CHALMERS

Arbitrary Accuracy Iterative Phase Estimation Algorithm as a Two Qubit Benchmark

Göran Johansson

M. Dobšíçek, G. J., V. S. Shumeiko, and G. Wendin, quant-ph/0610214 (2006)

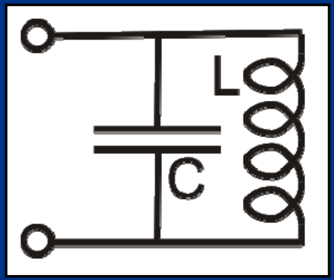
QMFPA 2006, 4th of December, Bertinoro

MC2

Outline

- Superconducting Quantum Circuits and Quantum Bits
- The Iterative Phase Estimation Algorithm
  - 2-qubit Benchmark Implementations
  - Repeated Measurements and Fault Tolerance
- Summary

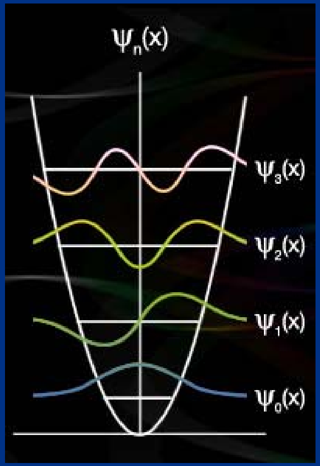
Quantum Mechanics and Electrical Circuits



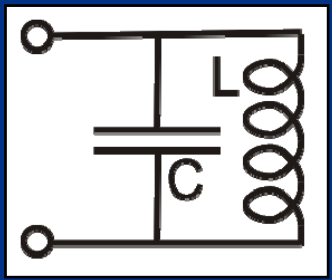
L=10 nH, C=1 pF $\Rightarrow$ ω_0=1.6 GHz

T=20 mK $\Rightarrow$ $k_B T/\hbar\omega_0$=0.15

(No dissipation)



Quantum Mechanics and Electrical Circuits



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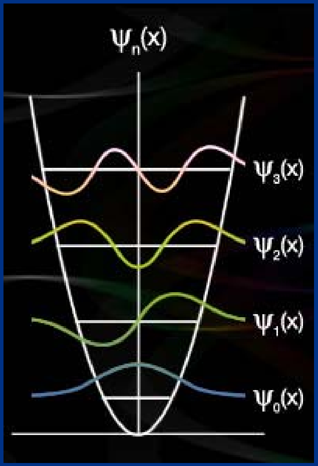


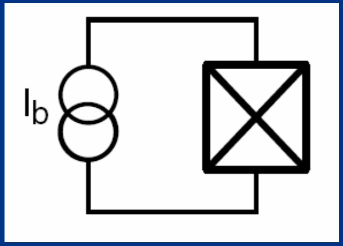
Non-trivial effects with non-linear circuit elements.
Josephson junction: non-linear

