Linear entropy as entanglement fermionic measure for an electronic scattering in 2D semiconductor system

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Bertinoro, 5 December, QMFPA

Entanglement in systems of identical particles

- *"Accessible Entanglement"*

Wiseman and Vaccaro PRL 91, 097902 (2003).

- *Use of density matrix mode-occupation represention*

Zanardi PRA 65,042101(2002).

- *Slater rank of the state*

Schliemann et al. PRA, 64, 022303 (2001).

Linear entropy

- *Slater rank*
- *Linear entropy of reduced statistical operators*

The aim of this talk is to show that the linear entropy is an efficient entanglement measure for a two-fermion system and its evalutation in binary collitions is easier than the ones performed with other approaches

Any "pure" two-particle fermionic state can be written as

$$|\Psi_F\rangle = \sum_{i,j}^{2N} w_{ij} a_i^\dagger a_j^\dagger |0\rangle \quad , \quad \{a_i, a_j^\dagger\} = \delta_{ij}$$

where W is a complex and antisymmetric matrix (2N X 2N) while $2N$ is the number of possible modes for each particle.

From the state $|\Psi_F\rangle$ we can move to one-particle density matrix in this way

$$|\Psi_F\rangle \Rightarrow \rho_F = |\Psi_F\rangle\langle\Psi_F| \Rightarrow \rho_{\nu,\mu} = \frac{\mathrm{Tr}(\rho_F a_\mu^\dagger a_\nu^\dagger)}{\mathrm{Tr}(\rho_F \sum_\mu a_\mu^\dagger a_\mu^\dagger)} = (W^\dagger W)_{\mu,\nu}$$

The eingenvalues of the matrix ρ are $|z_i|^2$

Linear entropy

$$\varepsilon_L = 1 - \mathrm{Tr}\rho^2$$

We note that the linear entropy can be expressed in terms of matrix W:

$$\varepsilon_L = 1 - \sum_{i,j}^{2N} \left| \sum_{k}^{2N} w_{ik} w_{kj}^{*} \right|^2$$

In order to calculate ε_L the definition and the allocation of the matrix ρ is not required. In this sense the linear entropy is easier to calculate than von Neumann entropy since no diagonalization of the matrix ρ is needed

Anyway by using the trace operation we can put ε_L in terms of eigenvalues of one particle density matrix

$$\varepsilon_L = 1 - \sum_{k}^{N} 2|z_k|^4$$

while the expression of von Neumann entropy is given by

$$\varepsilon_{vN} - \ln 2 - \sum_{k}^{N} 2|z_k|^2 \ln 2|z_k|^2$$

Again about linear entropy

Nonentangled states: $\varepsilon_L = 1/2$ Entangled states: $1/2 < \varepsilon_L \leq 1 - 1/(2N)$

The minimum value *1/2* indicates the unavoidable quantum correlations related to exchange simmetry as for the case of von Neumann entropy

$\varepsilon_L > 1/2$ ⇒ Additional ignorance naturally connected with genuine entanglement

Evaluation of the linear entropy and von Neumann entropy in a simple system

Let us consider the state of a simple two-fermion system with 2N degrees of freedom

$$|\chi\rangle = \sqrt{\frac{1+(N-1)(1-\alpha)^2}{2N}}\Big(|1\,2\rangle - |2\,1\rangle\Big) + \ldots \sqrt{\frac{\alpha(2-\alpha)}{2N}}\Big(|2k{-}1\,2k\rangle - |2k\,2k{-}1\rangle\Big)\ldots$$

where $2 \le k \le N$ and α is a real paramater ranging between 0 and 1. The matrix *W* is a block diagonal matrix with 2x2 blocks

$$W_1 = \begin{pmatrix} 0 & \sqrt{\frac{1+(N-1)(1-\alpha)^2}{2N}} \\ -\sqrt{\frac{1+(N-1)(1-\alpha)^2}{2N}} & 0 \end{pmatrix}$$

$$W_k = \begin{pmatrix} 0 & \sqrt{\frac{\alpha(2-\alpha)}{2N}} \\ -\sqrt{\frac{\alpha(2-\alpha)}{2N}} & 0 \end{pmatrix}$$

