

Quantum ratchets for periodically kicked cold atoms and Bose-Einstein condensates

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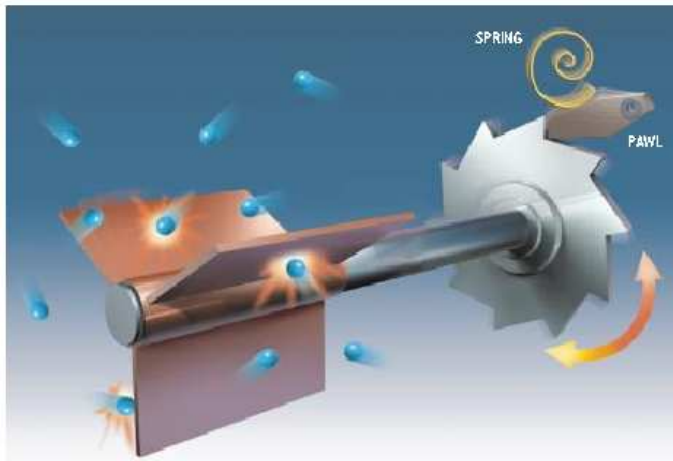
Dario Poletti, Baowen Li (Singapore)

Motivations and Outline

- Study the effect of **quantum noise** on **open quantum chaotic** systems (also motivated by technological progress)
- Possibilities opened by **optical lattices** for the experimental investigations of complex systems
 - 1) A model for **quantum** directed transport in a periodic **chaotic** systems with dissipation, in presence of lattice asymmetry and unbiased driving
Possible experimental implementation with cold atoms in **optical lattices**
 - 2) The role of **atom-atom interactions**: Many-body Hamiltonian quantum ratchet in a **Bose-Einstein condensate**

The Feynman ratchet

Can useful work be extracted out of unbiased microscopic random fluctuations if all acting forces and temperatures gradients average out to zero?



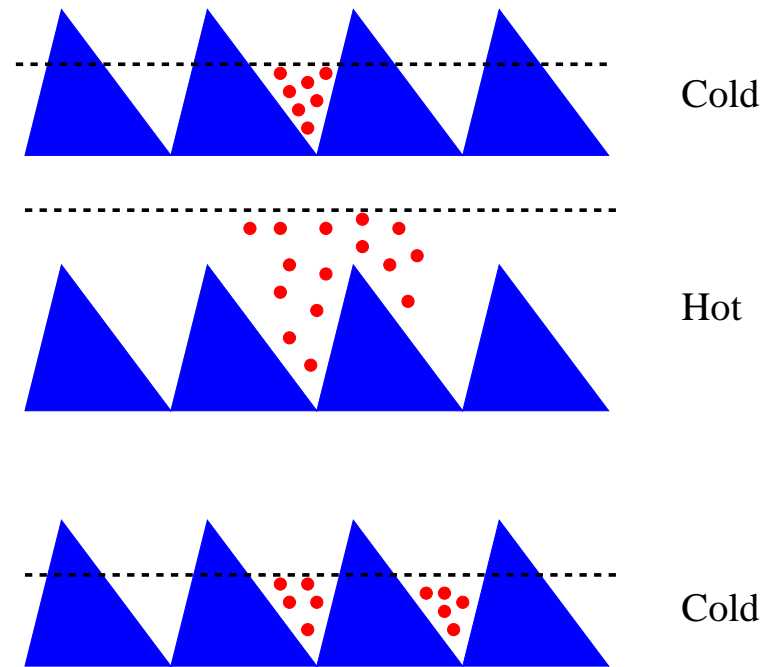
(taken from D.Astumian, Scientific American, July 2001)

Thermal equilibrium: the gas surrounding the paddles and the ratchet (plus the pawl) are at the same temperature

In spite of the built **asymmetry** no preferential direction of motion is possible. Otherwise, we could implement a **perpetuum mobile**, in contradiction with the second law of thermodynamics

