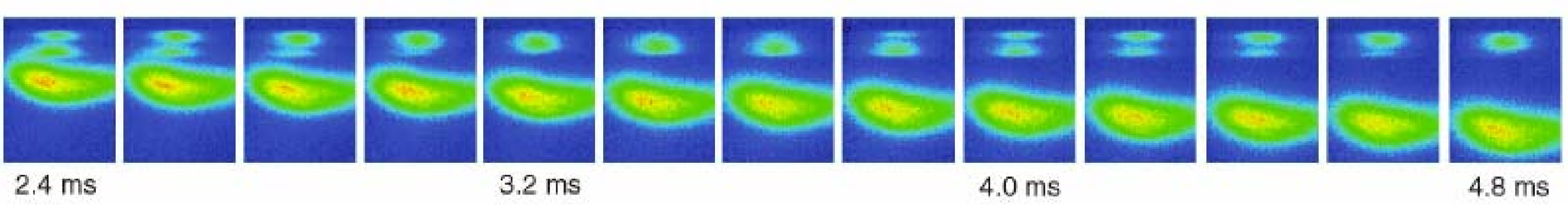
Carusotto, Antezza, Pitaevskii, Stringari

# Some High Precision Experiments on Bloch Oscillations with Cold atoms

- M. Ben Dahan *et al*., Phys. Rev. Lett. **76**, 4508 (1996)
- E. Peik *et al.*, Phys. Rev. A **55**, 2989 (1997)
- R. Battesti *et al*., Phys. Rev. Lett. **92**, 253001 (2004)
- G. Roati *et al*., Phys. Rev. Lett. **92**, 230402 (2004)
- P. Lemonde, and P. Wolf, Phys. Rev. A **72**, 033409 (2005)
- G. Ferrari *et al*., Phys. Rev. Lett. **97**, 060402 (2006)

# Bloch oscillations with thermal Sr atoms





Obtained sensitivity $5*10^{-6}$ g with thermal sample of $^{88}$Sr in 12 seconds

G. Ferrari, N. Poli, F. Sorrentino, and G. M. Tino, PRL **97**, 060402 (2006)

# Induced dipole and field

$$\vec{d}^{\,ind}[\omega] = \alpha(\omega)\vec{E}^{\,tot}[\omega;\vec{r}\,] \approx \alpha(\omega)\vec{E}^{\,fl}[\omega;\vec{r}\,]$$

$$\vec{E}^{\,ind}[\omega;\vec{r}\,] = G[\omega;\vec{r},\vec{r}\,']\vec{d}^{\,tot}[\omega] \approx G[\omega;\vec{r},\vec{r}\,']\vec{d}^{\,fl}[\omega]$$

## At thermal equilibrium, the average value of the fluctuating dipole and field : Fluctuations Dissipation Theorem

$$\left\langle d_i^{\,fl}[\omega]d_j^{\,fl+}[\omega']\right\rangle_S = 2\pi\delta_{ij}\hbar\ \coth\left(\frac{\hbar\omega}{2k_BT}\right)\ \alpha''(\omega)\ \delta(\omega-\omega')$$

$$\left\langle E_i^{\,fl}[\omega;\vec{r}\,]E_j^{\,fl+}[\omega',\vec{r}\,']\right\rangle_S = 2\pi\hbar\ \coth\left(\frac{\hbar\omega}{2k_BT}\right)\mathrm{Im}\,G_{ij}[\omega;\vec{r},\vec{r}\,']\ \delta(\omega-\omega')$$

# Rytov (1953), Lifshitz (1955), Polder&van Hove (1971),…..

## Fluctuation Dissipation Theorem

Imaginary part of the dielectric function (response function)

$$\left\langle P_i\,[\omega;\vec r]P_j^{+}[\omega',\vec r']\right\rangle_S = \frac{\hbar\varepsilon''(\omega)}{2}\ \coth\left(\frac{\hbar\omega}{2k_BT}\right)\ \delta(\omega-\omega')\delta(\vec r-\vec r')\delta_{ij}$$

Polarization (dipole moment per unit of volume)

Locality (small locality length)

## Electric Field

$$\vec E\,[\omega;\vec r]=\int_V \overline{G}\,[\omega;\vec r,\vec r']\bullet\vec P\,[\omega]\quad d\vec r$$

What for E ??