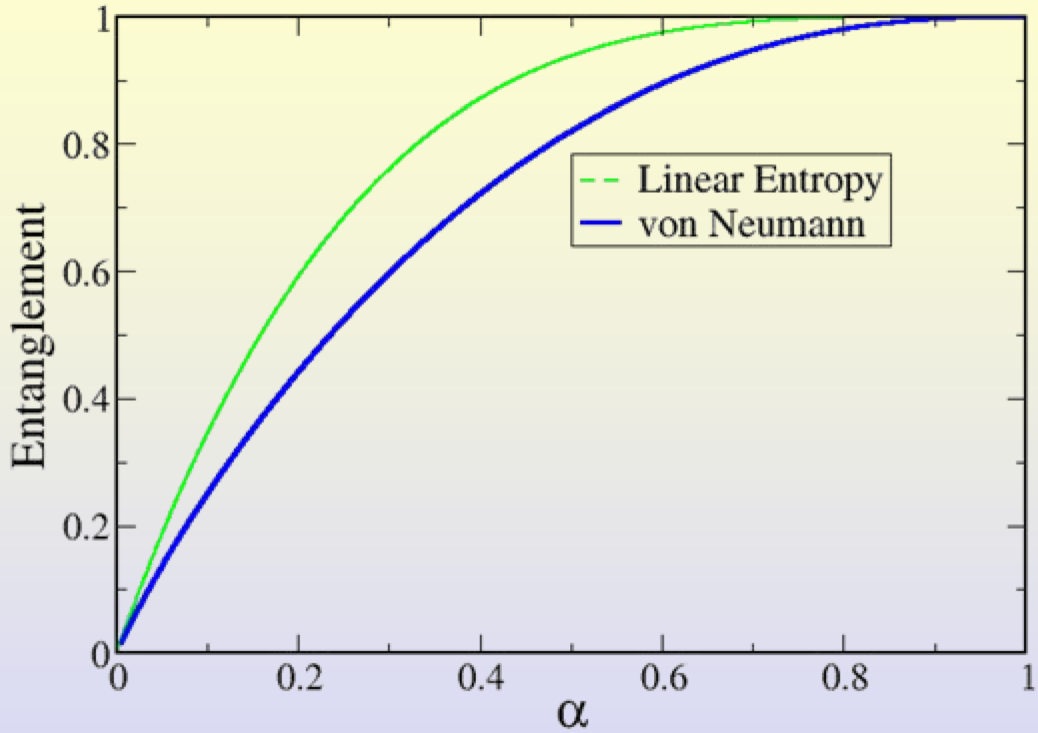
The linear entropy as a function of the parameter α takes this form

$$\tilde{\varepsilon}_L^{\chi} = 1 - (1-\alpha)^4$$

where we normalized ε_L so that its values lie in the interval between 0 and 1.

The expression for the normalized von Neumann entropy is given by

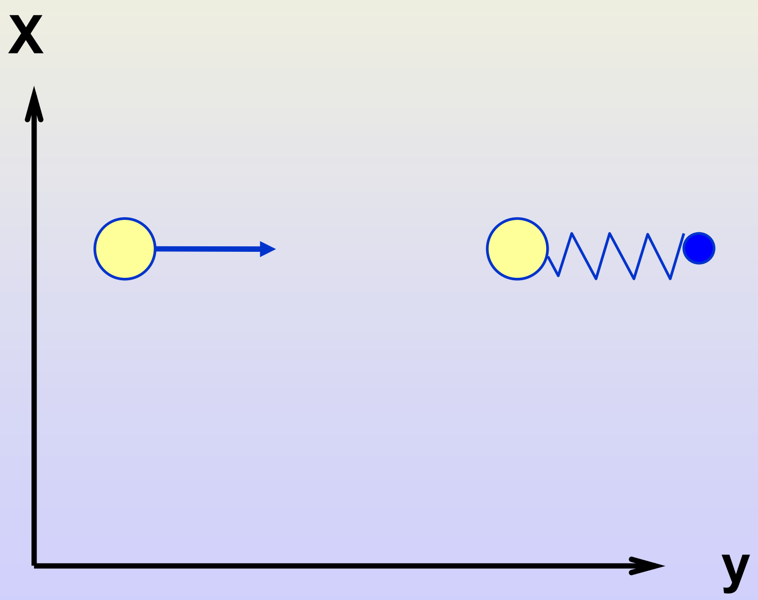
$$\tilde{\varepsilon}_{vN}^{\chi} = \frac{-1}{N \ln N}\Bigg((N-1)\alpha(2-\alpha)\ln\frac{\alpha(2-\alpha)}{N} + \left(1+(1-\alpha)^2(N-1)\right)\ln\frac{1+(1-\alpha)^2(N-1)}{N}\Bigg)$$