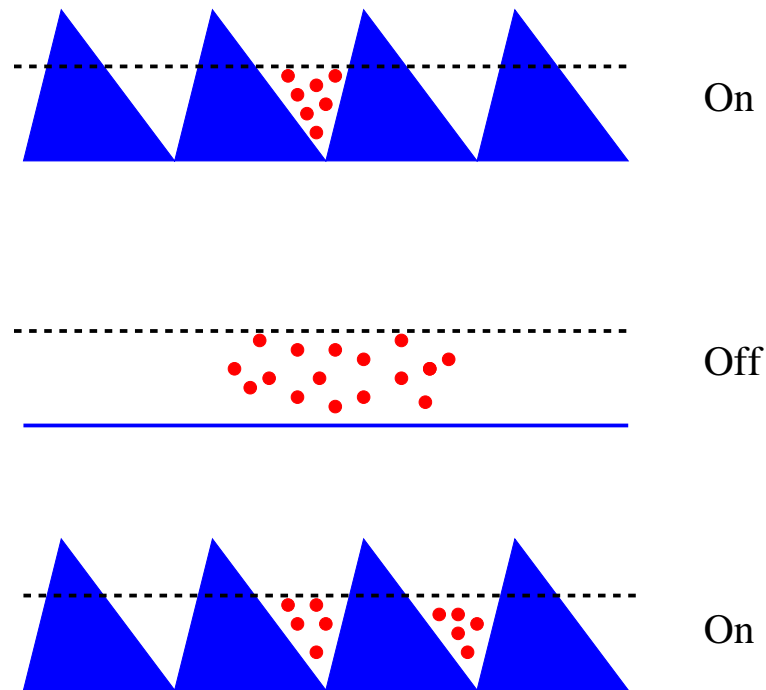
Brownian motors

To build a Brownian motor drive the system **out of equilibrium**

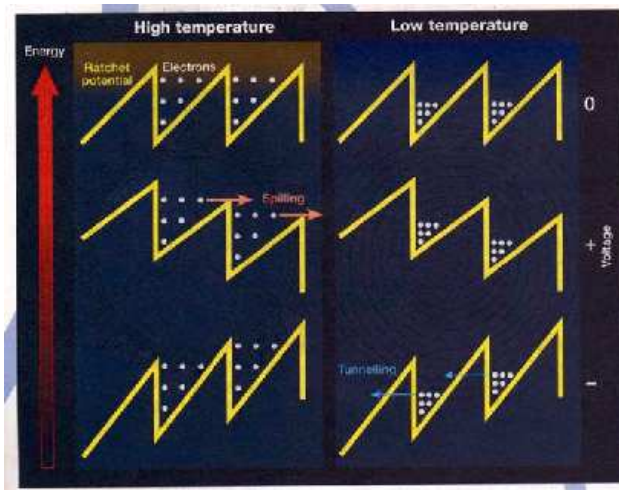


Working principle of a Brownian motor driven by **temperature oscillations**

Another model of Brownian motor: a pulsating (flashing) ratchet



Quantum ratchets



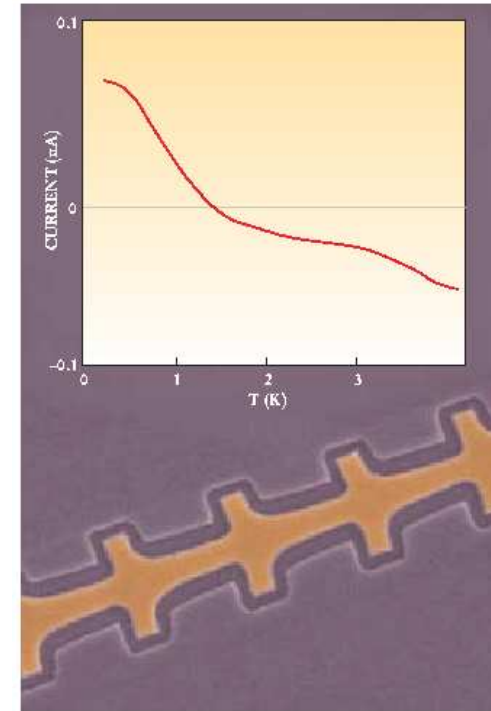
A **rocking ratchet**: the ratchet potential is tilted symmetrically and periodically

Due to the asymmetry of the barriers, a **thermally activated** net current (to the right) is generated (after averaging over both tilt directions)

Tunneling electrons, however, prefer the thinner barriers that are the result of tilt to the left

Electrons powered by ac signals could **run against a static electric field** (“electrons going **uphill**”)

Quantum tunneling provides a second mechanism (the first being the thermal activation) to overcome energy barriers and lead to directed motion



(String of triangular quantum dots, Linke *et al* experiments, Science, 1999)

Rectification of fluctuations in optical lattices

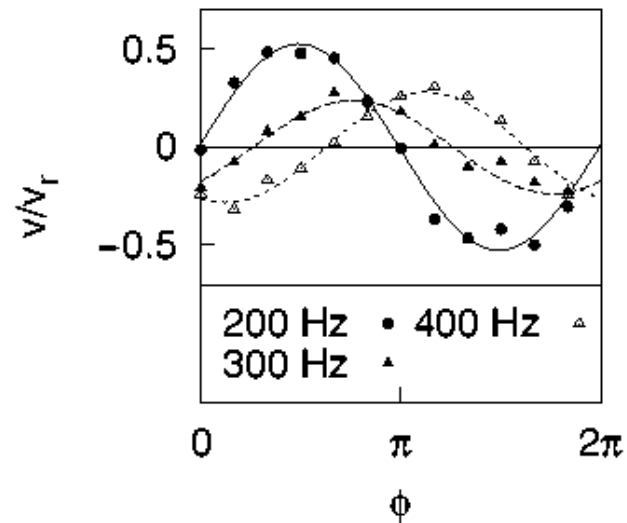
Optical pumping: transition between two ground state sublevels of atoms in optical lattices - As this is a **stochastic process**, fluctuations in the atomic dynamics are introduced, resulting in a **random walk** through the optical lattice

Apply a zero-mean ac force breaking all relevant system's symmetry:

$$F(t) = F_0[A \cos(\omega t) + B \cos(2\omega t - \phi)]$$

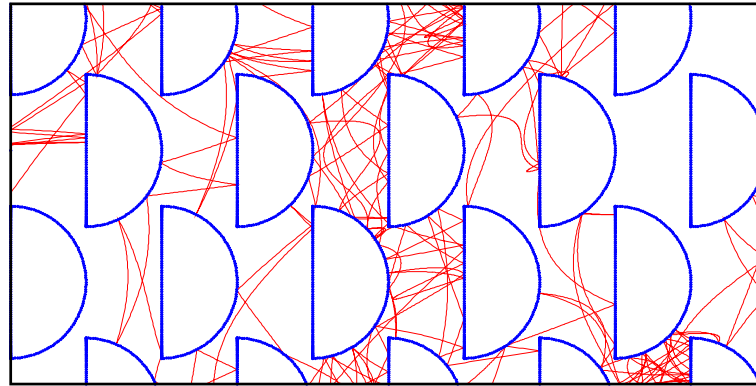
This force is obtained (in the accelerated frame in which the optical lattice is stationary) by means of a phase-modulated beam:

$$\alpha(t) = \alpha_0 \left[A \cos(\omega t) + \frac{B}{4} \cos(2\omega t - \phi) \right], \quad F_0 \propto \alpha_0$$



[R. Gommers, S. Bergamini, F. Renzoni, PRL 95, 073003 (2005)]

