QUANTUM MECHANICS:

FROM FUNDAMENTAL PROBLEMS TO APPLICATIONS,

Bertinoro, 4-7 December, 2006

QUANTUM IMAGING
AND THE DETECTION OF WEAK OBJECT

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Exp. in Como: Ottavia Jedrkiewicz, Paolo Di Trapani (Insubria Univ.)

Exp. in Torino: Marco Genovesi (Ist. Galileo Ferrari)

Outline

Measurement for the detection of weak amounts of absorbtion

- Principle of operation of differential measurement

- Beating the SQL by using sub-shot noise correlation of twin beams

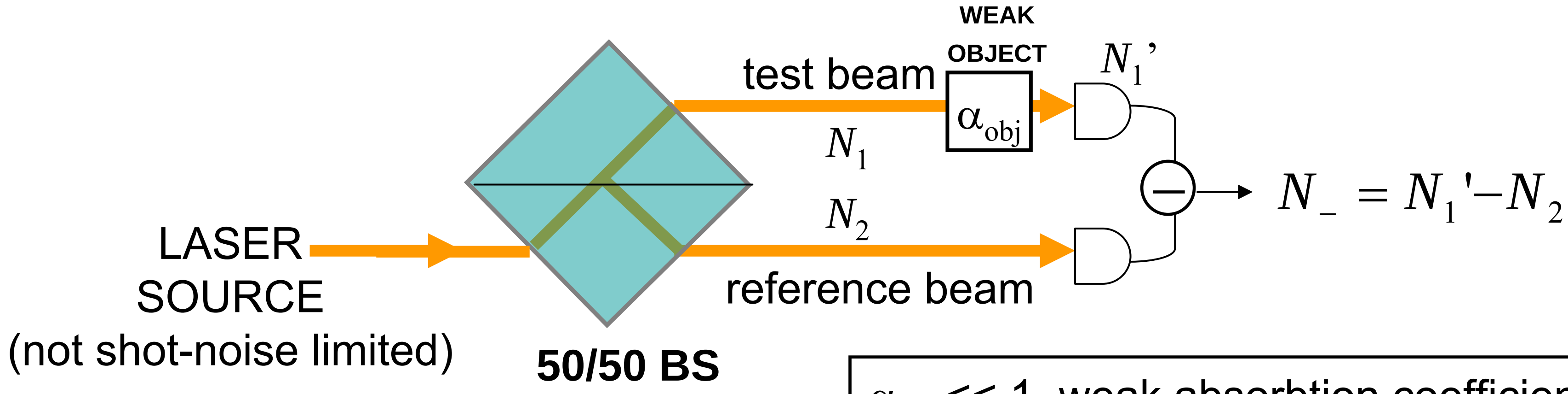
Question: can we also image a weak object using the same technique ?

- Demonstration of quantum correlation in the spatial domain in the high gain regime of spontaneous parametric down-conversion (exp. of Como).

- Improvement of SNR with respect to the SQL, simulation of the experiment with realistic parameters

- concluding remarks

Differential measurement of a weak absorbtion coefficient
(a) classical source



$\alpha_{obj} << 1$ weak absorbtion coefficient

Information on is retrieved from $\langle N_- \rangle = \alpha_{obj} \langle N_2 \rangle$

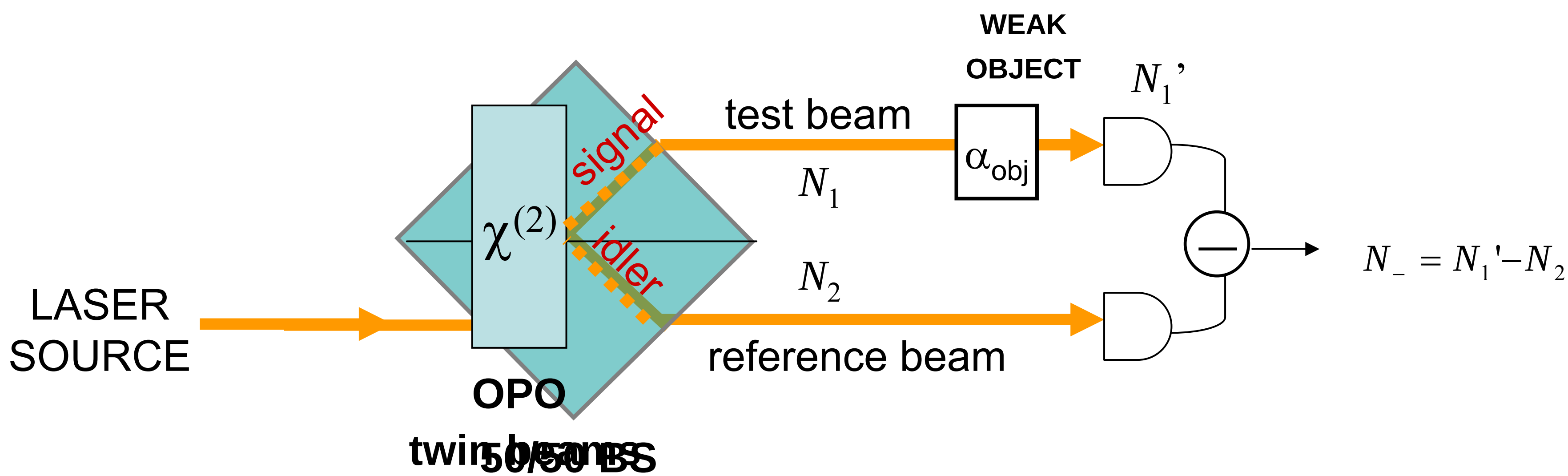
The signal-to-noise ratio depends on the source excess noise $E_{noise} = \dfrac{\langle \delta N_1^2 \rangle - \langle N_1 \rangle}{\langle N_1 \rangle}$

$$SNR_{class} \equiv \frac{\langle N_- \rangle}{\sqrt{\langle \delta N_-^2 \rangle}} = \frac{\alpha_{obj}\sqrt{\langle N_2 \rangle}}{\sqrt{\alpha_{obj}^2 E_{noise} + 2 - \alpha_{obj}}} \approx \alpha_{obj}\sqrt{\frac{\langle N_2 \rangle}{2}} \qquad \text{for } \alpha_{obj} << 1,\ E_{noise} << 1/\alpha_{obj}^2$$

standard quantum limit

Cancellation of excess noise effect - measurement limited only by shot-noise.

Differential measurement of weak absorbtion coefficient
(b) quantum source



Sub-shot noise signal-idler correlation: $\sigma^2_{1-2} = \dfrac{\left\langle (\delta N_2 - \delta N_1)^2 \right\rangle}{\langle N_1 + N_2 \rangle} < 1$

$$SNR_{\text{twin beams}} \equiv \frac{\langle N_- \rangle}{\sqrt{\langle \delta N_-^2 \rangle}} = \frac{\alpha_{\text{obj}} \sqrt{\langle N_2 \rangle}}{\sqrt{\alpha_{\text{obj}}{}^2 E_{noise} + 2\sigma^2_{1-2}\left(1-\alpha_{\text{obj}}\right) + \alpha_{\text{obj}}}} > SNR_{class}$$

Note: for $\sigma_{1\text{-}2}$=1 we recover the SQL result

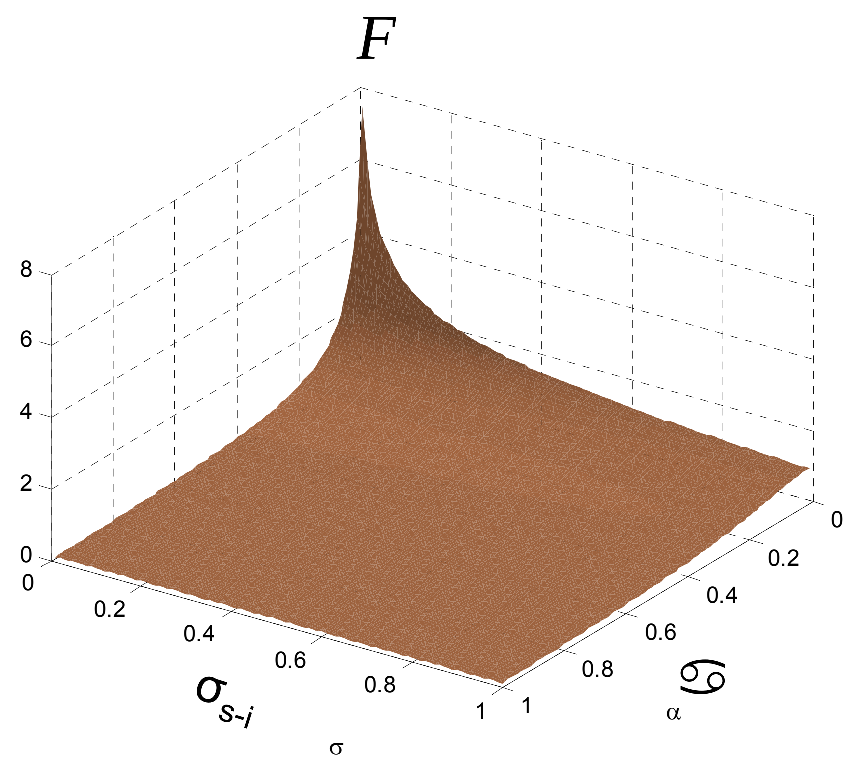
The improvement in SNR using the PDC source (with respect to the SQL) is

$$F = \frac{SNR_{\text{twin beams}}}{SNR_{SQL}} = \sqrt{\frac{2-\alpha_{\text{obj}}}{\alpha_{\text{obj}}{}^{2} E_{noise} + 2\sigma^{2}_{1-2}\left(1-\alpha_{\text{obj}}\right) + \alpha_{\text{obj}}}}$$

In the weak absorbtion limit

$$F = \frac{1}{\sigma_{1-2}} \qquad \text{for } \alpha_{\text{obj}} << 1,\ E_{noise} << 1/\alpha^{2}_{\text{obj}}$$

The signal-idler field need to be well below shot-noise if a substantial SNR improvement is desired



The technique has been used with **single-mode** twin beams generated by an OPO**:**

Souto Ribeiro, Schwob, Maitre, Fabre, Opt. Lett. **22**, 1893 (1997) (2dB noise reduction, for two-photon transition)

Jiangrui Gao et al., Opt.Lett. **23**, 870 (1998) (4dB noise reduction)

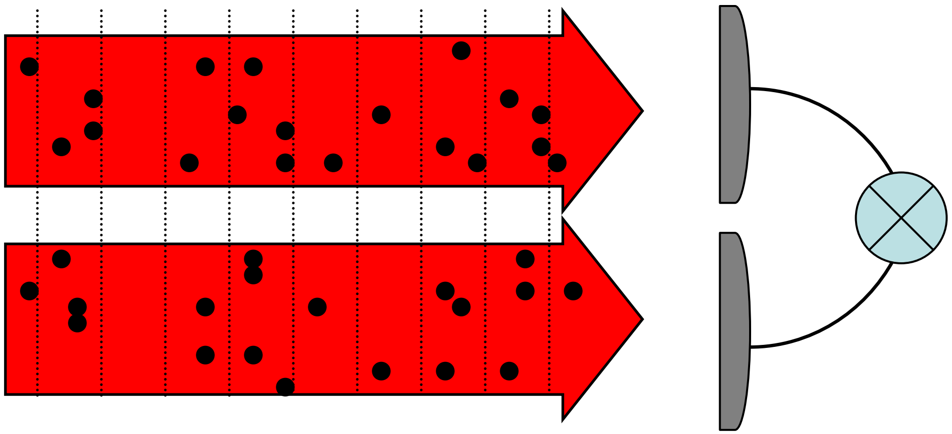
Question: can we use the same technique to detect the spatial distribution of the absorbtion coefficient $\alpha_{\mathrm{obj}}(x)$, i.e the image of a weak object ?