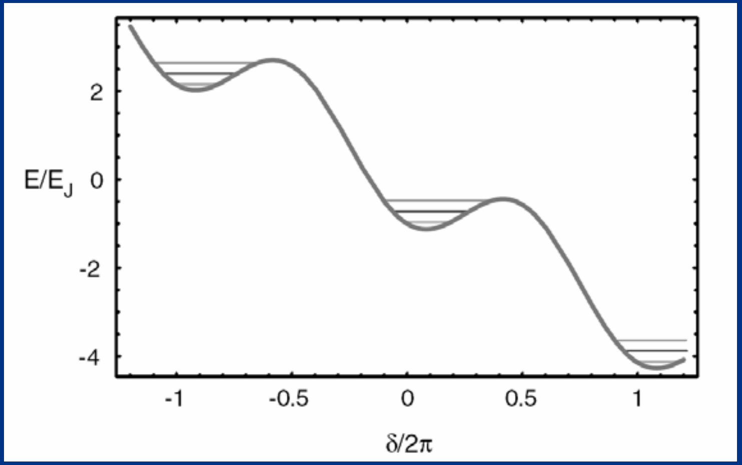
$$i(t) = \frac{1}{L}\Phi(t)$$

Inductor

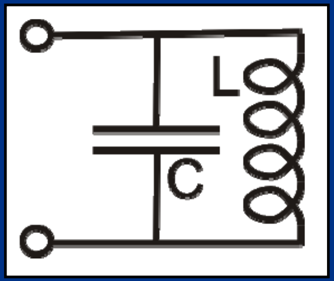
$$i(t) = I_0 \sin\left[\frac{2e}{\hbar}\Phi_J(t)\right]$$

Josephson junction



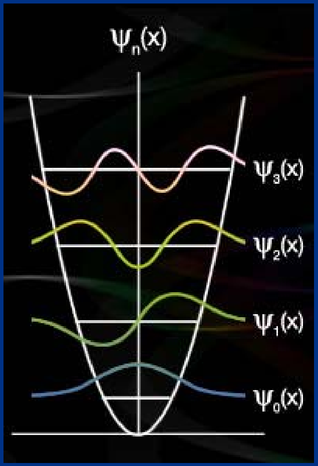


Quantum Mechanics and Electrical Circuits



$L=10$ nH, $C=1$ pF $\Rightarrow \omega_0=1.6$ GHz

$T=20$ mK $\Rightarrow k_BT/\hbar\omega_0=0.15$



Non-trivial effects with non-linear circuit elements.
Josephson junction: non-linear and non-dissipative

$$i(t) = \frac{1}{L}\Phi(t)$$

Inductor

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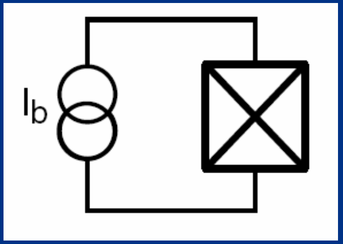
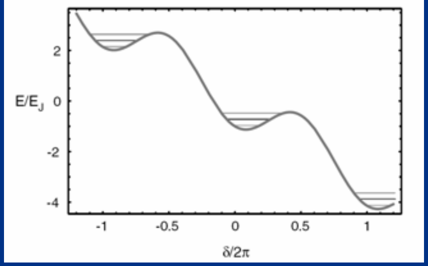
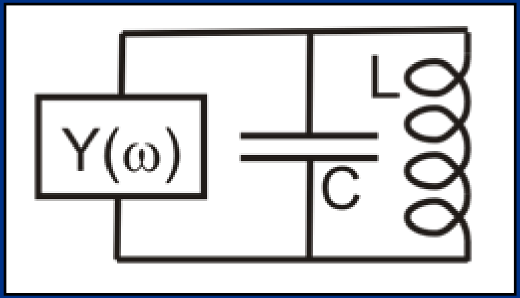
Josephson junction





What about dissipation?

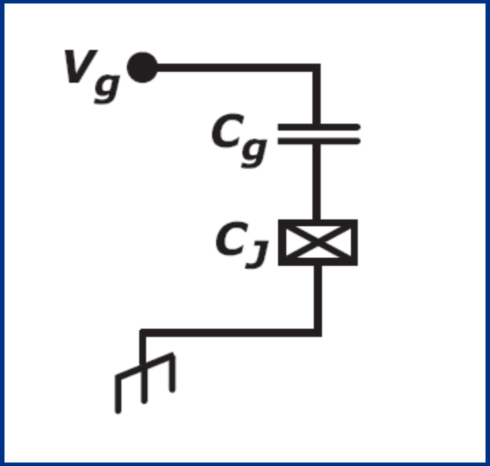
Quantum if level width is smaller than level splitting Q>1.