From Macroscopic Maxwell equation
+ Langevin type sources

⇩

$$\left\{\nabla\wedge\nabla\wedge -\ k^2\ \varepsilon(\omega;\vec r)\right\}\ \overline{G}\,[\omega;\vec r,\vec r']\ = 4\pi\,k^2\ \overline{I}\ \ \delta(\vec r-\vec r')$$

+ b.c. => retarded Green function

# Fluctuations effects (Maxwell stress-tensor) are bilinear in E.M. field

$$\left\langle E_i\,[\omega;\vec{r}\,]E_j^{\;+}[\omega',\vec{r}\,']\right\rangle_S = \frac{\hbar\varepsilon''(\omega)}{2}\;\coth\left(\frac{\hbar\omega}{2k_BT}\right)\delta(\omega-\omega')\int_V \mathrm{d}\,\vec{r}\,''\;G_{il}[\omega;\vec{r},\vec{r}\,'']\;G_{lj}^{*}[\omega;\vec{r}\,'',\vec{r}\,']$$

$$\left\langle P_i\,[\omega;\vec{r}\,]P_j^{\;+}[\omega',\vec{r}\,']\right\rangle_S$$

Volume containing the sources (polarizations)

# Additivity Theorem

$$\int_\Omega \mathrm{d}\,\vec{r}\,''\,\varepsilon''(\omega,\vec{r}\,'')\,G_{il}[\omega;\vec{r},\vec{r}\,'']\;G_{lj}^{*}[\omega;\vec{r}\,'',\vec{r}\,']=4\pi\;\mathrm{Im}\,G_{ij}[\omega;\vec{r},\vec{r}\,']$$

Volume on which Green functions vanish

# Fluctuations Dissipation Theorem for E.M. fields

At thermal equilibrium: ⇒ $V = \Omega =$ the whole space

⇓

Additivity theorem

⇓

$$\left\langle E_i\,[\omega;\vec{r}\,]E_j{}^{+}[\omega',\vec{r}\,']\right\rangle_S = 2\pi\hbar\ \ \coth\left(\frac{\hbar\omega}{2k_BT}\right)\mathrm{Im}\,G_{ij}[\omega;\vec{r},\vec{r}\,']\ \delta(\omega-\omega')$$

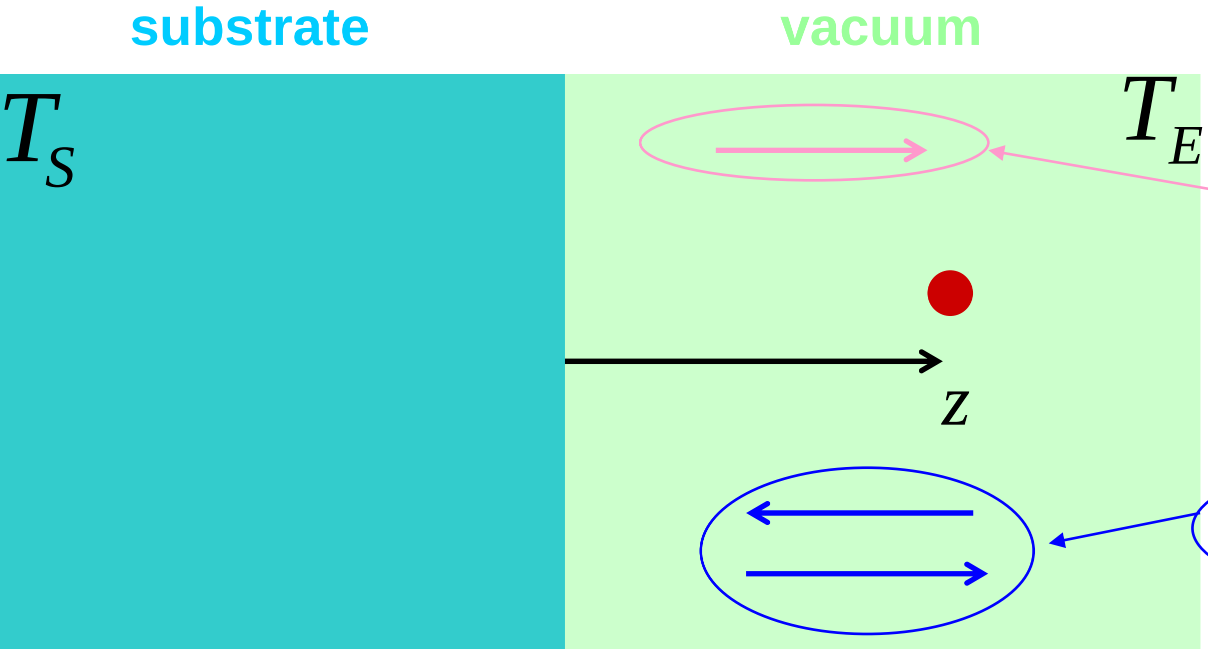
Out of thermal equilibrium: ⇒ $V = \Omega_1 + \Omega_2 + \Omega_3 + \ldots.. =$