- **No** if the twin beams are single mode.

- **Yes**, if we use multi-mode twin beams, e.g. as those generated by spontaneous parametric down-conversion (PDC) - we need sub-shotnoise correlation between small regions of the signal and idler cross-section, i.e. quantum correlation in the spatial domain.

Ordinary **(single-mode)** twin beams:

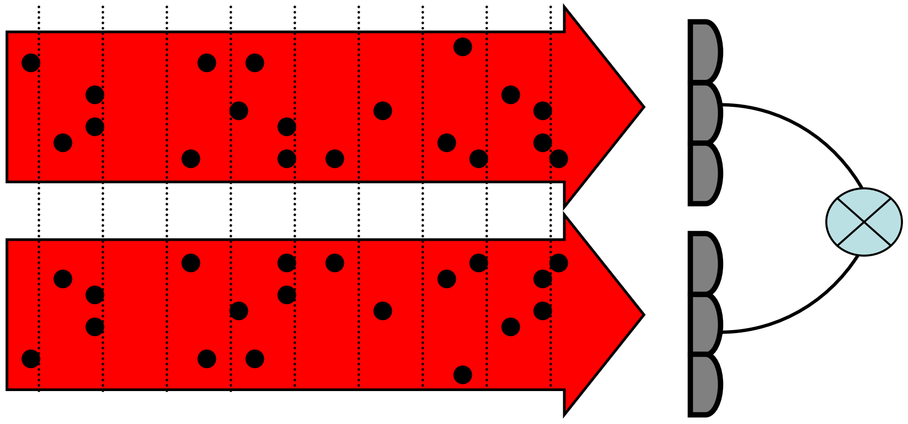
photon number correlated in time, but uncorrelated in space



sub-shot noise correlation
<u>only</u> with bucket detector

Spatially entangled **(multi-mode)** beams:

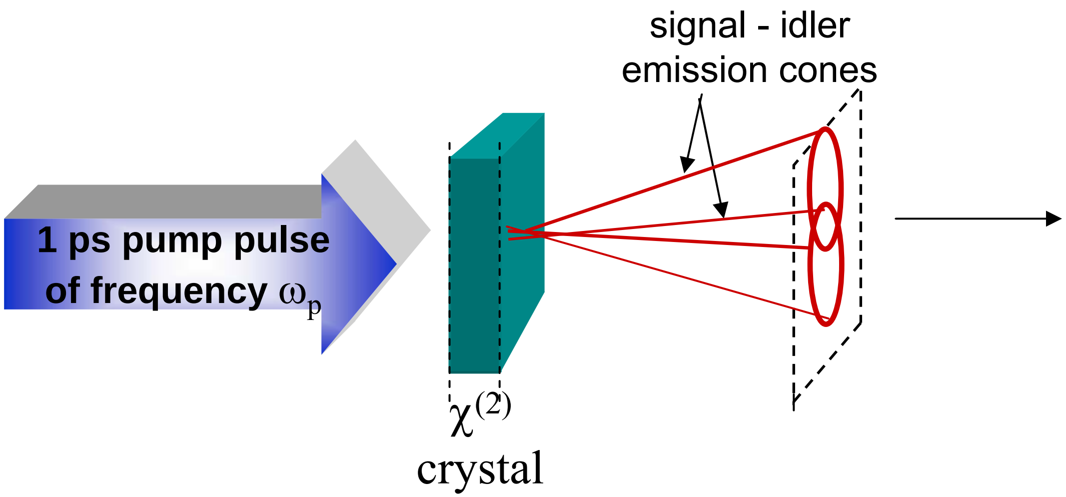
photon numbers correlated in time and in the beam cross sections

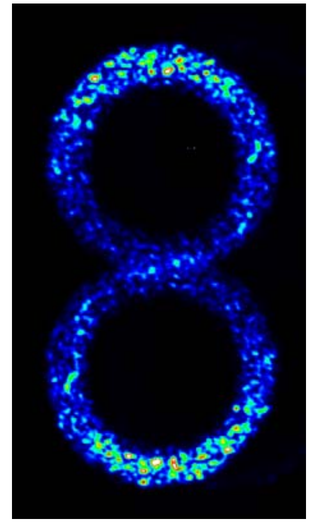


sub-shot noise Correlation with local detection

Process of spontaneous parametric down-conversion In the high gain regime (experiment of Como)

BBO crystal - type II phase-matching

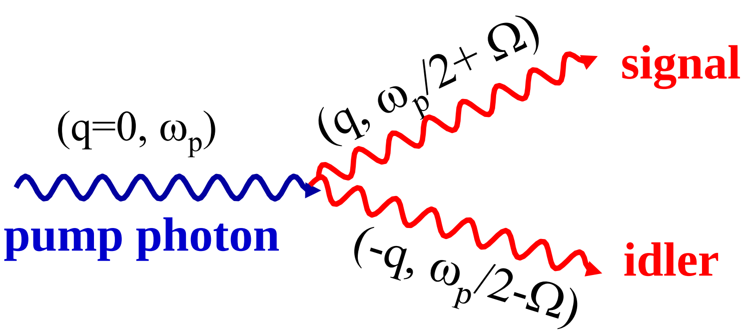




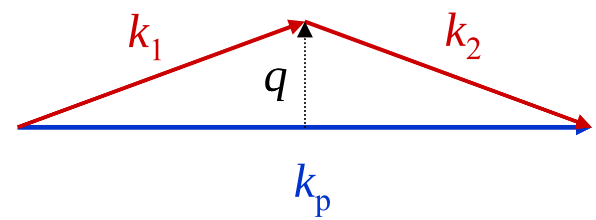
Far field pattern
from a single pump pulse

O. Jedrkiewicz et al.,
PRL **93**, 243601(2004)

Microscopic process: conservation of momentum:

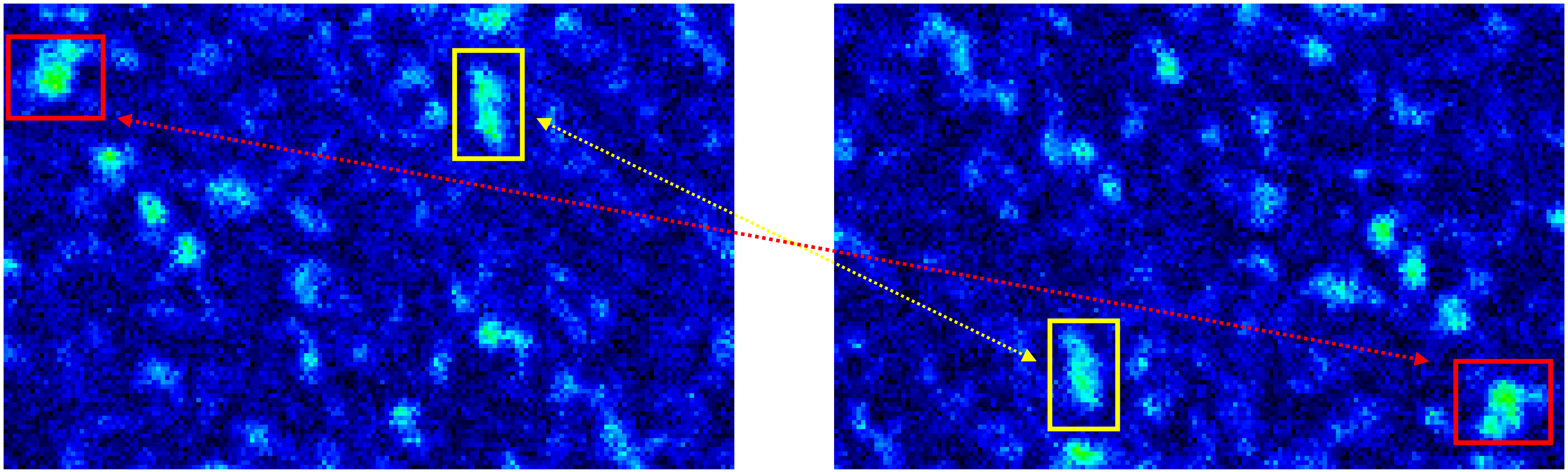


q = photon transverse momentum



Close to degeneracy the signal and idler photons are emitted symmetically

Evidence of twin beam effect in the spatial domain of high gain regime of PDC



zoomed signal **zoomed idler**

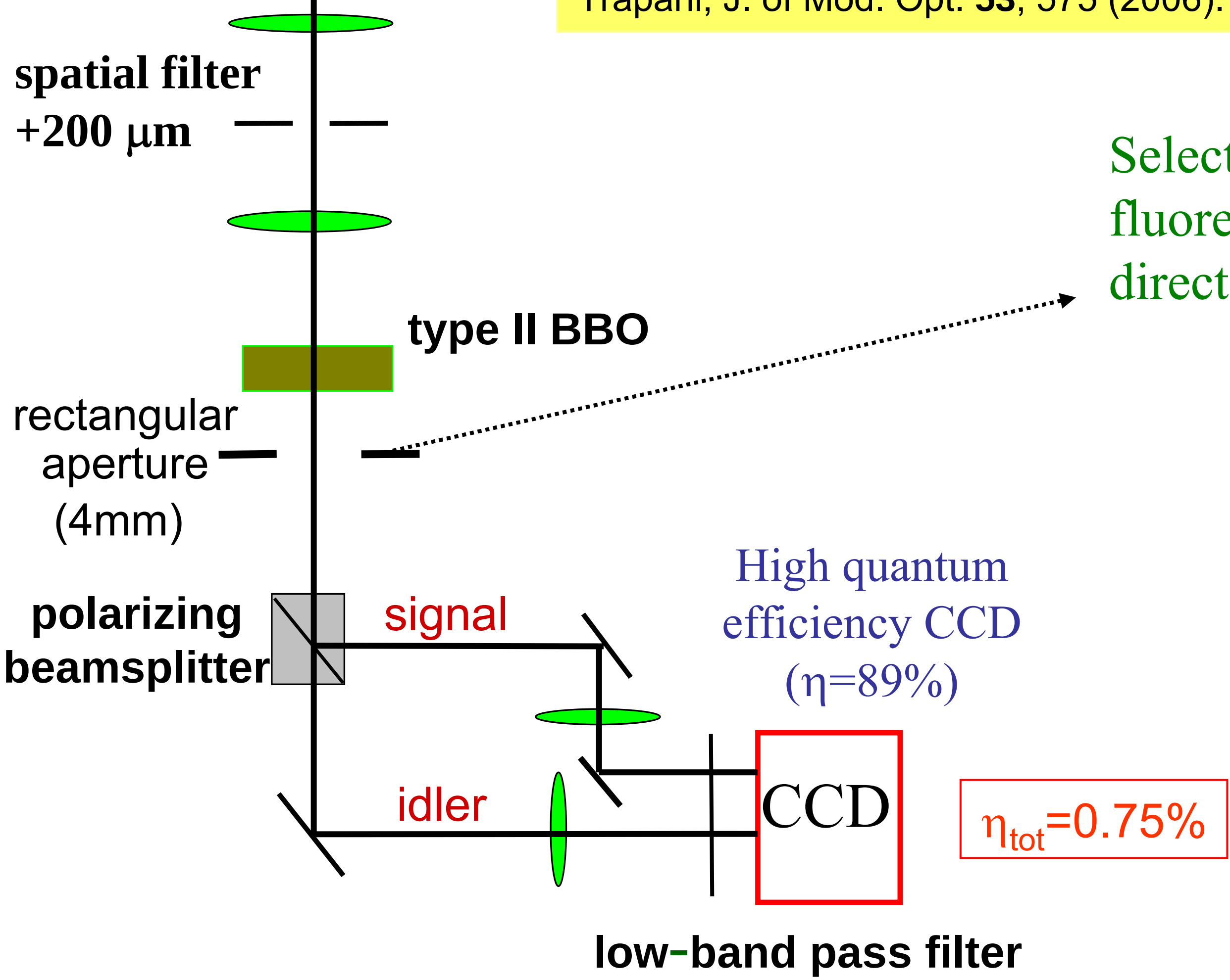
→ evident strong spatial correlation between the two symmetrical images