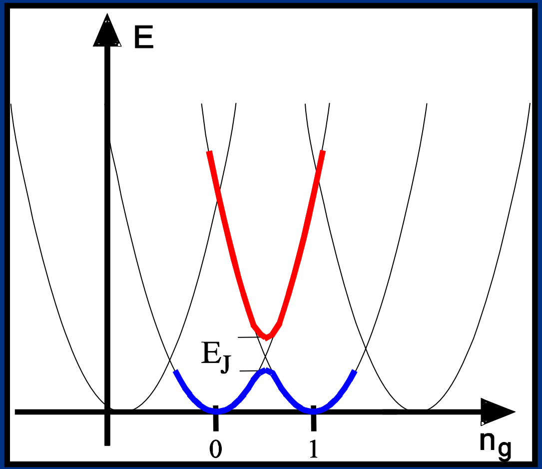


Superconductivity!

M. Devoret, Les Houches (1995)
"Quantum network theory", Yurke and Denker PRA (1984)

The Superconducting Charge Qubit





$E_J << E_C$

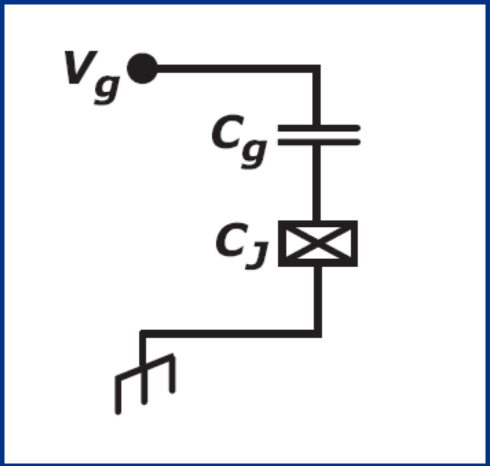
$$n_g = \frac{V_g C_g}{2e}$$

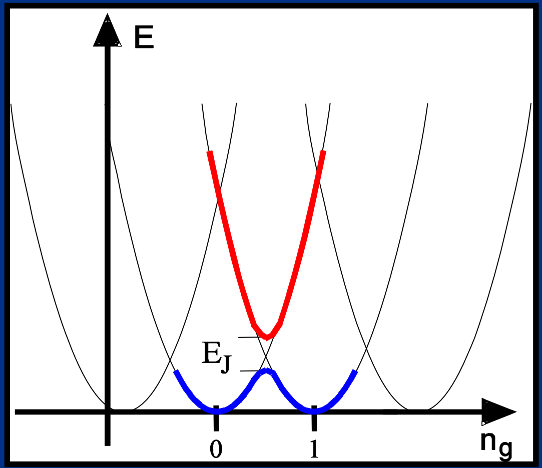
$$H = \sum_n \frac{4e^2(n-n_g)^2}{2(C_J+C_g)} |n><n| - \frac{1}{2} E_J \left(|n><n+1| + |n+1><n| \right)$$

M. Büttiker, PRB (1985)
A. Shnirman, G. Schön, and Z. Hermon, PRL (1997)
V. Bouchiat et al., Phys. Scr. (1998)

|n> is the state with n (extra) Cooper-pairs on the island

The Superconducting Charge Qubit





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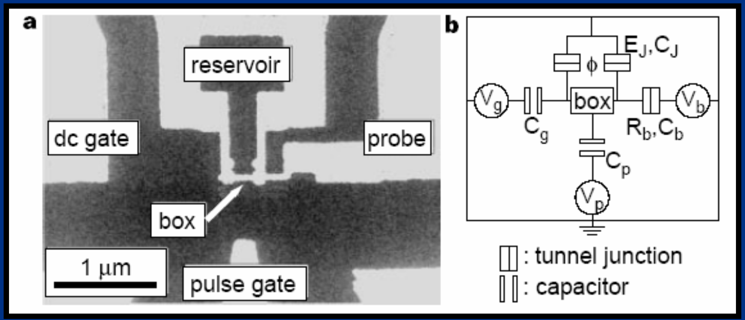
Coherent control of macroscopic quantum states in a single-Cooper-pair box