Directed transport in asymmetric antidot lattices



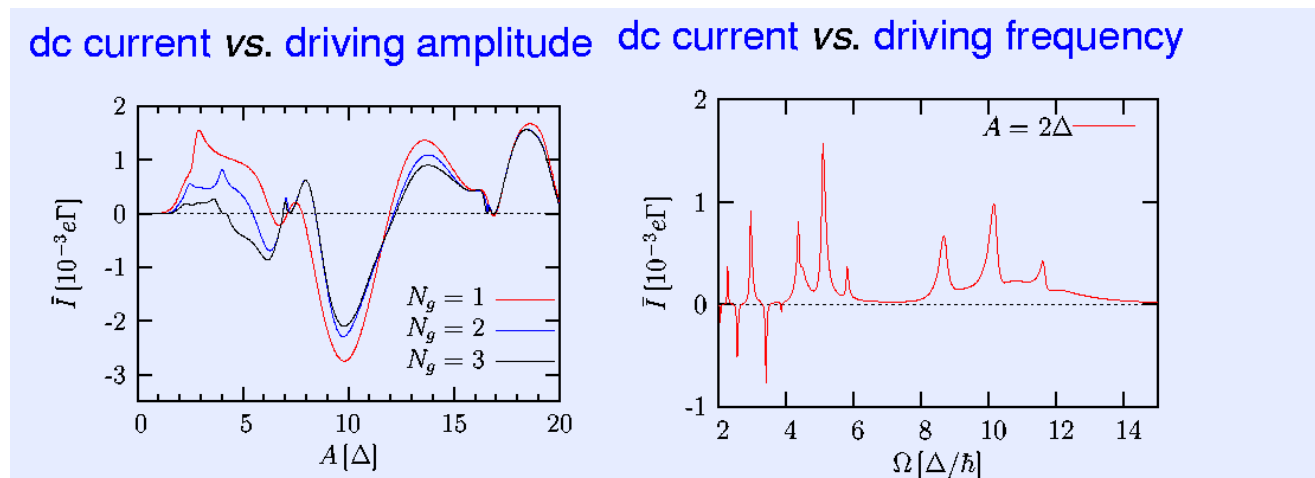
[A.D. Chepelianskii and D.L. Shepelyansky, PRB **71**, 052508 (2005)]

The semidisk Galton board (with chaotic classical dynamics) is subjected to **microwave polarized radiation**, at finite temperature

Directed transport with antidots of micron size up to about 100 GHz - possible application as new type of highly sensitive **detectors of polarized radiation**, useful for instance in the field of radioastronomy

Ratchet effect in molecular wires

Molecular wire in an asymmetric potential, subjected to effective dissipation from leads and to a laser field (Motivations: molecular electronics, self-assembly,...)



[J. Lehmann, S. Kohler, P. Hänggi, and A. Nitzan PRL **88**, 228305 (2002)]

Ratchet current exhibits resonances: coherent transport

The multiple current reversals open prospects to pump and shuttle electrons on the nanoscale in an *a priori* manner

A deterministic model of quantum chaotic dissipative ratchet

Particle moving in a kicked periodic asymmetric potential [$H = \frac{I^2}{2} + V(x, \tau)$]

$$V(x, \tau) = k \left[\cos(x) + \frac{a}{2} \cos(2x + \phi) \right] \sum_{m=-\infty}^{+\infty} \delta(\tau - mT),$$

Classical evolution in one period described by the map

$$\begin{cases} \bar{I} = (1 - \gamma)I + k(\sin(x) + a \sin(2x + \phi)), \\ \bar{x} = x + T\bar{I}, \end{cases}$$

$0 < \gamma < 1$ dissipation parameter (velocity proportional damping):

$\gamma = 1$ overdamping – $\gamma = 0$ Hamiltonian evolution

Introducing the rescaled momentum variable $p = TI$, one can see that classical dynamics depends on the parameter $K = kT$ (not on k and T separately)

Study of the quantized model

Quantization rules: $x \rightarrow \hat{x}$, $I \rightarrow \hat{I} = -i(d/dx)$ (we set $\hbar = 1$)

Since $[\hat{x}, \hat{p}] = [\hat{x}, T\hat{I}] = iT$, the effective Planck constant is $\hbar_{\text{eff}} = T$

In order to simulate a dissipative environment in the quantum model we consider a master equation in the Lindblad form for the density operator $\hat{\rho}$ of the system:

$$\dot{\hat{\rho}} = -i[\hat{H}_s, \hat{\rho}] - \frac{1}{2} \sum_{\mu=1}^2 \{\hat{L}_\mu^\dagger \hat{L}_\mu, \hat{\rho}\} + \sum_{\mu=1}^2 \hat{L}_\mu \hat{\rho} \hat{L}_\mu^\dagger$$

$\hat{H}_s = \hat{I}^2/2 + V(\hat{x}, \tau)$ system Hamiltonian

\hat{L}_μ Lindblad operators

$\{, \}$ denotes the anticommutator

The dissipation model

We assume that dissipation is described by the lowering operators

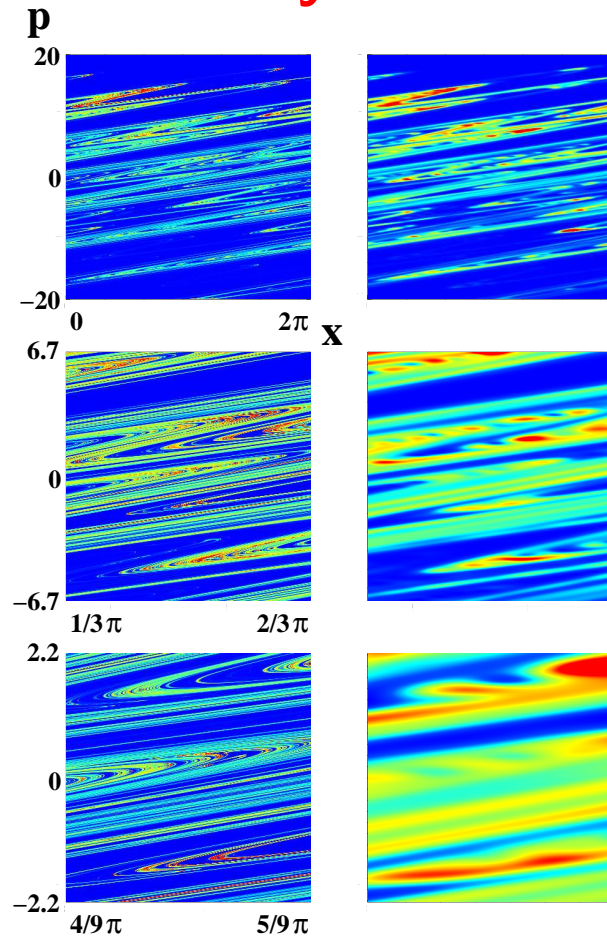
$$\begin{aligned}\hat{L}_1 &= g \sum_I \sqrt{I+1} |I\rangle \langle I+1|, \\ \hat{L}_2 &= g \sum_I \sqrt{I+1} |-I\rangle \langle -I-1|, \quad I = 0, 1, \dots\end{aligned}$$