- pump beam waist ~ 1 mm
- gain 10-1000 photon/pixel

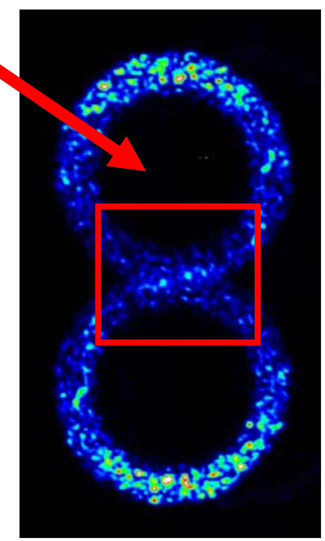
Experimental set-up to measure $\sigma^2_{1-2} = \frac{\langle \delta N_-^2 \rangle}{\langle N_1 + N_2 \rangle}$

PUMP PULSE
λ_p= 352nm, τ_p=1ps

O. Jedrkiewicz, Y..K. Jiang, E. Brambilla, A. Gatti, M. Bache, L. A. Lugiato, and P. Di Trapani, Phys. Rev. Lett. **93**, 243601 (2004).
O. Jedrkiewicz, E. Brambilla, M. Bache, A. Gatti, L. A. Lugiato, and P. Di Trapani, J. of Mod. Opt. **53**, 575 (2006).



Selection of a portion of PDC fluorescence around collinear direction

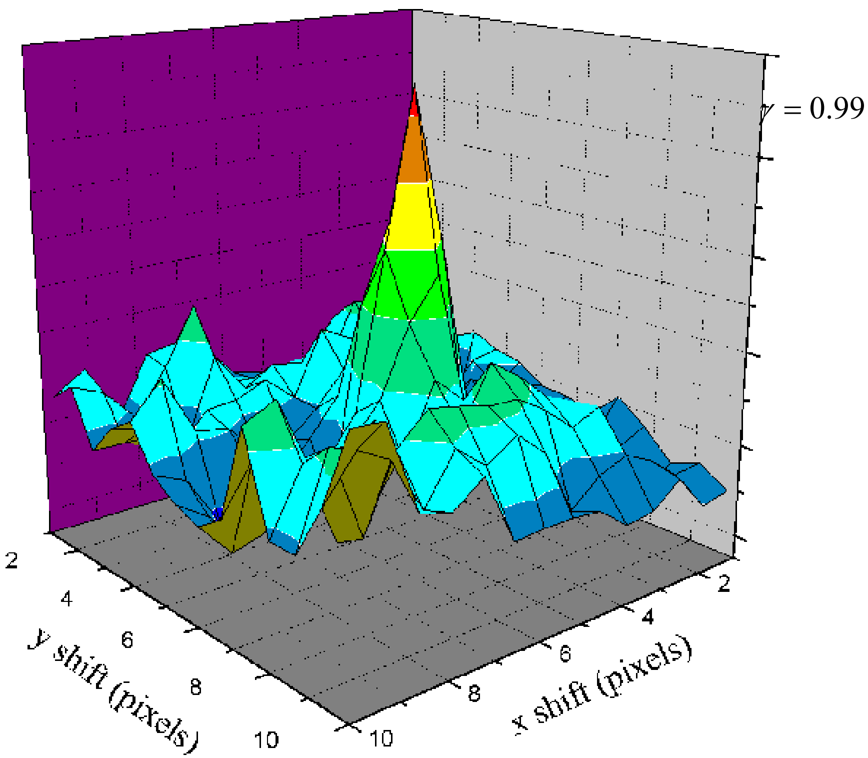


$\lambda_1 \sim \lambda_2 \sim$ 704nm

Local character of the signal-idler correlation

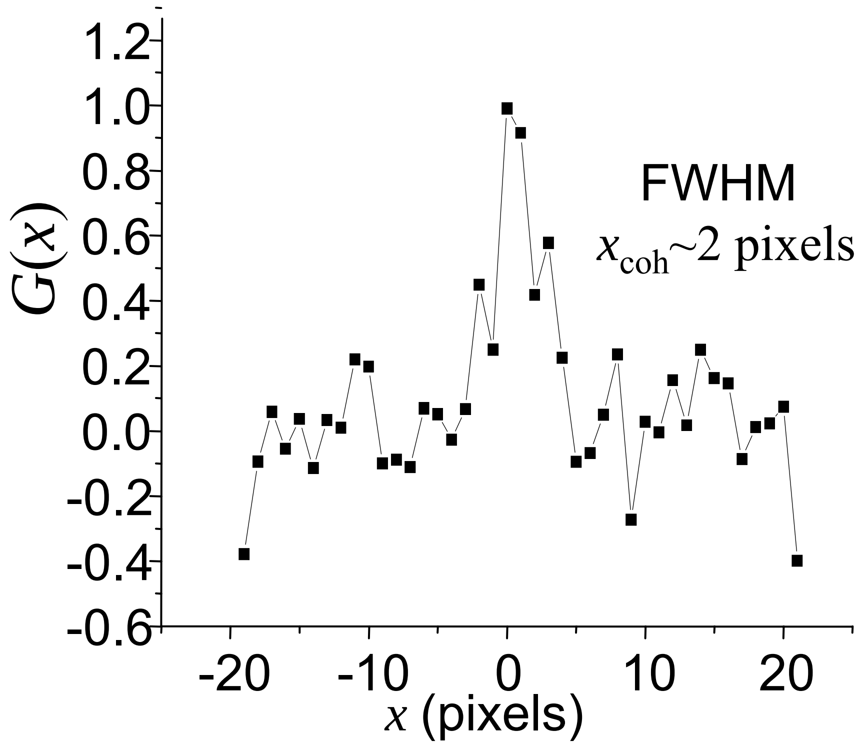
Correlation of signal-idler intensity fluctuations:

$$G(x) = \frac{\langle \delta N_1(x_0)\delta N_2(-x_0 + x\rangle}{\sqrt{\langle \delta N_1^2\rangle\langle \delta N_2^2\rangle}}$$



peak value $\gamma \equiv G(x=0) = 1$ for perfect correlation

$\langle N_1 + N_2 \rangle \approx 8$

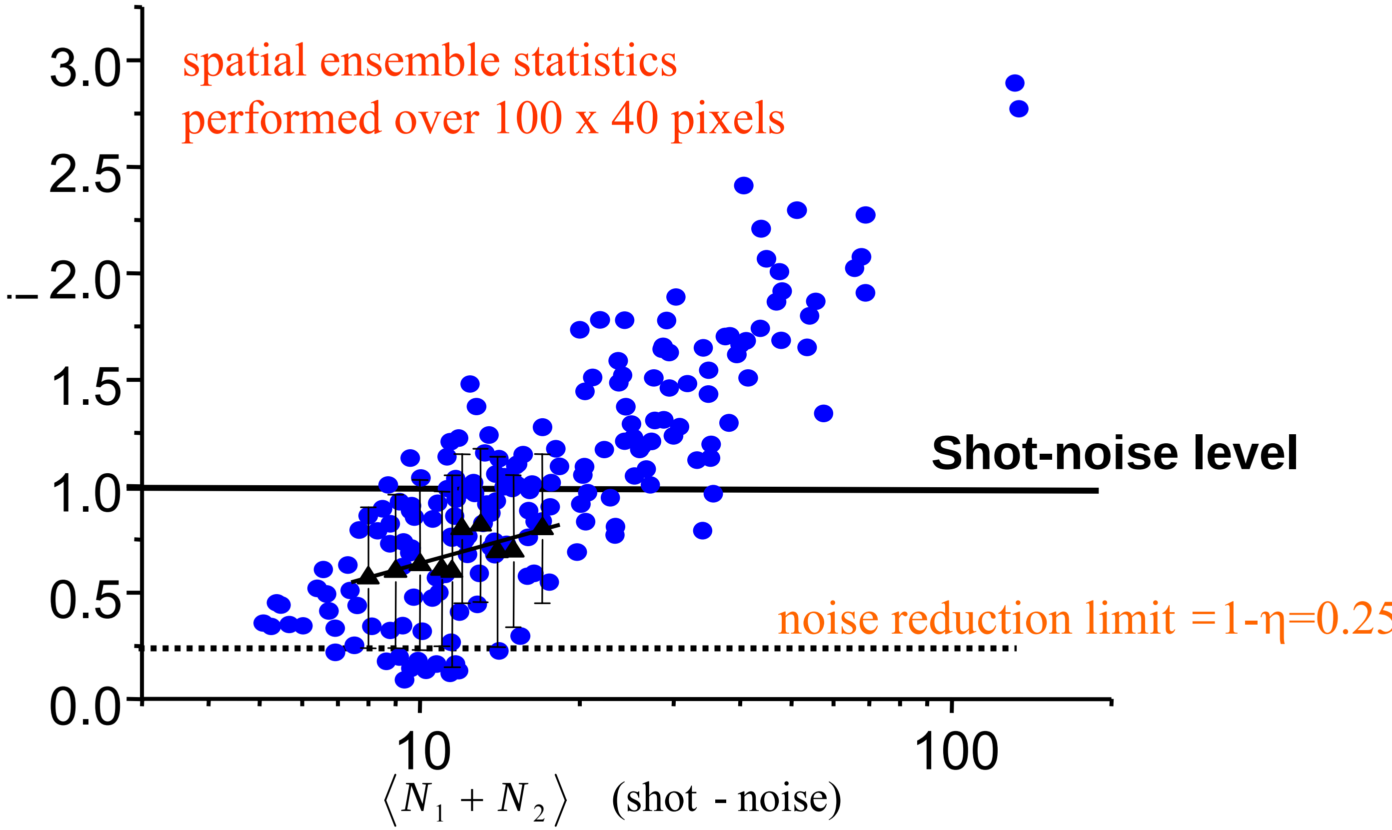


Correlation function profile

measurement of sub-shot noise correlation

$$\sigma^2_{1-2} = \frac{\langle \delta N_-^2 \rangle}{\langle N_1 + N_2 \rangle}$$

Spread of data due to CCD noise (~7ph. pixel).