Y. Nakamura*, Yu. A. Pashkin† & J. S. Tsai*



NATURE | VOL 398 | 29 APRIL 1999

Superconducting qubits

Charge qubit

T_2=2 ns, Q=25

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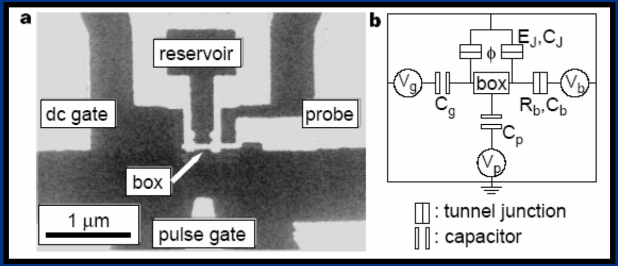

MC2

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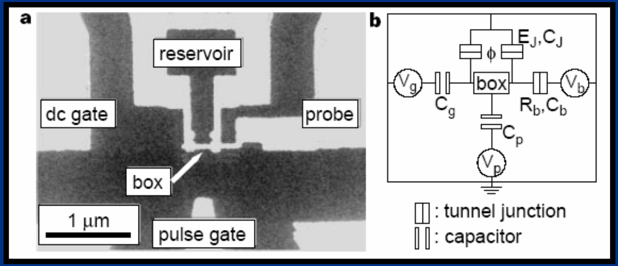
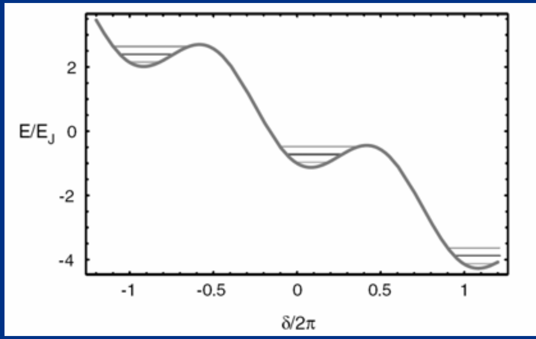


Phase Qubit

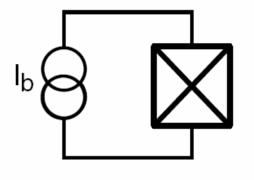


Rabi Oscillations in a Large Josephson-Junction Qubit

John M. Martinis, S. Nam, and J. Aumentado* C. Urbina†

PHYSICAL REVIEW LETTERS 9 SEPTEMBER 2002

T_2= 80 ns, Q=460



Coherent Temporal Oscillations of Macroscopic Quantum States in a Josephson Junction

Yang Yu,[1] Siyuan Han,[1*] Xi Chu,[2†] Shih-I Chu,[2] Zhen Wang[3]

SCIENCE VOL 296 3 MAY 2002

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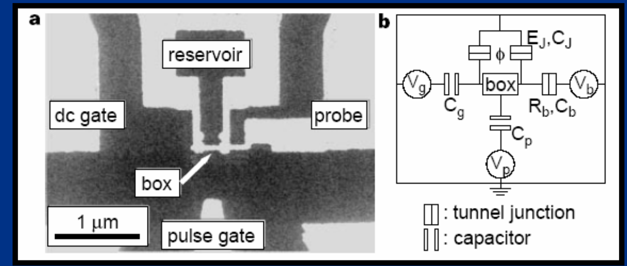
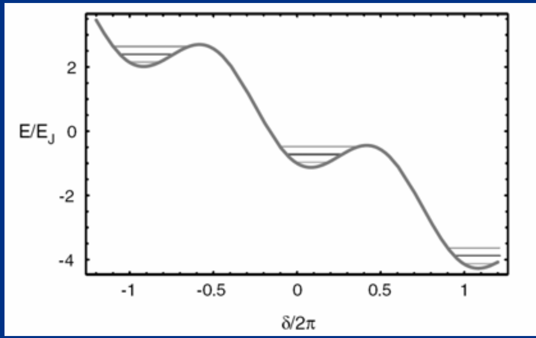