These Lindblad operators can be obtained by considering the interaction between the system and a bosonic bath. The master equation is then derived, at zero temperature, in the usual weak coupling and Markov approximations

Requiring that at short times $\langle p \rangle$ evolves like in the classical case, as it should be according to the Ehrenfest theorem, we obtain $e^{-g^2} = 1 - \gamma$

Simulation of quantum dissipation with quantum trajectories

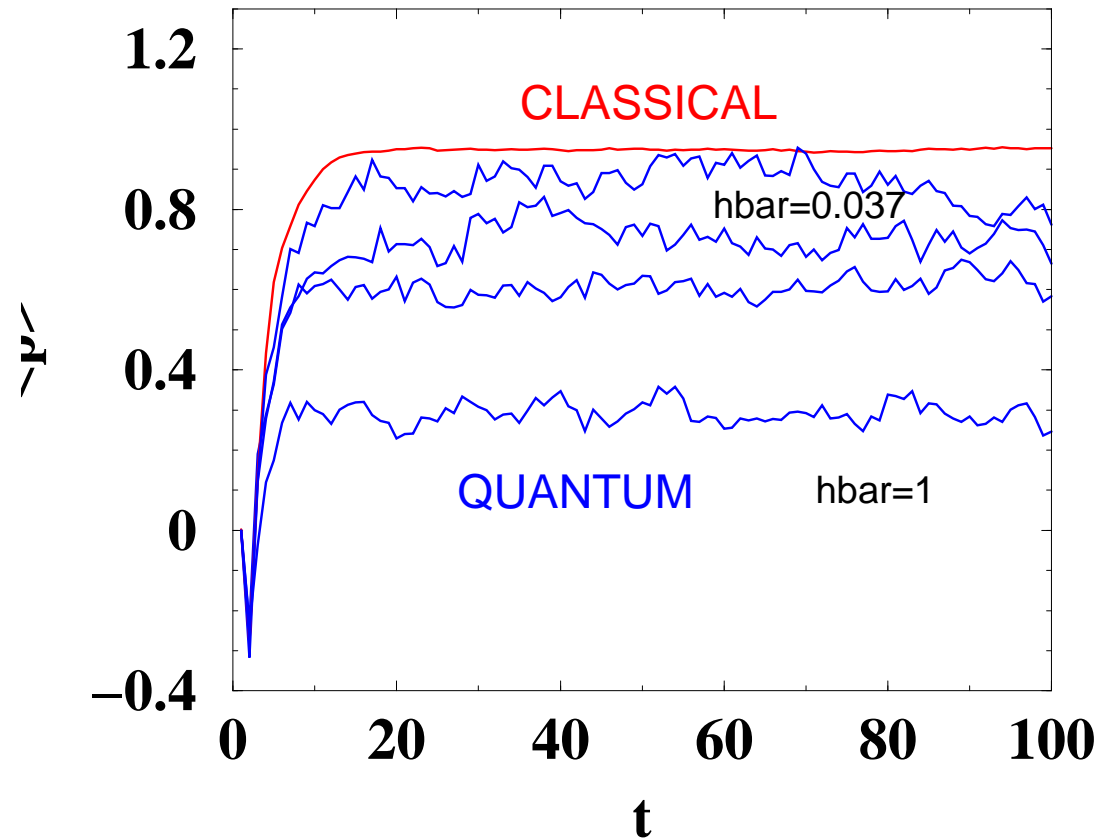
Asymmetric quantum strange attractor



Phase space pictures for $K = 7$,
 $\gamma = 0.3$, $\phi = \pi/2$, $a = 0.7$, after
 100 kicks: classical Poincaré sections
 (left) and quantum Husimi functions
 at $\hbar_{\text{eff}} = 0.012$ (right)

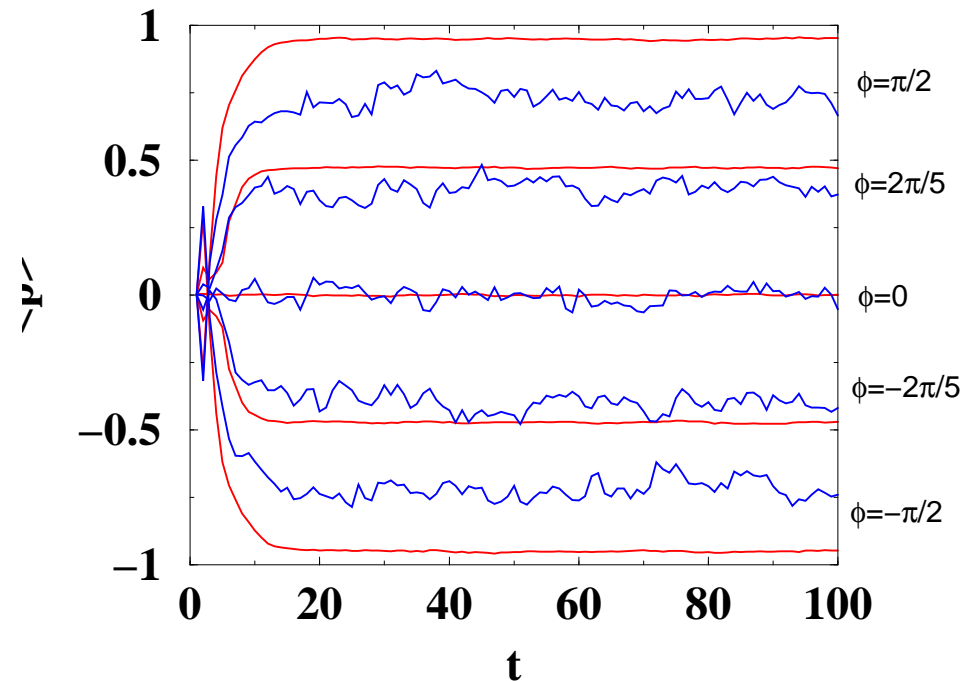
$p = TI$ rescaled momentum
 $K = Tk$ rescaled kicking strength

Ratchet effect



Average momentum $\langle p \rangle$ as a function of time t (measured in number of kicks)