The comparison between linear entropy and von Neumann entropy for the state $|\chi\rangle$
We see that LE entropy is always greater than vNE apart from the initial and the final values when they coincide

Physical model in 2D

An electron propagating in a 2D system interacting through a Coulomb potential with a electron bound to a specific site by a harmonic potential



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The Hamiltonian

$$\mathcal{H} = -\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_A} - \frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_B} + \frac{e^2}{\xi|\mathbf{r}_A - \mathbf{r}_B|} + \frac{1}{2}m\omega^2(\mathbf{r}_A - \mathbf{r}_0)^2 + \frac{1}{2}m\omega^2(\mathbf{r}_B - \mathbf{r}_0)^2$$

It does not contain spin terms

Initial condition

Incoming electron

$$\psi(\mathbf{r}, t_0) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(\mathbf{r} - \mathbf{r}_0)^2}{4\sigma^2} + i\mathbf{k}\cdot\mathbf{r}\right)$$

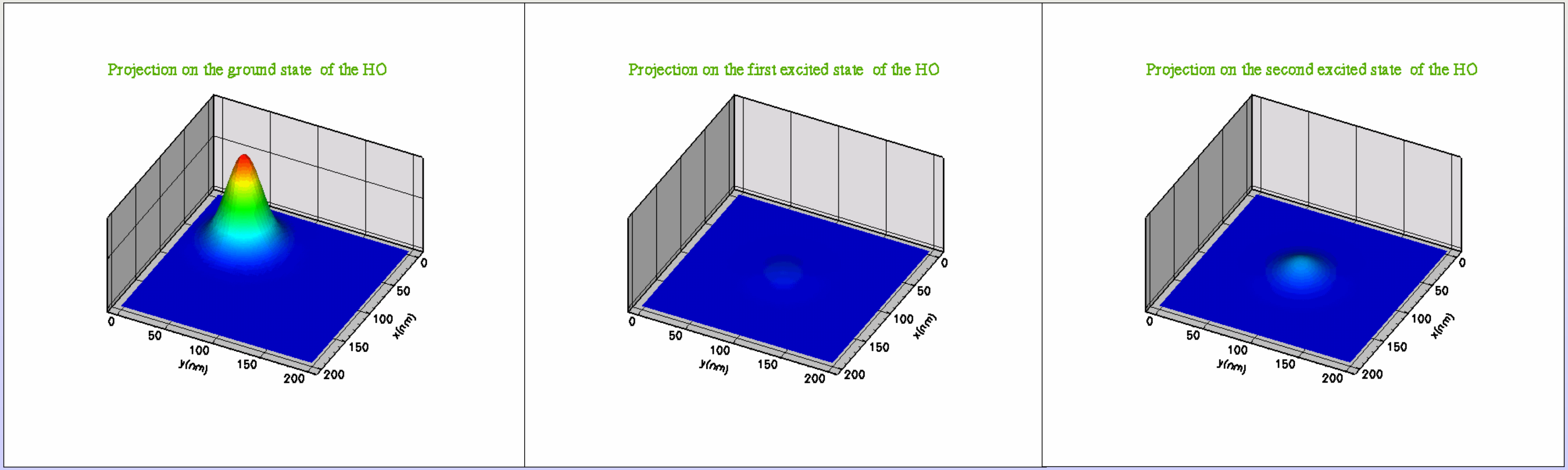
Minimum uncertainity wave packet

Bound electron

$$\phi(\mathbf{r}, t_0) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\exp\left(-\frac{m\omega(\mathbf{r} - \mathbf{r}_1)^2}{2\hbar}\right)$$