Phase Qubit



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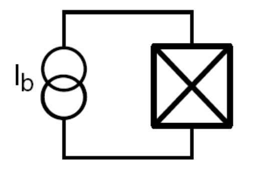

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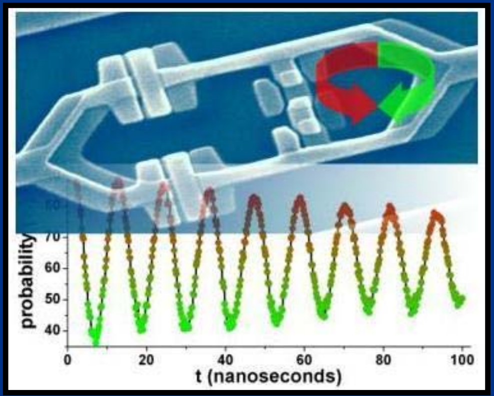
SCIENCE VOL 296 3 MAY 2002

Flux qubit

T_2=120 ns, T_{echo}=3.9 µs, Q=23400



Coherent Quantum Dynamics of a Superconducting Flux Qubit

I. Chiorescu,[1]* Y. Nakamura,[1,2] C. J. P. M. Harmans,[1] J. E. Mooij[1]

SCIENCE VOL 299 21 MARCH 2003

I_p=0.57 µA

Millions of electrons

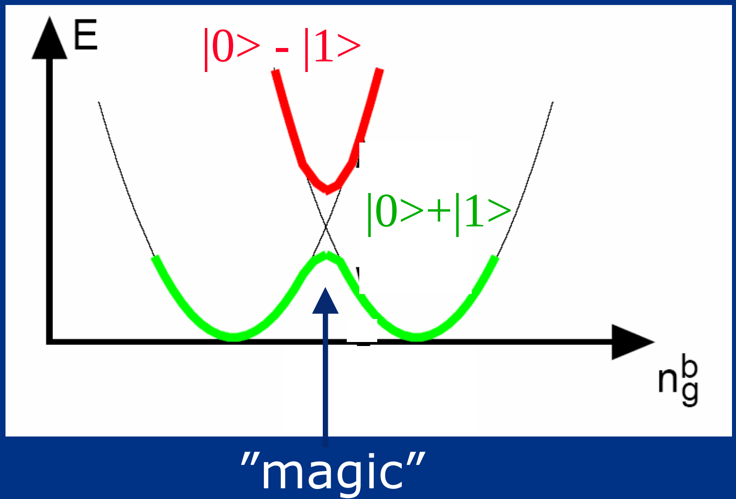
Progress of Charge Qubit Readout

Single shot readout possible using the single-electron transistor (SET).

A. Aassime, G. J., G. Wendin, R. J. Schoelkopf, and P. Delsing, PRL (2001)
G. J., A. Käck, and G. Wendin, PRL (2002)
A. Käck, G. Wendin, and G. J. PRB (2003)

Charge qubits decohere quickly away from the degeneracy (”magic”) point due to 1/f charge noise.

The SET measures charge.
Nothing to measure at degeneracy.