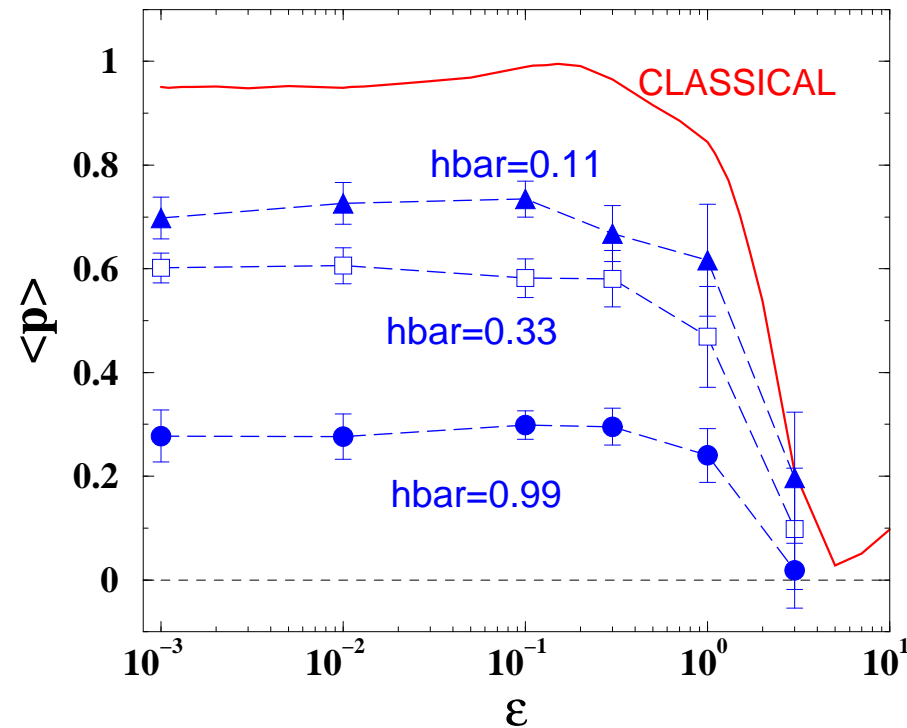
Control the direction of transport



Zero net current for $\phi = n\pi$, due to the **space symmetry** $V(x, \tau) = V(-x, \tau)$

In general $\langle p \rangle_{-\phi} = -\langle p \rangle_{\phi}$, due to the symmetry $V_{\phi}(x, \tau) = V_{-\phi}(-x, \tau)$

Stability under noise effects



Memoryless fluctuations in the kicking strength: $K \rightarrow K_\epsilon(t) = K + \epsilon(t)$, $\epsilon(t) \in [-\epsilon, +\epsilon]$

The ratchet effect survives up to a noise strength ϵ of the order of the kicking strength K

Quantum ratchets in dissipative chaotic systems,
Phys. Rev. Lett. **94**, 164101 (2005)

A note about possible implementations

Possible experimental implementations with cold atoms in a periodic standing wave of light

Values $K = 7$, $\hbar_{\text{eff}} \sim 1$ used in the experimental implementations of the kicked rotor model

Ex: From Raizen's group, PRL **75**, 4598 (1995) (sodium atoms in a laser field):

$\lambda_L = 589$ nm laser field wave length

$K = \sqrt{2\pi}\alpha\Omega_{\text{eff}}\omega_r T^2$ classical chaos parameter

$\hbar_{\text{eff}} = 8\omega_r T$

T pulse periodicity

Ω_{eff} effective Rabi frequency

$\omega_r = \hbar k_L^2 / 2M$ recoil frequency ($k_L = 1/\lambda_L$, M atomic mass)

α fraction of Gaussian pulse duration in units of pulse period

$\Omega_{\text{eff}}/2\pi = 75$ MHz, $T = 0.8 \mu\text{s}$, $\alpha = 0.05$ give $\hbar_{\text{eff}} \approx 1$, $K \approx 5$

Quantum ratchets in dissipative chaotic systems,
Phys. Rev. Lett. **94**, 164101 (2005)

The $\cos(x) + \cos(2x + \phi)$ potential has been recently implemented in optical lattices by the group of Martin Weitz (cond-mat/0512018)

Friction force can be implemented by means of Doppler cooling techniques

For sodium a dissipation rate $2\beta \approx 4 \times 10^5 \text{ s}^{-1}$ [Raab et al., PRL **59**, 2631 (1987)] gives $\gamma \approx 0.3$

State reconstruction techniques [Bienert et al., PRL **89**, 050403 (2002)] could in principle allow the experimental observation of a quantum strange ratchet attractor

The ratchet effect is robust when noise is added; due to the presence of a strange attractor, the stationary current is independent of the initial conditions

Experimental proposal

The ratchet effect can be realized with two series of spatially periodic kicks:

$$H(t) = \frac{p^2}{2} + V_{\phi, \xi}(x, t), \quad V_{\phi, \xi} = k \sum_{n=-\infty}^{+\infty} [\delta(t - nT) \cos(x) + \delta(t - nT - \xi) \cos(x - \phi)]$$

We can break all relevant symmetries and induce the ratchet effect with a **purely Hamiltonian model** (see also Monteiro *et al.*, PRL **89**, 194102 (2002))

We are interested in symmetries that leave invariant the equations of motion but change the sign of p (see Flach *et al.*, PRL **84**, 2358 (2000)):

$$(I) \quad x \rightarrow -x + \alpha, \quad t \rightarrow t + \beta,$$

$$(II) \quad x \rightarrow x + \alpha, \quad t \rightarrow -t + \beta.$$

Symmetry (I) is broken for $\phi \neq 0, \pi$, symmetry (II) is broken for $\xi \neq 0, T/2$

It should be remarked that fluctuations in the rectified current grow with time in the Hamiltonian case for this model, while they saturate in the dissipative case when the strange attractor sets in

Many-body quantum ratchet in a Bose-Einstein condensate

Quantum Hamiltonian ratchets are relevant in systems such as cold atoms in which the high degree of quantum control may allow experimental implementations *near to the dissipationless limit*

The realization of Bose-Einstein condensates of dilute gases has opened new opportunities for the study of dynamical systems in the presence of many-body interactions: it is possible to prepare initial states with high precision and to tune over a wide range the many-body atom-atom interaction