Sub-shot-noise correlation up to $\langle N_1 + N_2 \rangle$ ~ 20 photons corresponding to 100 ph. per mode (transverse size of the coherence areas in that regime about 2 pixels)

Problem: $\sigma_{s\text{-}i}$ increases with gain and for the weak object detection we have to consider $\langle N_1 \rangle >>$ CCD noise

Deterioration of signal-idler correlation with gain

Important: it is not predicted by the basic PDC theory:

$$\sigma_{1-2} \approx 1 - \eta \qquad \text{if size of detectors} > x_{\mathrm{coh}}$$

unbalance due to different losses: suppose $\eta_1 \neq \eta_2$, with $\eta_1 \approx \eta_2$

$$\sigma_{1-2} = 1 - \overline{\eta} + \frac{(\eta_1 - \eta_2)^2}{2\overline{\eta}} E_{noise}$$

$$E_{noise} \equiv \frac{\langle \delta N_1^2 \rangle - \langle N_1 \rangle}{\langle N_1 \rangle} = \frac{\langle N_1 \rangle}{M} \quad \text{for the thermal beams of PDC}$$

M = degeneracy factor ~ number of modes collected by the detector

~ 2.5 in the experiment of Como (few temporal mode since $\tau_{\mathrm{coh}} \sim \tau_{\mathrm{pump}} \sim$ 1ps)

$(\eta_1 - \eta_2)^2 \ll 1$ in the experiment -> small contribution of the term αE_{noise}

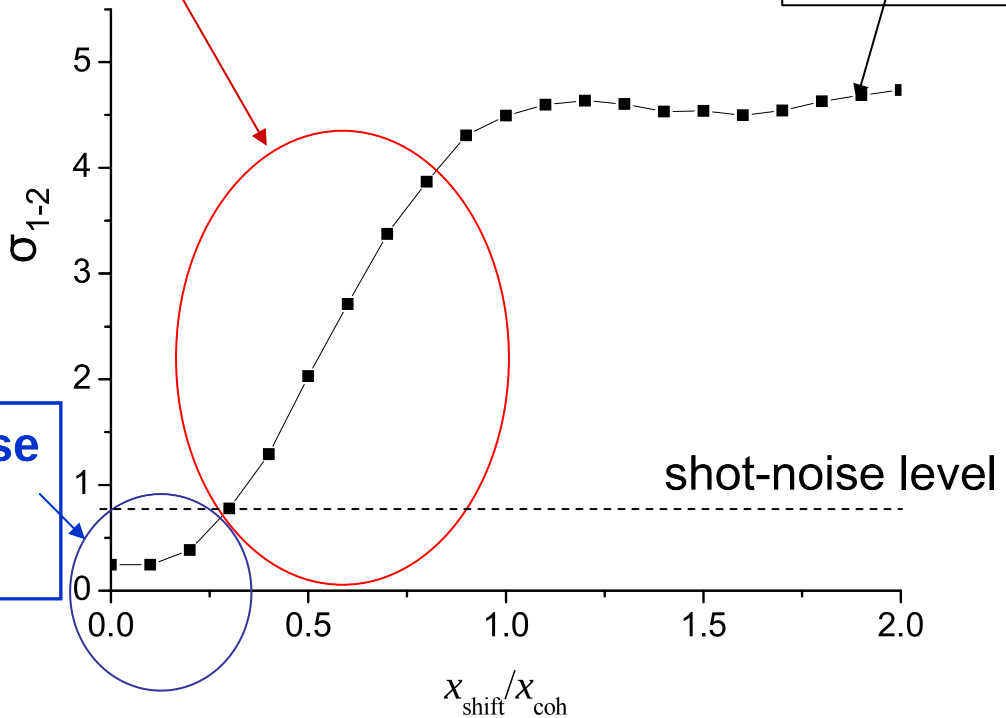
Inacuracy in the determination of the symmetry center

Unbalance due to an error x_{shift} in determining the center of symmetry of the far field pattern ($x_{\text{shift}} \sim$ size of CCD pixel = 20μm, $x_{\text{coh}} \sim$ 40 μm)

$$\sigma^2_{1-2} \approx \frac{x_{\text{shift}}}{x_{\text{coh}}} E_{noise} \qquad \text{for } x_{\text{shift}} < x_{\text{coh}}$$

$$\sigma^2_{1-2} = 1 + \eta\, E_{noise} \qquad \text{for } x_{shift} > x_{coh}$$

(uncorrelated pixel pairs)



sub-shot noise Correlation
$\sigma^2_{1-2} = 1 - \eta$

numerical simulation

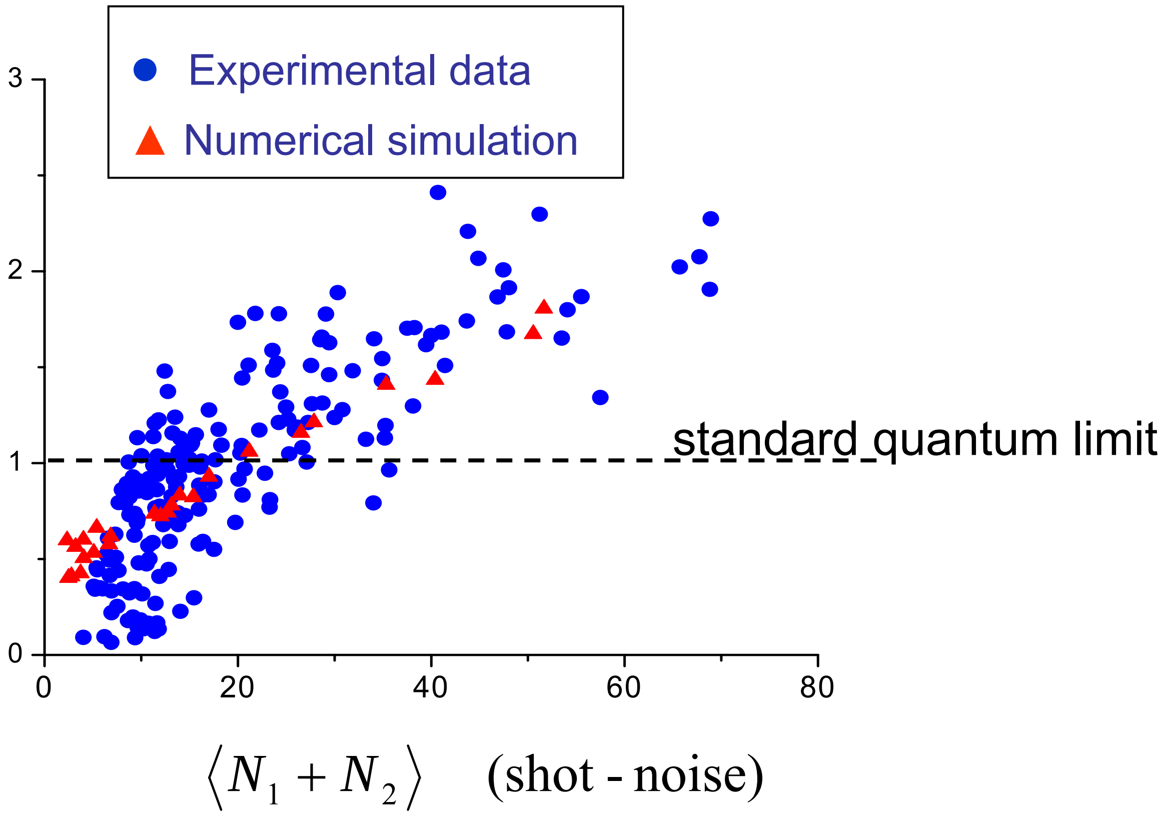
sub-shot noise correlation are extremely sensitive to x_{shift} when E_{noise} is large

Effect of the inacuracy x_{shift} in the center of symmetry

Comparison between experimental data and numerical simulation

(x_{shift}=3μm in the numerical)

$$\sigma_{1-2}^{2} = \frac{\left\langle \delta N_{-}^{2} \right\rangle}{\left\langle N_1 + N_2 \right\rangle}$$



Transition from quantum to classical correlation:

$$\sigma^2_{1-2} > 1 \qquad \text{for} \quad x_{\text{shift}} \gtrsim \frac{x_{\text{coh}}}{1+\eta\, E_{noise}}$$

<u>more tolerance</u> on the error in the symmetry center x_{shift} if E_{noise} is small

$$E_{noise} = \frac{\langle N_1 \rangle}{M} \qquad \text{where } M \propto \frac{\tau_{\text{pump}}}{\tau_{\text{coh}}} \qquad \text{degeneracy factor}$$

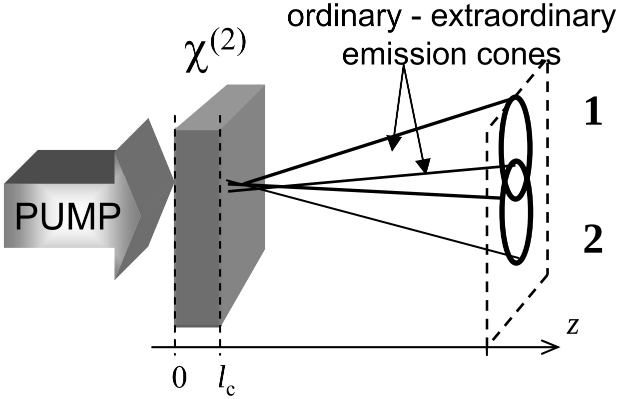
We need $\langle N_1 \rangle >>$ CCD noise in the weak absorbtion experiment,

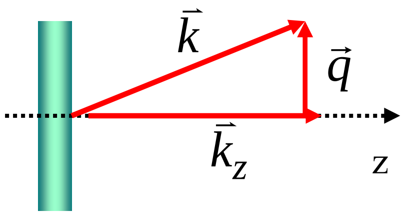
Solution: we can make M large by increasing the pump pulse time

$$\langle \delta N_1^2 \rangle = \langle N_1 \rangle + \frac{\langle N_1 \rangle^2}{M} \rightarrow \langle N_1 \rangle \text{ for } M >> 1$$

signal and idler statistics become poissonian

Numerical modelling of parametric down-conversion (any gain regime)





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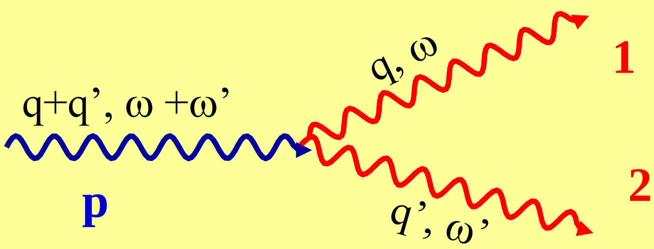
p pump
1 signal
2 idler

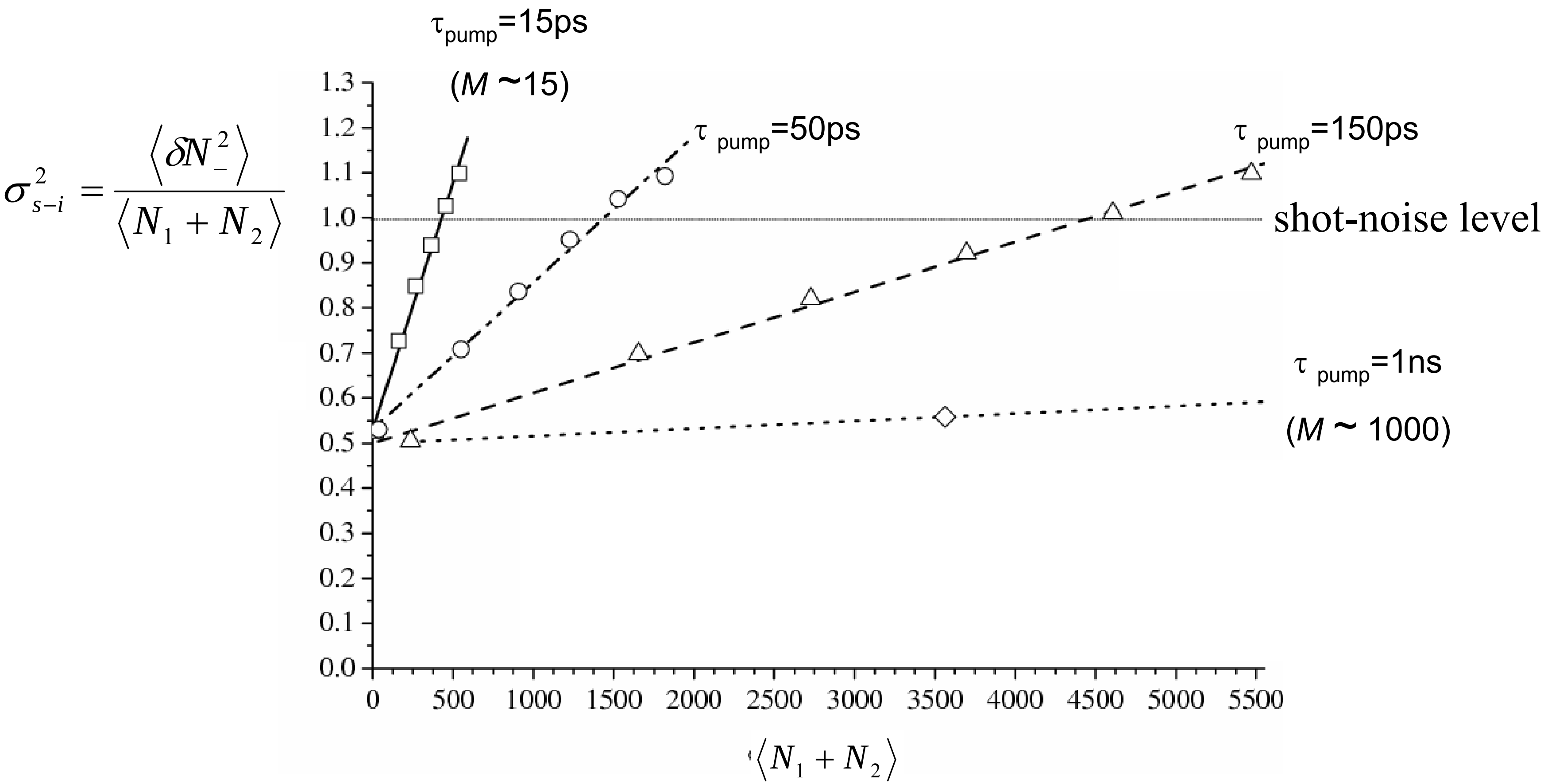
$A_p(q,\omega)$ spatio-temporal Fourier transform of a classical, undepleted, pump (parametric regime)