Ground state of the harmonic potential

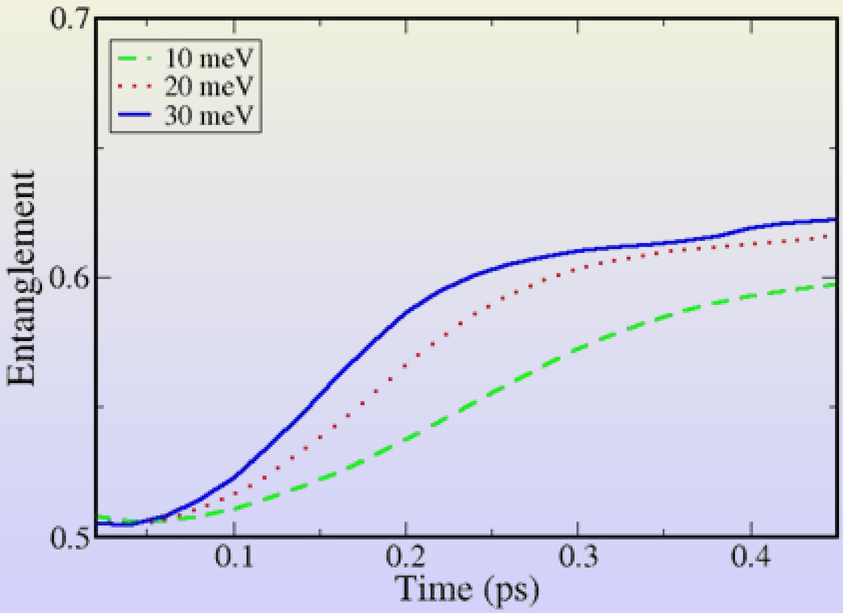
As a consequence of the scattering the state of the bound particle changes. Here we plot the time evolution the square modulus of projection the antisymmetrized two particle wavefunction $\Psi(\mathbf{r},\mathbf{r}') = \psi(\mathbf{r})\,\phi(\mathbf{r}') - \phi(\mathbf{r})\,\psi(\mathbf{r}')$ on the eingenstates of the harmonic oscillator



Two fermions, same spin.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big(|\psi\,\phi\rangle - |\phi\,\psi\rangle\Big)|\uparrow\uparrow\rangle$$

ħω=2 meV

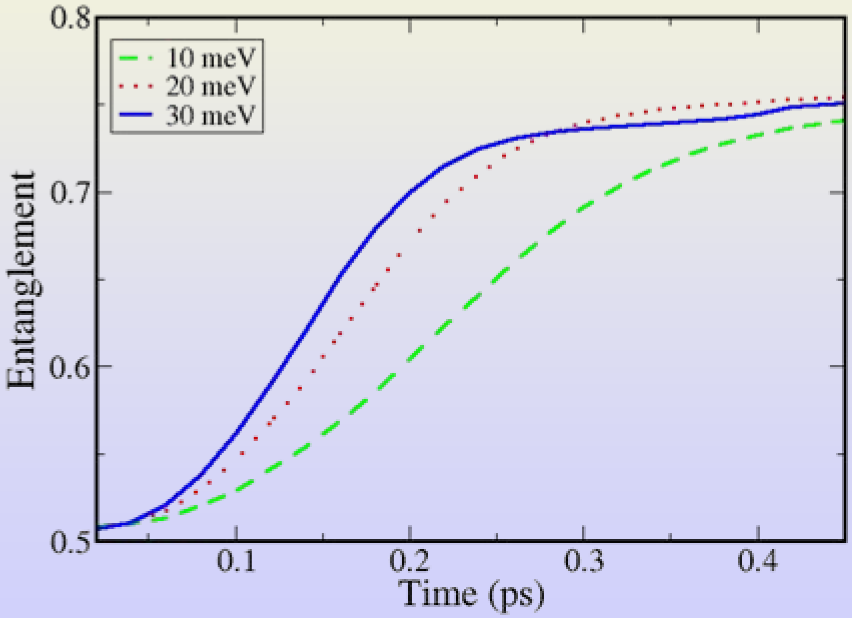


- Time evolution of the entanglement for different values of the incoming electron initial energy E_K.
- The stationary value of the entanglement is higher for higher energies