Study directed transport in many-body quantum system

The model: a kicked BEC

We consider N condensed atoms confined in a toroidal trap of radius R and cross section πr^2 ($r \ll R$, one-dimensional motion)

The $T = 0$ motion of a dilute BEC in a pair of periodically kicked optical lattices is described by the Gross-Pitaevskii nonlinear equation

$$i \frac{\partial}{\partial t} \psi(\theta, t) = \left[-\frac{1}{2} \frac{\partial^2}{\partial \theta^2} + g |\psi(\theta, t)|^2 + V(\theta, \phi, t) \right] \psi(\theta, t)$$

θ azimuthal angle

$g = 8NaR/r^2$ scaled strength of the repulsive ($g > 0$) nonlinear interaction (a s -wave scattering length)

$$V(\theta, \phi, t) = \sum_n [V_1(\theta) \delta(t - nT) + V_2(\theta, \phi) \delta(t - nT - \xi)]$$

$$V_1(\theta) = k \cos \theta, \quad V_2(\theta, \phi) = k \cos(\theta - \phi)$$

k kicking strength, T period of the kicks

The noninteracting limit ($g = 0$)

When $\phi \neq 0, \pi$ and $\xi \neq 0, T/2$ space-time symmetries are broken and there is directed transport, both in the classical limit and, in general, in quantum mechanics

However, if $T = 6\pi$ and $\xi = 4\pi$, then the quantum motion, independently of the kicking strength k , is periodic of period $2T$

$$\psi(\theta, 4\pi^+) = \exp[-iV_1(\theta)]\psi(\theta, 0)$$

$$\psi(\theta, 6\pi^+) = \exp[-iV_2(\theta, \phi)]\psi(\theta + \pi, 4\pi^+) = \exp\{-i[V_2(\theta, \phi) - V_1(\theta)]\}\psi(\theta + \pi, 0)$$

$$\psi(\theta, 10\pi^+) = \exp[-iV_1(\theta)]\psi(\theta, 6\pi^+) = \exp(-iV_2(\theta, \phi))\psi(\theta + \pi, 0)$$

$$\psi(\theta, 12\pi^+) = \exp[-iV_2(\theta, \phi)]\psi(\theta + \pi, 10\pi^+) = \psi(\theta, 0)$$

If the initial wave function $\psi(\theta, 0) = 1/\sqrt{2\pi}$, then directed transport is absent

The momentum $\langle p(t) \rangle = -i \int_0^{2\pi} d\theta \psi^*(\theta, t) \frac{\partial}{\partial \theta} \psi(\theta, t)$ also changes periodically with period $2T = 12\pi$ (4 kicks)

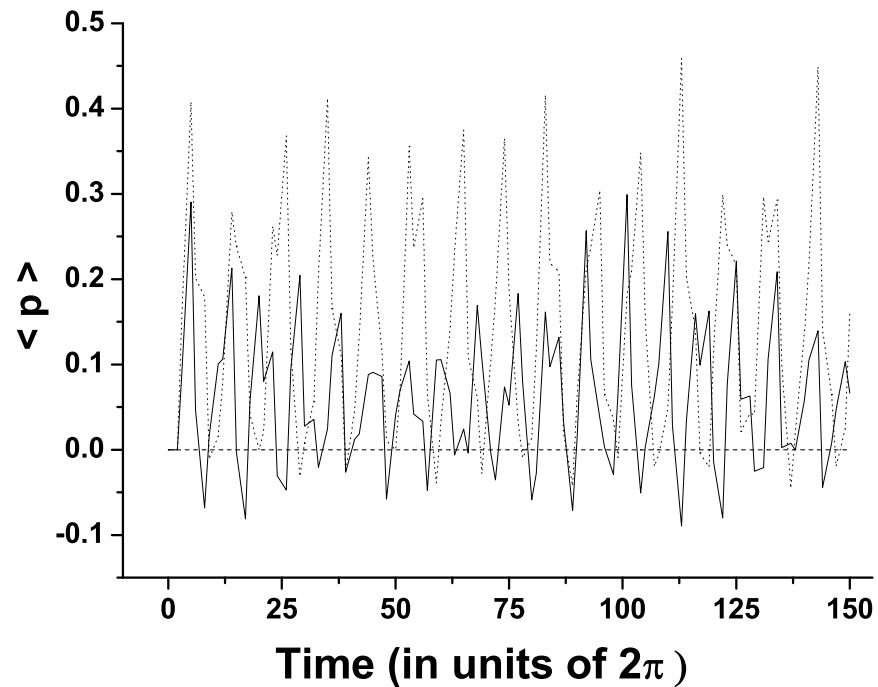
Therefore, the average momentum $p_{\text{av}} \equiv \lim_{t \rightarrow \infty} \bar{p}(t)$, ($\bar{p}(t) \equiv \frac{1}{t} \int_0^t dt' \langle p(t') \rangle$) is obtained after averaging the momentum over the period $2T$:

$$p_{\text{av}} = \langle p(0) \rangle + \frac{k}{2} \int_0^{2\pi} (\sin(\theta) - \sin(\theta - \phi)) |\psi(\theta, 0)|^2 d\theta,$$