$$\Delta_{12}(\vec{q},\vec{q}';\omega,\omega') = k_{1z}(\vec{q},\omega) + k_{2z}(\vec{q}',\omega') - k_{0z}(\vec{q}+\vec{q}',\omega+\omega')$$

Phase mismatch function: includes the effects of temporal and spatial walk-off, diffraction, dispersion

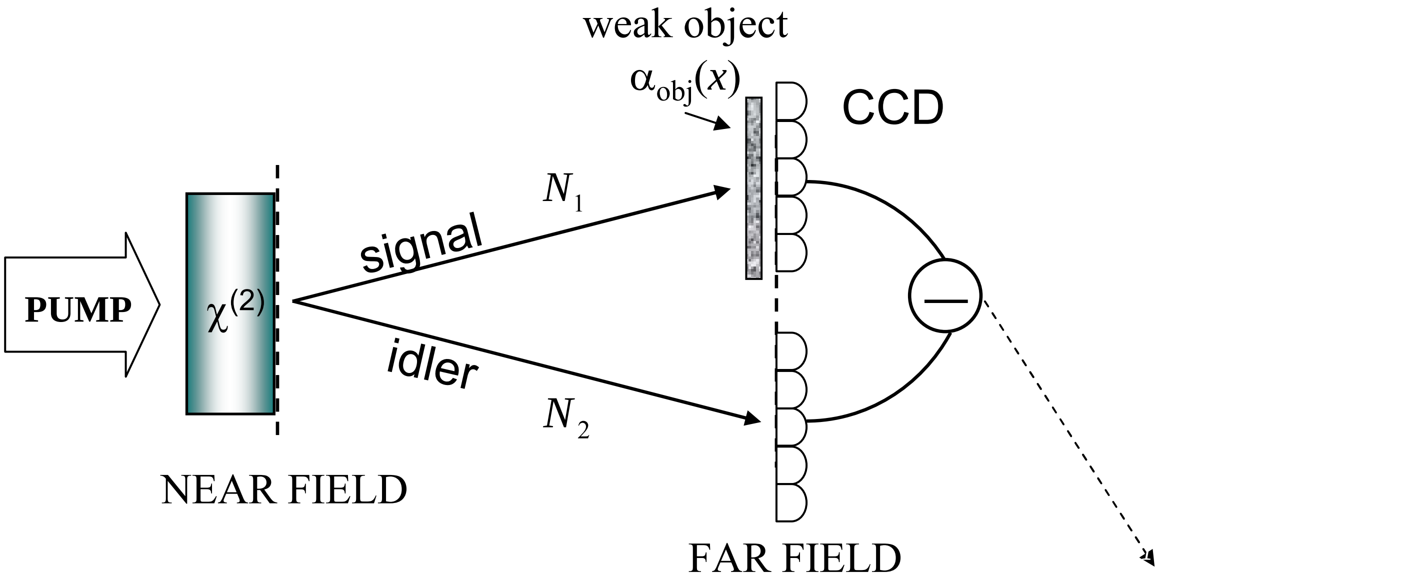
Numerical simulation with diferent values of the pump pulse time assuming x_{shift}=6µm



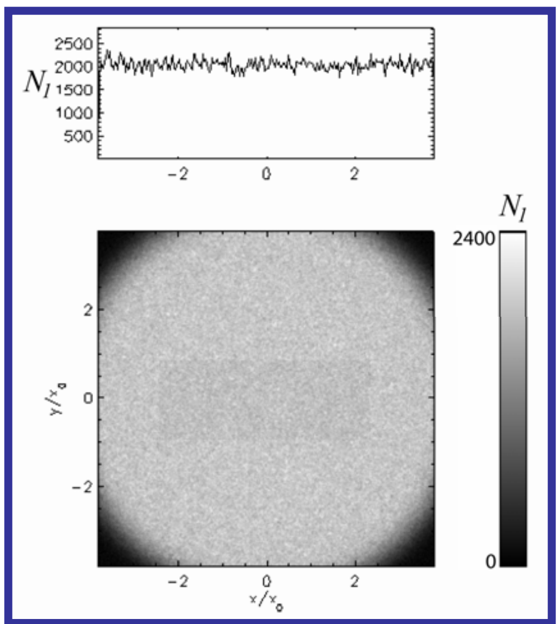
By increasing τ_{pump} α M, the slope of σ_{s-i} decreases

→ sub-shot noise correlation also for large photon number

Numerical simulation for the detection of a weak object $\alpha_{obj}(x)$



$N_1(x)$



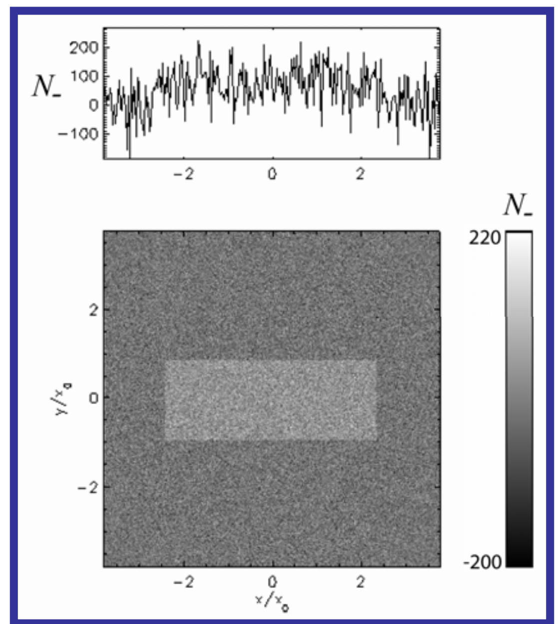
Simulation

τ_{pump}=5ns

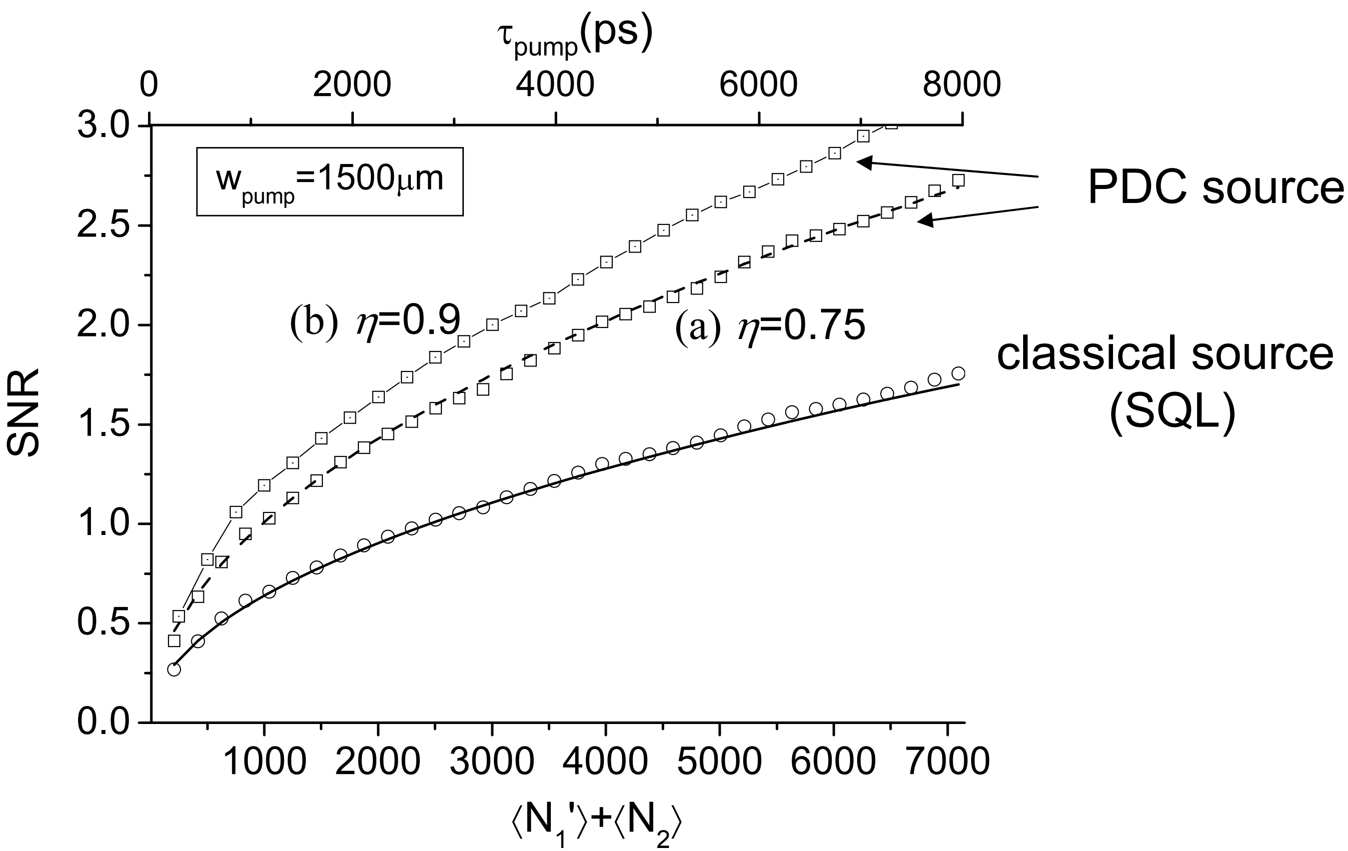
w_{pump}=1500 μm

α_{obj}=0.04

$N_-(x) = N_2'(x) - N_1(-x)$



SNR as a function of the pump pulse time τ_{pump} (the gain/mode is kept constant).



$$\text{(a)}\quad F_{\eta=0.75} = \frac{SNR_{PDC}}{SNR_{SQL}} = 1.55, \quad \sigma^2_{1-2} = 0.4 \qquad\qquad \text{(b)}\quad F_{\eta=0.9} = 1.82, \quad \sigma^2_{1-2} = 0.3$$

CONCLUSIONS

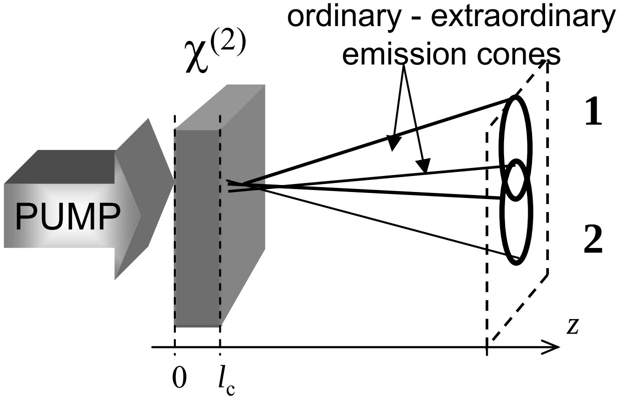
-The detection of weak images beating the SQL can be achieved using multi-mode twin beams generated by spontaneous parametric down-conversion.