Two fermions, different spin, non factorized state.

$$|\Upsilon\rangle = \frac{1}{\sqrt{2}}\Big(|\psi\,\phi\rangle|\uparrow\downarrow\rangle - |\phi\,\psi\rangle|\downarrow\uparrow\rangle\Big)$$

$\hbar\omega$=2 meV



The final value of the entanglement reaches its maximum value for E_K= 20 meV

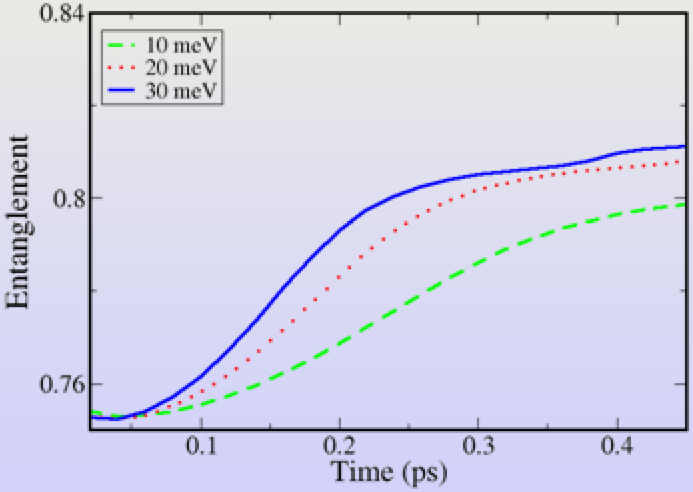
Two fermions, different spin, factorized state.

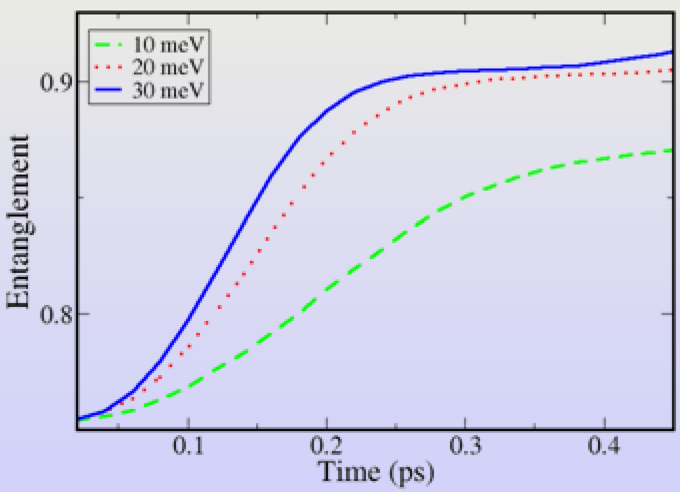
$$|\Xi\rangle = \frac{1}{2}\Big(|\psi\,\phi\rangle - |\phi\,\psi\rangle|\Big)\Big(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big)$$

$$|\Phi\rangle = \frac{1}{2}\Big(|\psi\,\phi\rangle + |\phi\,\psi\rangle\Big)\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big)$$

Triplet spin state

Singlet spin state



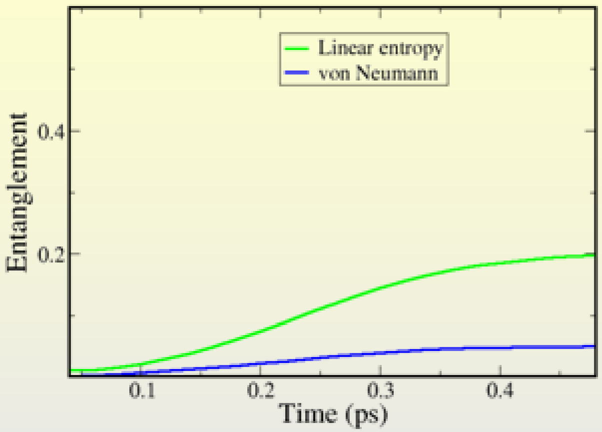


The linear entropy of triplet spin state can be obtained from the one of the same spin as $\varepsilon^{\Xi} = 0.5*(1+\varepsilon^{\Psi})$