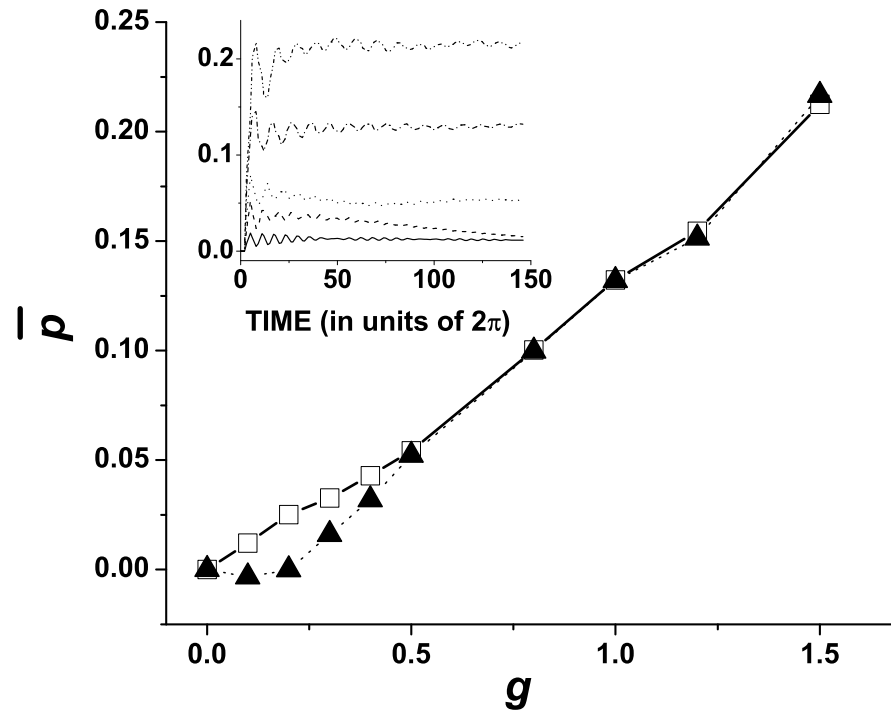
For $\psi(t, 0) = 1/\sqrt{2\pi}$ (ground state of a particle in the trap) the momentum is always zero at any time:

This initial condition has an important physical meaning, as it corresponds to the initial condition for a Bose-Einstein condensate

Ratchet effect in a BEC ($g \neq 0$)



Momentum versus time for different values of interaction strength g , at $k \approx 0.74$, $\phi = -\pi/4$:
 $g = 0$ (dashed curve),
 $g = 0.5$ (continuous curve),
 $g = 1$ (dotted curve)



Momentum averaged over the first 30 kicks (squares) and asymptotic momentum (triangles)

Inset: $g = 0.1, 0.2, 0.4, 1.0, 1.5$

The ratchet phenomenon can be used to measure atom interaction strength

STABILITY OF THE RATCHET TO PERTURBATIONS:

- Kicking period fluctuations of size $T/100$ generate, after 30 kicks, a current $\bar{p} = -0.007$
- Gaussian pulses of width $T/10$ lead to $\bar{p} = -0.01$

Why the interaction-induced ratchet effect?

For small g , approximate the free evolution of the BEC by a **split-operator method**:

$$\psi(\theta, \tau) \approx e^{-i\frac{1}{2}\frac{\partial^2}{\partial\theta^2}\frac{\tau}{2}} e^{-ig|\tilde{\psi}(\theta, \frac{\tau}{2})|^2\tau} e^{-i\frac{1}{2}\frac{\partial^2}{\partial\theta^2}\frac{\tau}{2}}\psi(\theta, 0), \quad \tilde{\psi}(\theta, t + \Delta t) = e^{-i\frac{1}{2}\frac{\partial^2}{\partial\theta^2}\Delta t}\psi(\theta, t)$$

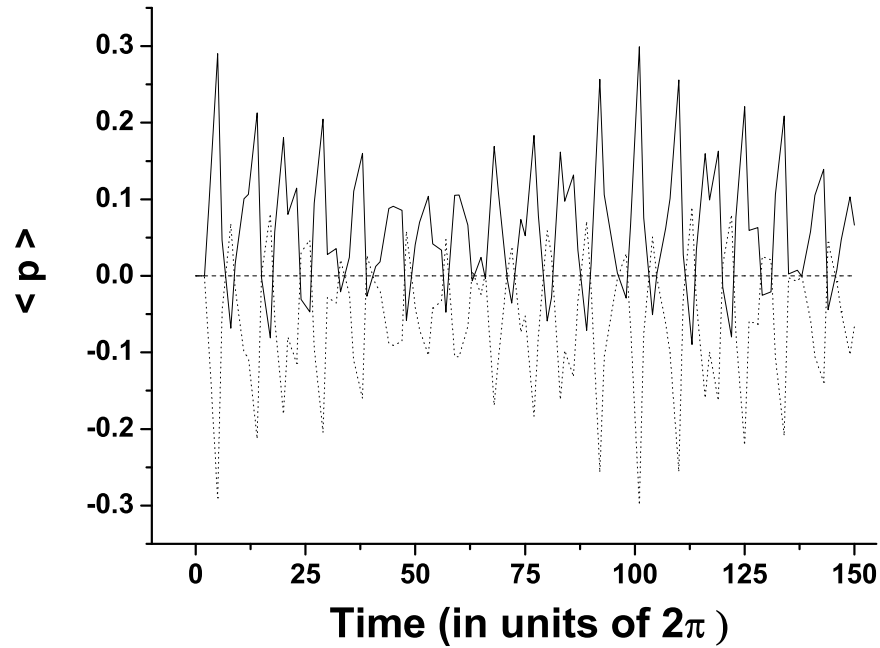
$$|\psi(\theta, 6\pi)|^2 \approx \frac{1}{2\pi} \{1 + g \sin[4V_1(\theta)]\}, \quad V_1(\theta) = k \cos \theta$$

At $t = 6\pi$ (before the second kick) the initial constant probability distribution is modified by a term symmetric under the transformation $\theta \rightarrow -\theta$

$$\langle p(6\pi^+) \rangle = - \int_0^{2\pi} d\theta V_2'(\theta, \phi) |\psi(\theta, 6\pi)|^2 \approx -gk \sin(\phi) J_1(4k), \quad V_2 = k \cos(\theta - \phi)$$

This current is in general different from zero, provided that $V_2(\theta, \phi)$ is not itself symmetric under $\theta \rightarrow -\theta$, that is, when $\phi \neq 0, \pi$

$\phi \rightarrow -\phi$ symmetry



$\phi = -\pi/4$ (continuous line),
 $\phi = 0$ (dashed line),
 $\phi = \pi/4$ (dotted line)

After substituting $\theta \rightarrow -\theta$ in the Gross-Pitaevskii equation, and taking into account that that $V(-\theta, \phi, t) = V(\theta, -\phi, t)$, we obtain

$$i\frac{\partial}{\partial t}\tilde{\psi}(\theta, t) = \left[-\frac{1}{2}\frac{\partial^2}{\partial\theta^2} + g|\tilde{\psi}(\theta, t)|^2 + V(\theta, -\phi, t) \right] \tilde{\psi}(\theta, t), \quad \tilde{\psi}(\theta, t) \equiv \psi(-\theta, t)$$