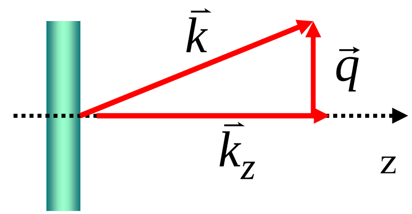
A substantial gain in SNR can be obtained if:

1) a careful balance of the test and the reference beam is achieved

2) The excess noise in the two beam is as small as possible (e.g. by considering long pump pulses)

Numerical modelling of parametric down-conversion (any gain regime)



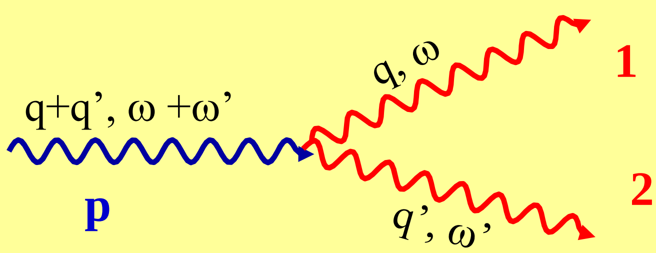


Starting point:propagation equation along the crystal for signal and idler envelope operators:

$$\frac{d}{dz}\hat{a}_1(\vec{q},\omega)=\chi\int d\vec{q}'\int d\omega' A_p(\vec{q}+\vec{q}',\omega+\omega')\hat{a}_2^+(\vec{q}',\omega')e^{-i\Delta_{12}(\vec{q},\vec{q}';\omega,\omega')z}$$

p pump
1 signal
2 idler

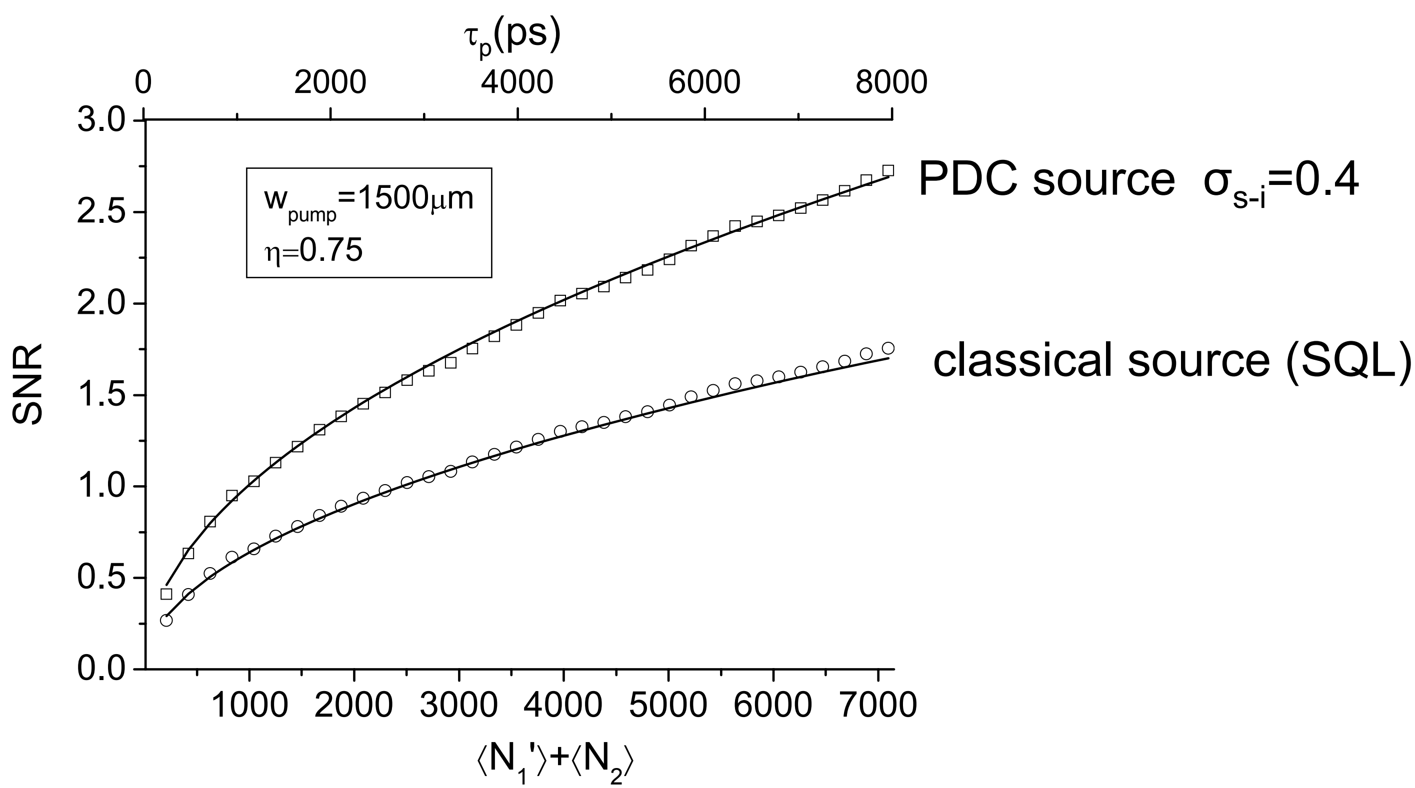
$A_p(q,\omega)$ spatio-temporal Fourier transform of a classical, undepleted, pump (parametric regime)



$$\Delta_{12}(\vec{q},\vec{q}';\omega,\omega')=k_{1z}(\vec{q},\omega)+k_{2z}(\vec{q}',\omega')-k_{0z}(\vec{q}+\vec{q}',\omega+\omega')$$

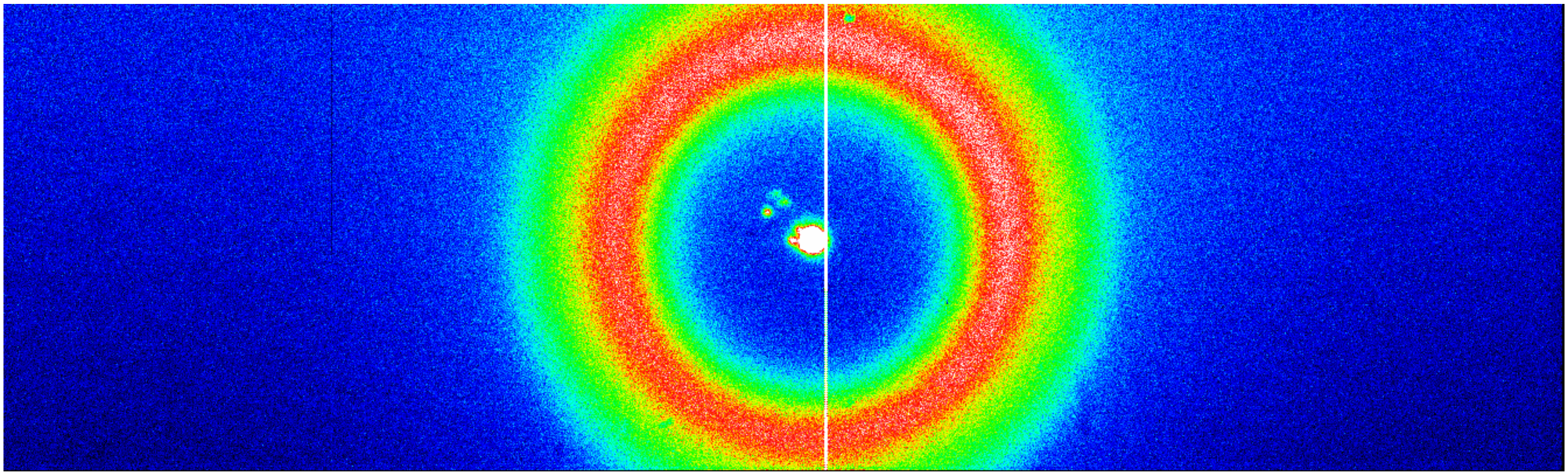
Phase mismatch function: includes the effects of temporal and spatial walk-off, diffraction, dispersion

SNR as a function of the pump pulse time ♦$_{pump}$ (the gain/mode is kept constant).



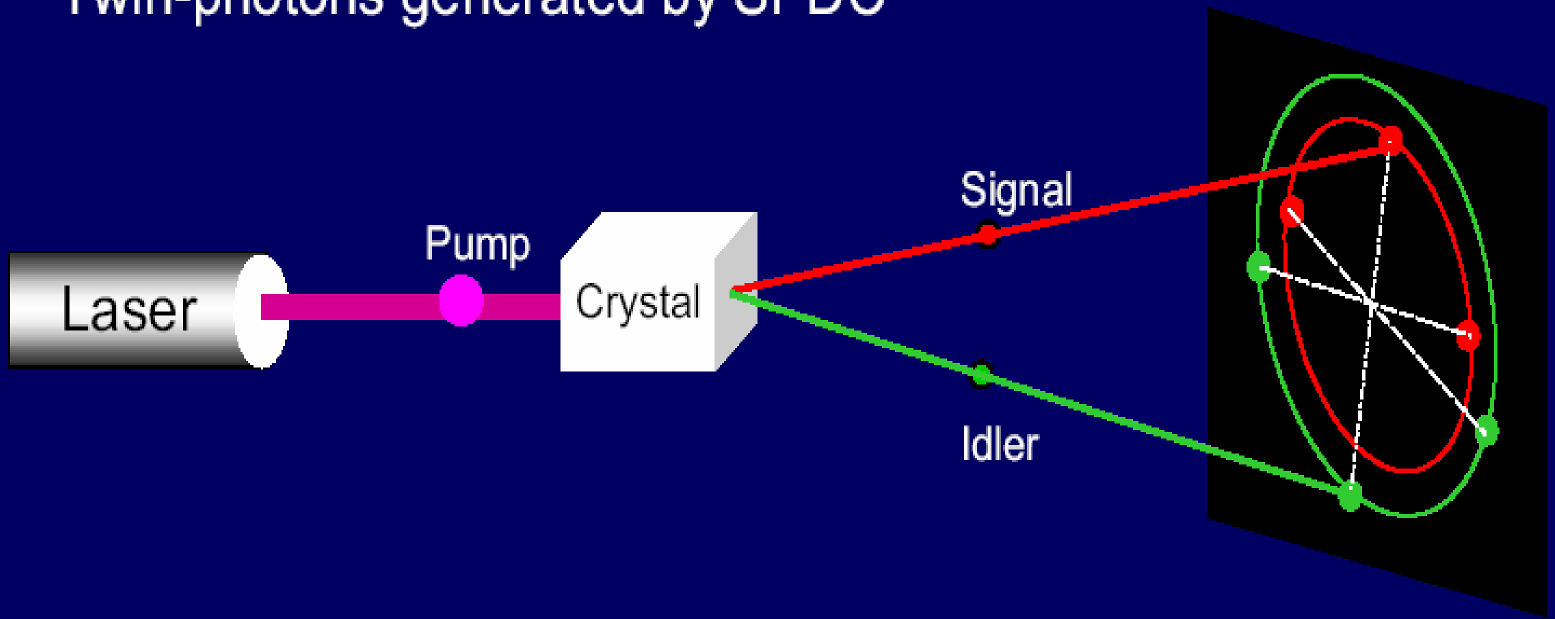
$$F = \frac{SNR_{PDC}}{SNR_{SQL}} = 1.55$$