Single shot dispersive readout at degeneracy by capacitive coupling to a slow oscillator. (Quantum capacitance)

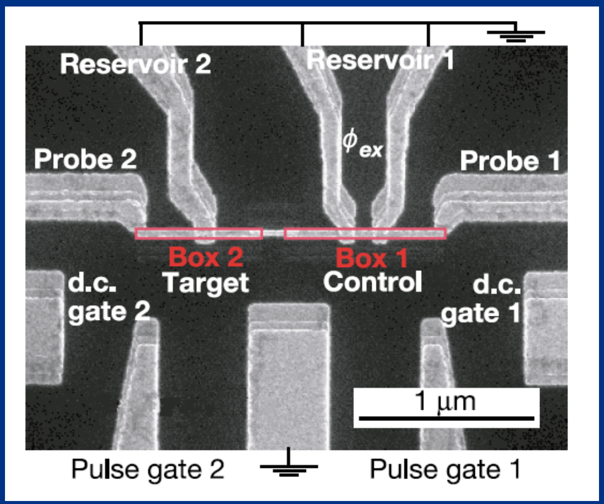
T. Duty, G. J., K. Bladh, D. Gunnarsson, C. Wilson, P. Delsing, PRL (2005)
G. J., L. Tornberg, V.S. Shumeiko and G. Wendin, JPCM (2006)
G. J., L. Tornberg, and C. Wilson, PRB(R) (2006)
L. Tornberg and G. J., JLTP (2006)

Superconducting 2-qubit systems

NATURE | VOL 425 | 30 OCTOBER 2003 |

Demonstration of conditional gate operation using superconducting charge qubits

T. Yamamoto[1,2], Yu. A. Pashkin[2*], O. Astafiev[2], Y. Nakamura[1,2] & J. S. Tsai[1,2]



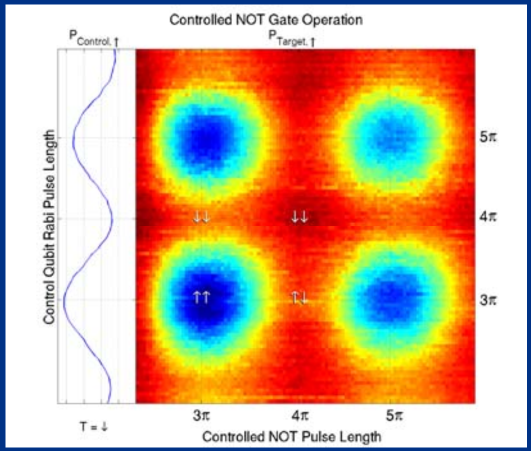
SCIENCE VOL 313 8 SEPTEMBER 2006

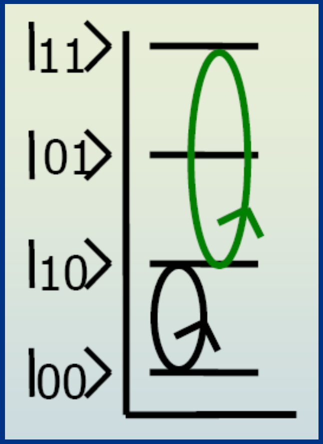
Phase Qubits

Measurement of the Entanglement of Two Superconducting Qubits via State Tomography

Matthias Steffen,* M. Ansmann, Radoslaw C. Bialczak, N. Katz, Erik Lucero, R. McDermott, Matthew Neeley, E. M. Weig, A. N. Cleland, John M. Martinis†

Flux qubit CNOT-gate in Delft (!)





What 2-qubit experiments should be performed?