Therefore, if $\psi(\theta, t)$ is a solution of the Gross-Pitaevskii equation, then also $\tilde{\psi}(\theta, t)$ is a solution, provided that we substitute $\phi \rightarrow -\phi$ in the potential V

The momentum $\langle\tilde{p}(t)\rangle$ of the wavefunction $\tilde{\psi}(\theta, t)$ is given by $\langle\tilde{p}(t)\rangle = -\langle p(t)\rangle$, where $\langle p(t)\rangle$ is the momentum of $\psi(\theta, t)$

Since we start with an even wavefunction, $\tilde{\psi}(\theta, 0) = \psi(-\theta, 0) = \psi(\theta, 0)$, then $\phi \rightarrow -\phi$ changes the sign at any later time

Evolution of non-condensed particles

When studying the dynamics of a kicked BEC, it is important to take into account the **proliferation of noncondensed atoms**: actually, strong kicks may lead to **thermal excitations out of equilibrium and destroy the condensate**, rendering the description by the Gross-Pitaevskii equation meaningless

Let us show that, for the parameter values considered in the previous figures, the number of noncondensed particles is *negligible compared to the number of condensed ones*

Linear stability analysis

At $T = 0$ the mean number of noncondensed particles is [see Castin and Dum, PRA57, 3008 (1998) and Zhang *et al*, PRL92, 054101 (2004)]

$$\delta N(t) = \sum_{j=1}^{\infty} \int_0^{2\pi} d\theta |v_j(\theta, t)|^2,$$

$$i \frac{\partial}{\partial t} \begin{bmatrix} u_j(\theta, t) \\ v_j(\theta, t) \end{bmatrix} = \begin{bmatrix} H_1(\theta, t) & H_2(\theta, t) \\ -H_2^*(\theta, t) & -H_1^*(\theta, t) \end{bmatrix} \begin{bmatrix} u_j(\theta, t) \\ v_j(\theta, t) \end{bmatrix},$$

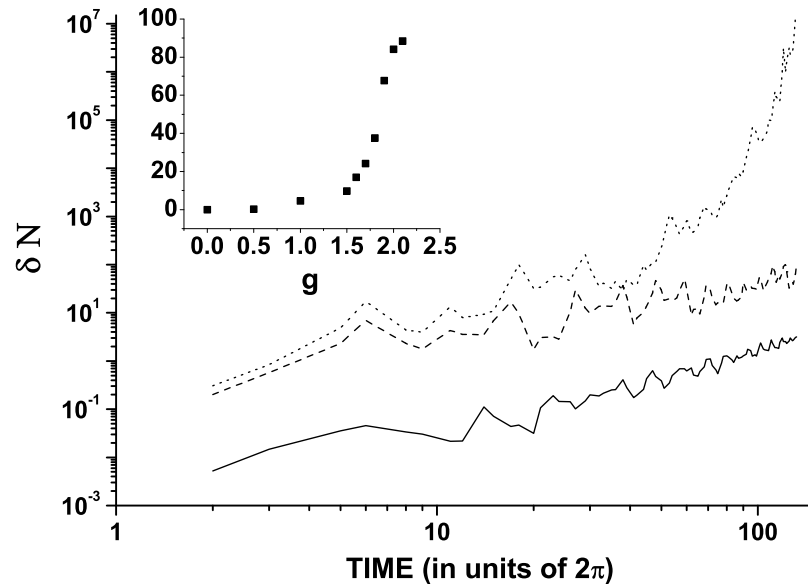
$$H_1(\theta, t) = H(\theta, t) - \mu(t) + gQ(t)|\psi(\theta, t)|^2Q(t)$$

$$H(\theta, t) = -\frac{1}{2}\frac{\partial^2}{\partial\theta^2} + g|\psi(\theta, t)|^2 + V(\theta, \phi, t) \text{ mean-field Gross-Pitaevskii Hamiltonian}$$

$$\mu(t) \text{ chemical potential } [H(\theta, t)\psi(\theta, t) = \mu(t)\psi(\theta, t)]$$

$$Q(t) = 1 - |\psi(t)\rangle\langle\psi(t)| \text{ projects orthogonally to } |\psi(t)\rangle$$

$$H_2(\theta, t) = gQ(t)\psi^2(\theta, t)Q^*(t)$$



From bottom to top: $g = 0.5, 1.5, 2.0$
Inset: δN vs. g after 30 kicks

The number δN of noncondensed particles, depending on the stability or instability of the condensate, grows polynomially or exponentially

The transition from stability to instability takes place at $g = g_c \approx 1.7$

At $g > g_c$, thermal particles proliferate exponentially fast, $\delta N \sim \exp(rt)$

At $g < g_c$, the growth rate $r = 0$

Remarks on experimental feasibility

Torus-like potential confining the BEC feasible by means of optical billiards

Kicks may be applied using a periodically pulsed strongly detuned laser beam with a suitably engineered intensity [Mieck and Graham, J. Phys. A **37**, L581 (2004)]

Optical traps such as the ^{87}Rb BEC in a quasi-one-dimensional optical box trap, with condensate length $\sim 80 \mu\text{m}$, transverse confinement $\sim 5 \mu\text{m}$, and number of particles $N \sim 10^3$ [T.P. Meyrath *et al.*, Phys. Rev. A **71**, 041604(R) (2005)]

Sequences of up to 25 kicks have been applied to a BEC of ^{87}Rb atoms confined in a static harmonic magnetic trap, with kicking strength $k \sim 1$ and in the quantum antiresonance case for the kicked oscillator model, $T = 2\pi$ [G.J. Duffy *et al.*, Phys. Rev. A **70**, 041602(R) (2004)]

The interaction strength g can be tuned using a Feshbach resonance

Conclusions and prospects

- Cold atoms and Bose-Einstein condensates exposed to time-dependent standing waves of light provide an **ideal test bed to explore complex quantum dynamics**
- Quantum dissipative ratchets: study the impact of **dynamical effects** such as bifurcations on the ratchet current
- Quantum many-body ratchet effect in a BEC: find different models (beyond periodic motion)
- From simple models to complex solid state and biological samples: understand charge-transfer phenomena in **molecular wires** and **biomolecules**: study conductance under laser excitation, current control, ...