$=$ the whole space

⇓

Additivity of the fluctuations but not Additivity theorem

# Field fluctuations out of thermal equilibrium



**Vacuum is described as a**

**dielectric with** $\boxed{\varepsilon_{vac}''(\omega) \to 0}$

$$E^{tot}[\omega; z] = E^{t}[\omega; z] + E^{i}[\omega; z] + E^{r}[\omega; z] + E^{l}[\omega; z]$$

C. Henkel et al.(2002)

$$F^{neq}(T_{S,}T_E, z) = F_0(z) + F_{th}^{neq}(T_{S,}T_E, z)$$

$$F_{th}^{neq}(T_S, 0, z) \propto \left\langle E^{t}[\omega, z] E^{t}[\omega', z'] \right\rangle$$

$$F_{th}^{neq}(0, T_E, z) \propto \left\langle \sum\nolimits_{a,b=i,r,l} E^{a}[\omega, z] E^{b}[\omega', z'] \right\rangle$$

**Averages are calculated using locally the Fluctuation Dissipation Theorem for the source currents in the substrate and in the vacuum**

# Surface-atom force out of thermal equilibrium

$$F(T_S,T_E,z)=F_0(z)+F_{th}(T_S,T_E,z)=F_0(z)+F_{th}(T_S,0,z)+F_{th}(0,T_E,z)$$

Additivity of substrate and environment thermal forces

$$F(T_S,T_E,z)=F_{eq}(T_E,z)+F_{th}(T_S,0,z)-F_{th}(T_E,0,z)$$

$$F_{th}^{neq}(T,0,z)=-\frac{2\sqrt{2}\hbar\alpha_0}{\pi c^4}\int_0^\infty d\omega\frac{\omega^4}{e^{\hbar\omega/k_BT}-1}\int_1^\infty dqq\ \ \sqrt{q^2-1}\ \ e^{-2z\sqrt{q^2-1}\omega/c}$$

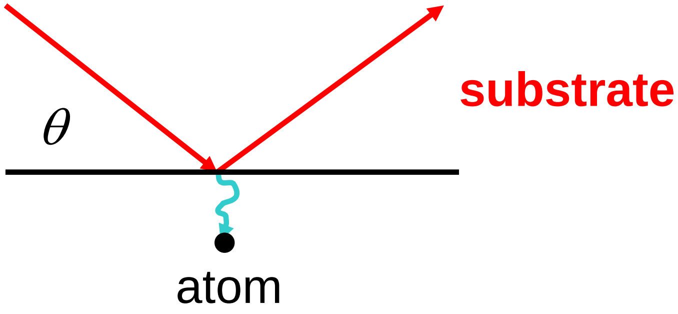
$$\times\sqrt{\left|\varepsilon-q^2\right|+\left(\varepsilon'-q^2\right)}\ \left(\frac{1}{\left|\sqrt{\varepsilon-q^2}+\sqrt{1-q^2}\right|^2}+\frac{\left(q^2+\left|\varepsilon-q^2\right|\right)\ (2q^2-1)}{\left|\sqrt{\varepsilon-q^2}+\varepsilon\sqrt{1-q^2}\right|^2}\right)$$

- Includes dispersive and absorption effects in substrate
- Neglects thermal atom radiation effects ( $k_BT_S,k_BT_E<<\hbar\omega_{at}$)

# Calculation of $F_{th}(T,0,z)$

$T_S = T$

$T_E = 0$



## Simple estimate of the long distance behaviour

- Force is produced by **evanescent waves** transmitted by substrate radiation into vacuum (only radiation undergoing total reflection contributes to the force)

- **Neglect absorption** and **dispersion** of **substrate** (static approximation) (radiation inside substrate is described in terms of ideal gas of photons)

- Leading contribution (at large distances) to force provided by incident waves with **angle close to angle of total reflection** $\sin\theta_r = 1/\sqrt{\varepsilon}$ . (at equilibrium are neglected by grazing waves)

- Relevant interval of solid angle is of order $-(\lambda_T / z)^2$
  Corresponding e.m. energy density in vacuum behaves like

$$\boxed{-(\lambda_T / z)^2 U_{black-body} \approx -T^2 / z^2} \Rightarrow$$

Force
- is attractive
- behaves like $T^2$
- decays like $1/z^3$