BBO type I
Shutter time 40ms
Pumped at 400nm by a continuous diode laser



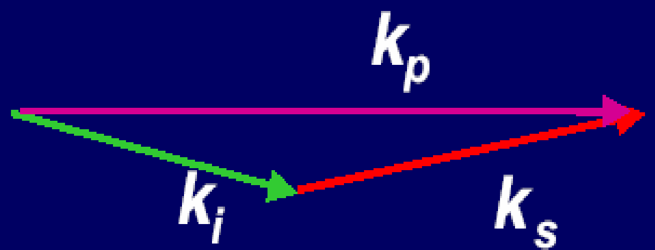
Entangled-Photon Imaging

Twin-photons generated by SPDC

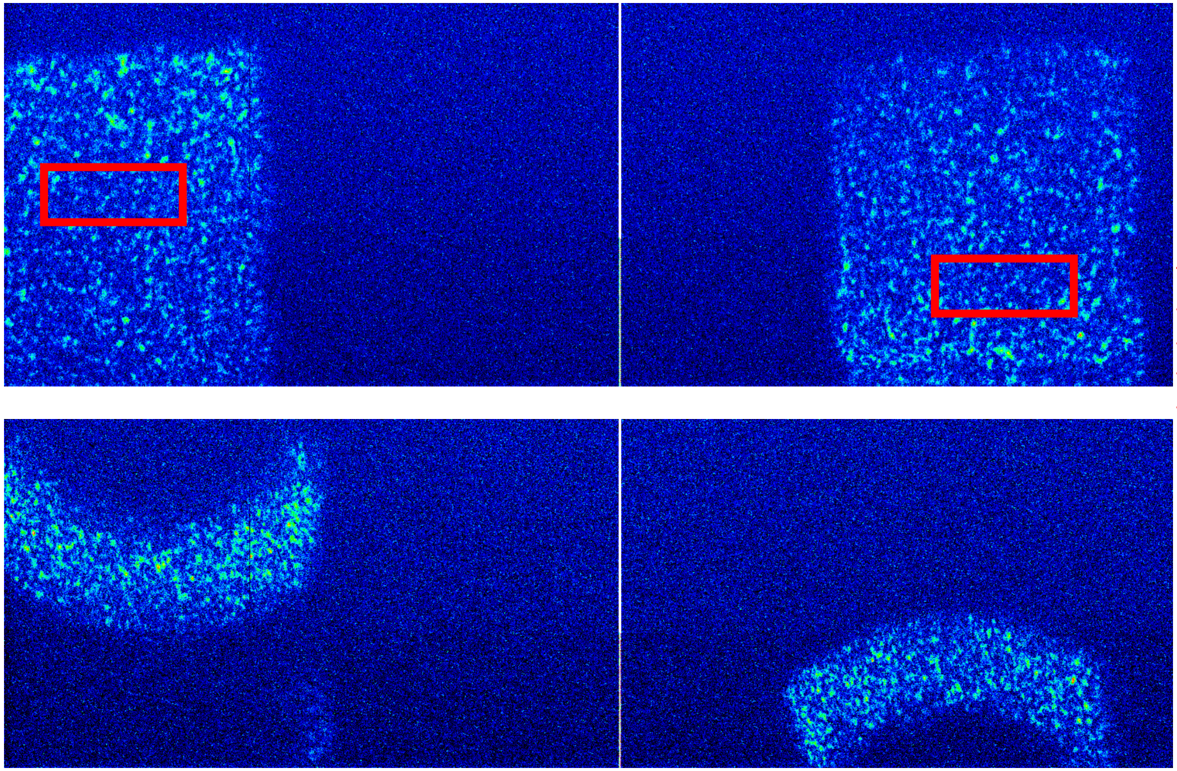


Conservation of energy $\omega_p = \omega_s + \omega_i$

Conservation of momentum $\mathbf{k}_p = \mathbf{k}_s + \mathbf{k}_i$



Far field image of the selected portion of PDC fluorescence



No temporal statistics is made.

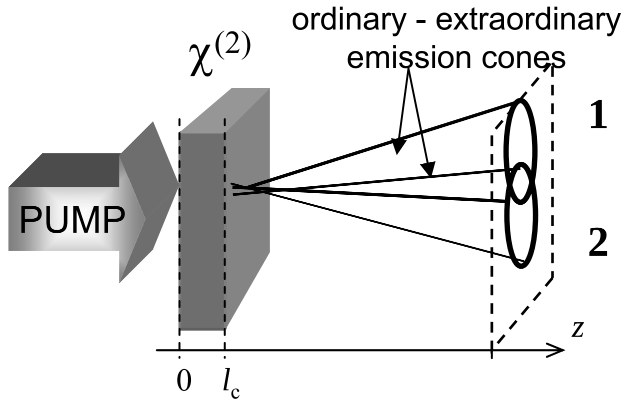
Statistical ensemble constituted only by *spatial replicas*.

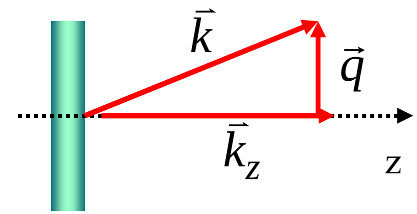
Averages are only *SPATIAL* performed inside box (4000 pix) for *each single laser pulse*

Inserting the 10nm IF allows to locate on the CCD the collinear degeneracy point

The generated pairs of signal and idler phase-conjugate modes propagate at symmetrical angles with respect to the pump direction in order to fulfill the phase-matching constraints, and each pair of symmetrical spots characterizing the far field represents a *spatial replica*.

Modelling parametric down-conversion (any gain regime)





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p pump
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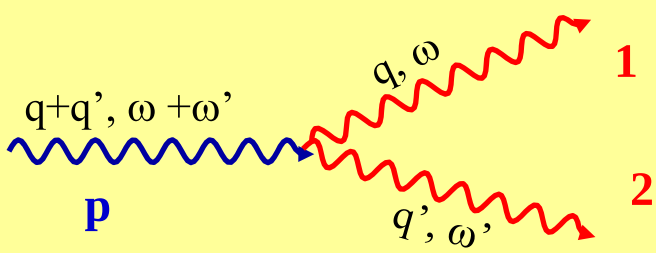
$A_p(q,\omega)$ spatio-temporal Fourier transform of a classical, undepleted, pump (parametric regime)



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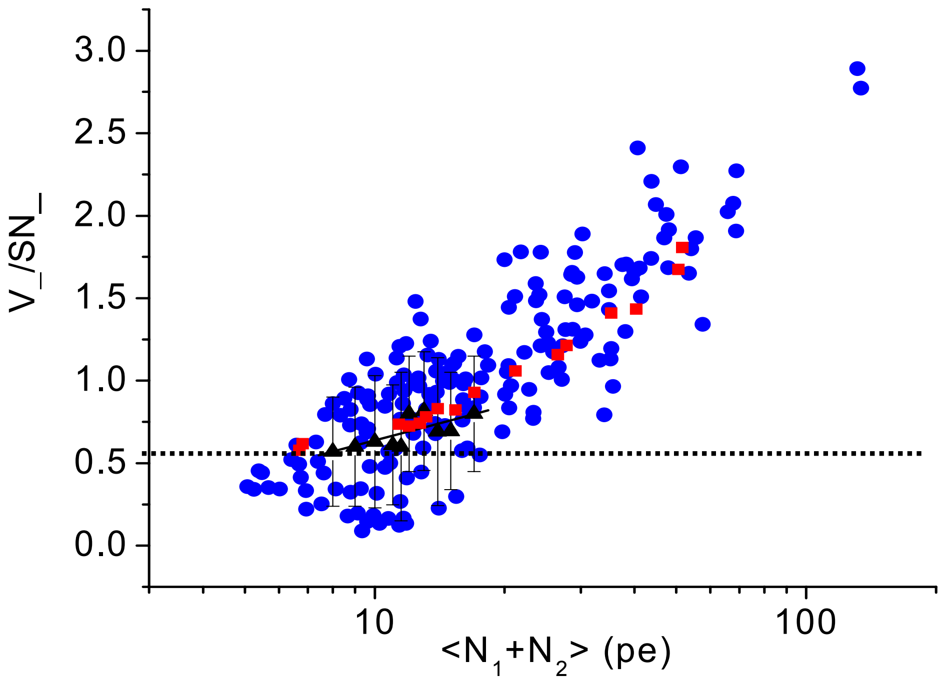
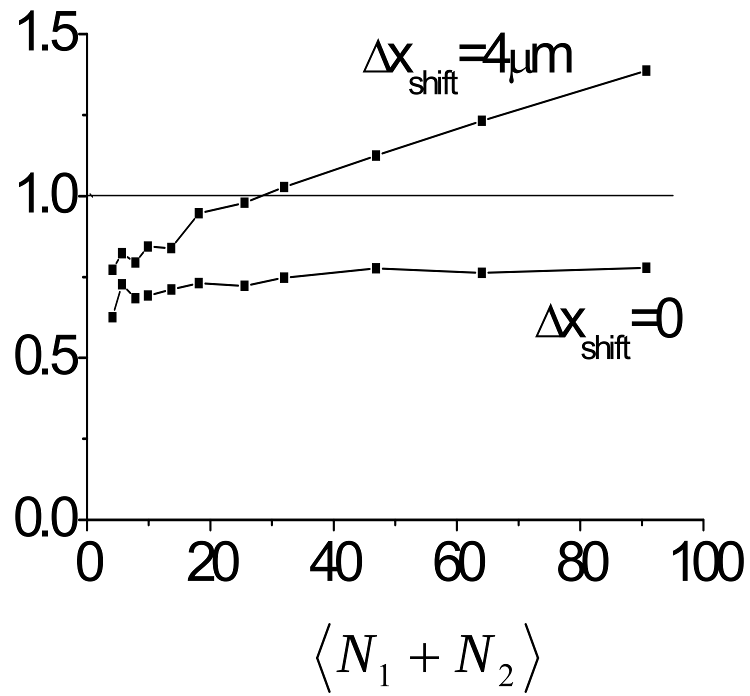
Phase mismatch function: includes the effects of temporal and spatial walk-off, diffraction, dispersion