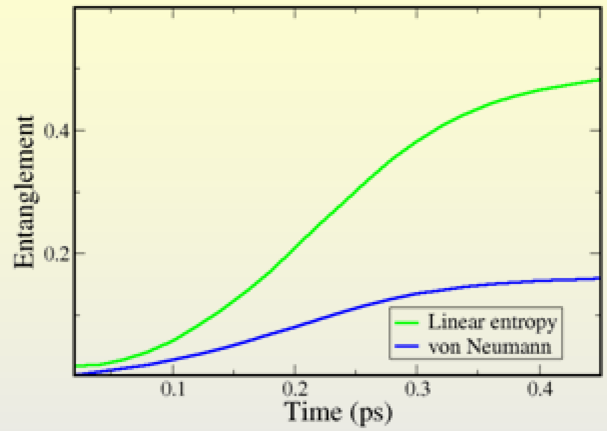
The initial value of the entanglement is 3/4. The triplet and singlet spin state are initially entangled

Comparison between normalized linear and von Neumann entropies

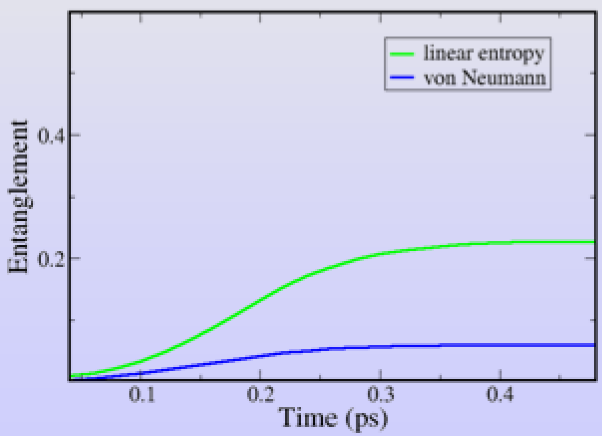
Same spin 10 meV



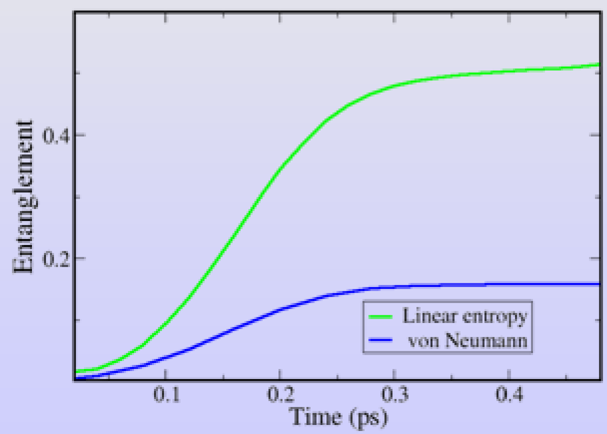
Diff spin 10 meV



Same spin 20 meV



Diff spin 20 meV



We observe that both measures increase with time but the linear entropy is greater than the von Neumann one. Time of entanglement formation is the same for the two notions.

Conclusions

- *We have analyzed one possible entanglement measure for two-fermion system based on the linear entropy of the one particle-density matrix. It agrees with Slater rank criterion*
- *We evaluate the entanglement as LE in an electron-electron scattering in a 2D semiconductor system. Our results confirm a dependence of the entanglement on spin components of the state. The times of entanglement formation correspond to the ones obtained in the previous work by using the vNE.*
- *From a computational point of view the evaluation of the entanglement in terms of LE is easier and much faster than one performed by means of vNE*

F.Buscemi,P.Bordone and A.Bertoni, PRA 73 052312 (2006)

F.Buscemi,P.Bordone and A.Bertoni, quant-ph/0611223