Effect of shift on correlation center

Comparison between experimental and numerical results

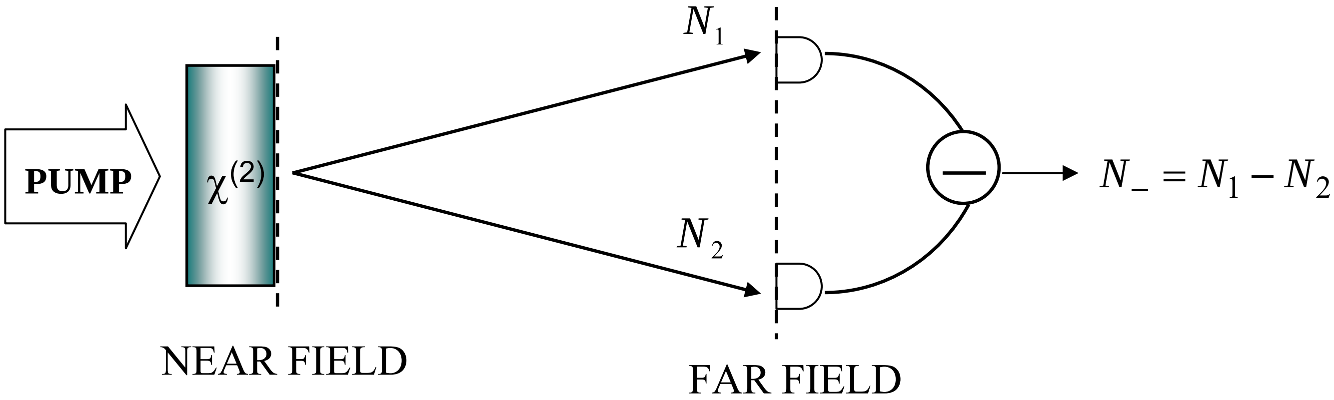
No scattering contribution
No background noise correction
taken into account in numerics





Shot-noise limit

Spatial entanglement in the far field of PDC (high gain regime)



For symmetry reason we have

$$\langle N_1 \rangle = \langle N_2 \rangle \quad \text{and} \quad \langle \delta N_1^2 \rangle = \langle \delta N_2^2 \rangle$$

In an ideal experiment (plane-wave pump, no detection losses), the theory predicts

$$\langle \delta N_-^2 \rangle = \langle (\delta N_2 - \delta N_2)^2 \rangle = 0$$

In a real experiment

$$\langle \delta N_-^2 \rangle < \langle N_1 + N_2 \rangle \equiv SN\_$$

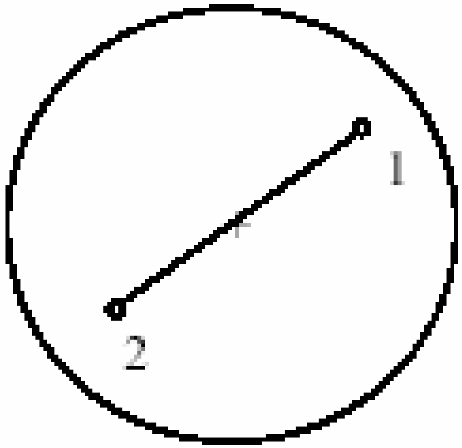
shot-noise level

more precisely $$\frac{\langle \delta N_-^2 \rangle}{SN\_} = 1 - \eta \quad \text{for} \quad \mathrm{S}_{\mathrm{det}} > \mathrm{S}_{\mathrm{diff}}$$

η = detection quantum efficiency

SPATIAL ENTANGLEMENT

FAR FIELD



2 SYMMETRICAL DETECTOR
(PIXELS OF A CCD CAMERA

N_1 = number of photons detected in 1

N_2 = number of photons detected in 2

Fluctuations $\delta N_i = N_i - \langle N_i \rangle, \qquad i = 1,2$

For symmetry reasons $\langle N_1 \rangle = \langle N_2 \rangle, \quad \langle \delta N_1^2 \rangle = \langle \delta N_2^2 \rangle$

Let us consider the difference $N_- = N_1 - N_2$

In the limit of <u>plane-wave pump</u> one has $\langle \delta N_-^2 \rangle = 0 \quad \Rightarrow \quad N_1 = N_2$, i.e, by measuring N_1
one immediately can <u>infer</u> the value of N_2 (entanglement).

Brambilla, Gatti, Lugiato, Kolobov, Eur. Phys. J. D **15**, 127 (2001)

NOTE: This result is due to the perfect correlation between the number of photons in 1 and 2.

$$\langle \delta N_-^2 \rangle = \langle \delta (N_1 - N_2)^2 \rangle = \langle \delta N_1^2 \rangle + \langle \delta N_2^2 \rangle - 2\langle \delta N_1 \delta N_2 \rangle$$

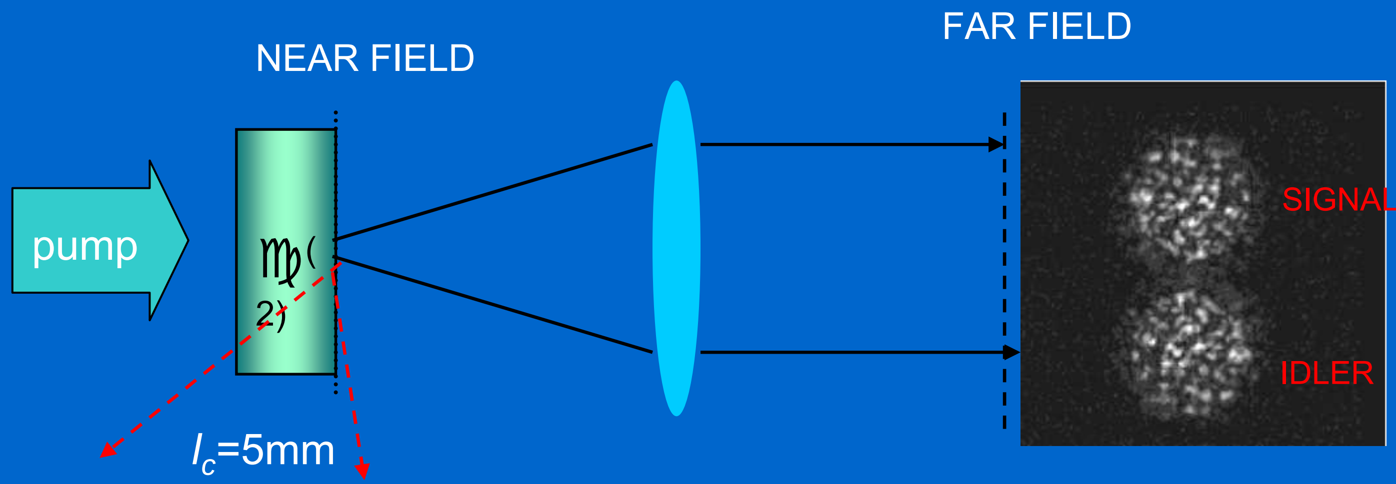
Since $\langle \delta N_1^2 \rangle = \langle \delta N_2^2 \rangle$ and $\langle \delta N_-^2 \rangle = 0$ one has

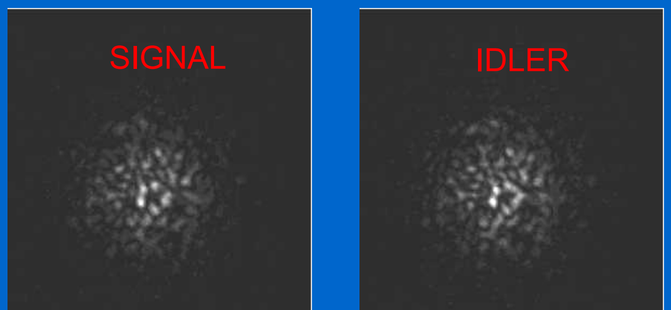
$$\boxed{C = \frac{\langle \delta N_1 \delta N_2 \rangle}{\sqrt{\langle \delta N_1^2 \rangle}\sqrt{\langle \delta N_2^2 \rangle}} = 1}$$

Perfect correlation

For the realistic case of a Gaussian pump, if the pixels are larger than the "spot size" in the far field, THE FLUCTUATIONS OF N_- LIE WELL BELOW THE SHOT NOISE LEVEL.

$$\langle \delta N_-^2 \rangle < \langle N_1 \rangle + \langle N_2 \rangle$$





•Finite crystal length--> uncertainty in the twin photon position due to diffraction spread

- $l_{coh} \sim 1/q_0 \sim \sqrt{\lambda l_c/\pi}$ uncertainty in the position of photon 1 from a measurement of the position of photon 2

•Perfect spatial intensity correlation for detection areas larger than l_{coh}^2

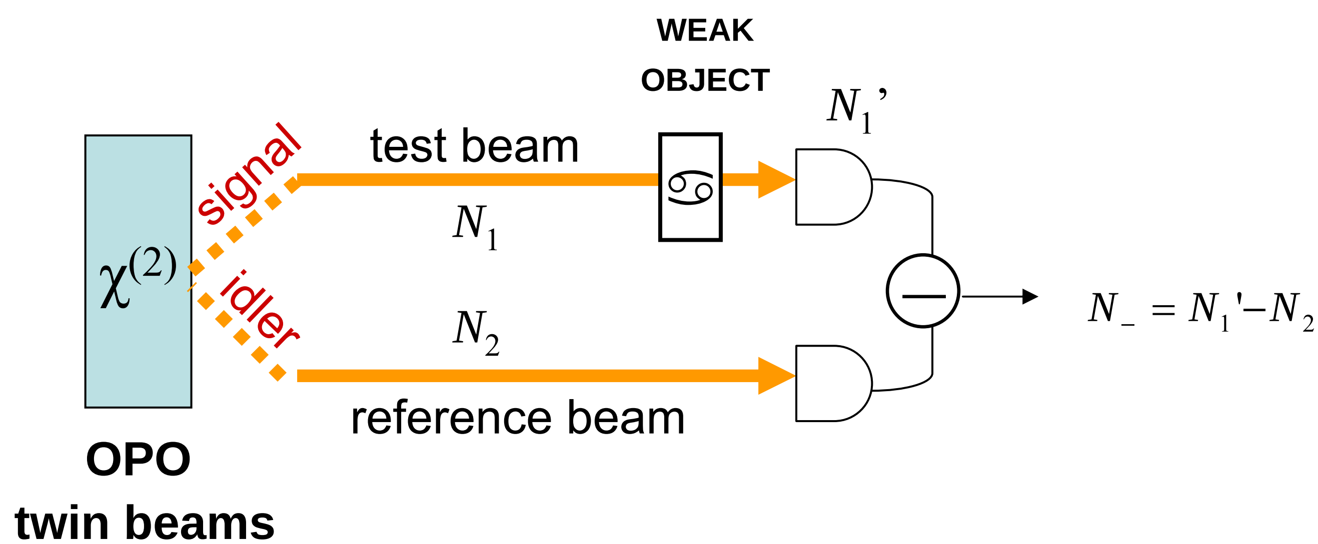
•Finite size of the pump waist w_P --> uncertainty in the propagation directions of twin photons

- $\delta q \approx 1/w_P$ uncertainty in the transverse momentum of photon 1 from a measurement of the momentum of photon 2

•Perfect intensity correlation recovered for detection areas larger than $\frac{(\lambda f)^2}{\pi w_p^2}$

Brambilla, Gatti, Bache and Lugiato, Phys. Rev. A 69, 023802 (2004)

Differential measurement of weak absorbtion coefficient
(b) quantum source



Sub-shot noise signal-idler correlation: $\sigma_{1-2} = \dfrac{\left\langle \left(\delta N_2 - \delta N_1\right)^2 \right\rangle}{\left\langle N_1 + N_2 \right\rangle} < 1$

$$SNR_{PDC} \equiv \frac{\left\langle N_- \right\rangle}{\sqrt{\left\langle \delta N_-^2 \right\rangle}} = \frac{\alpha_{\text{obj}} \sqrt{\left\langle N_2 \right\rangle}}{\sqrt{\alpha_{\text{obj}}{}^2 E_{noise} + 2\sigma_{1-2}\left(1 - \alpha_{\text{obj}}\right) + \alpha_{\text{obj}}}} > SNR_{class}$$

Note: for $\sigma_{1-2}=1$ we recover the SQL result