The phase estimation problem

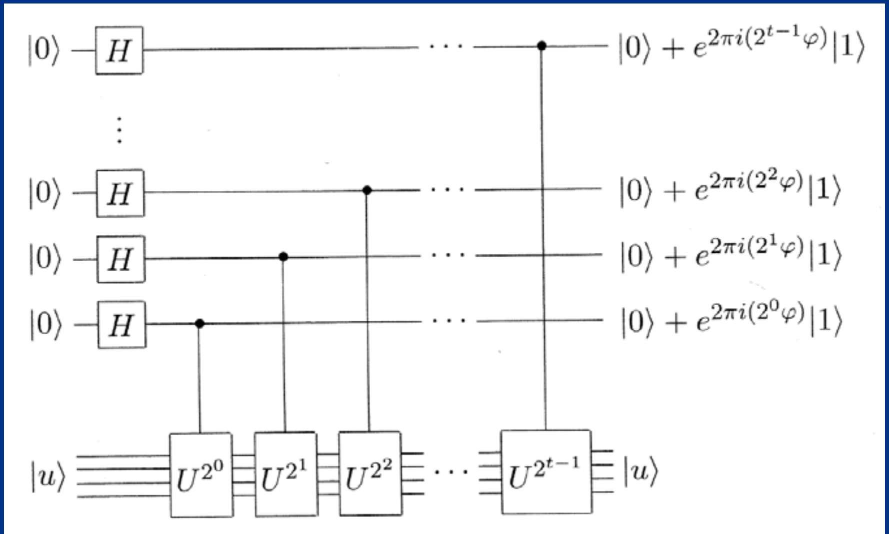
$$\hat{U}\left|\Psi\right\rangle = e^{i2\pi\phi}\left|\Psi\right\rangle$$

Determine ϕ, for given unitary operator Û and eigenstate |Ψ>.
Natural and important problem suitable for quantum computing.
Û can model a physical system Hamiltonian, ϕ can be $E_0 t/\hbar$.

Simulated Quantum Computation of Molecular Energies

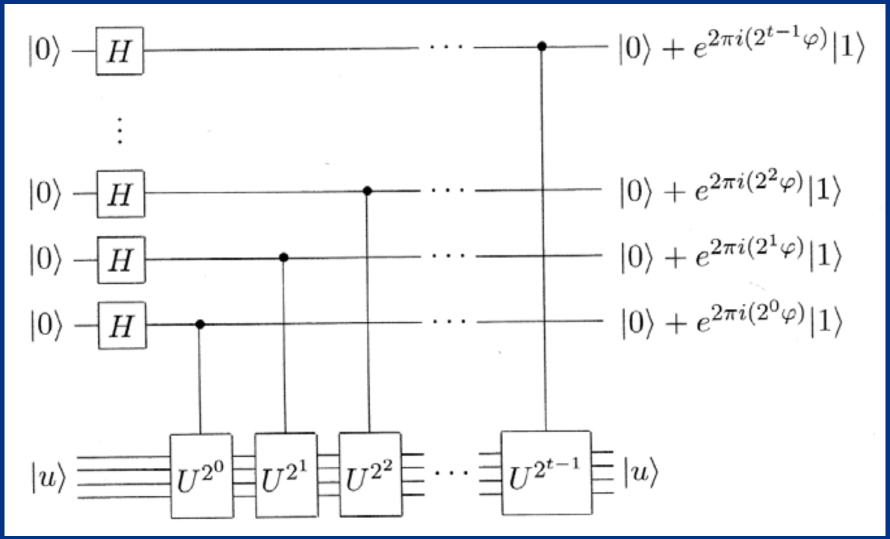
Alán Aspuru-Guzik,[1*†] Anthony D. Dutoi,[1*] Peter J. Love,[2] Martin Head-Gordon[1,3]

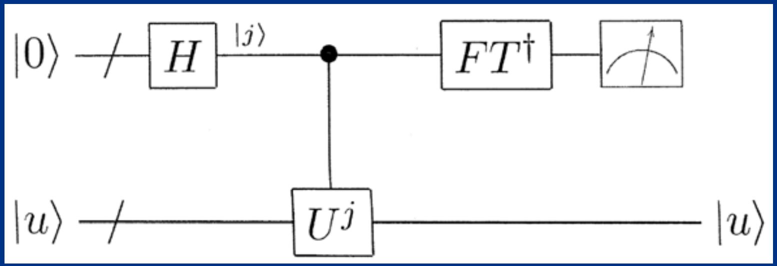
9 SEPTEMBER 2005 VOL 309 SCIENCE



Textbook (Nielsen & Chuang); use inverse Quantum Fourier Transform

MC2





Phase estimation using inverse Quantum Fourier Transform:

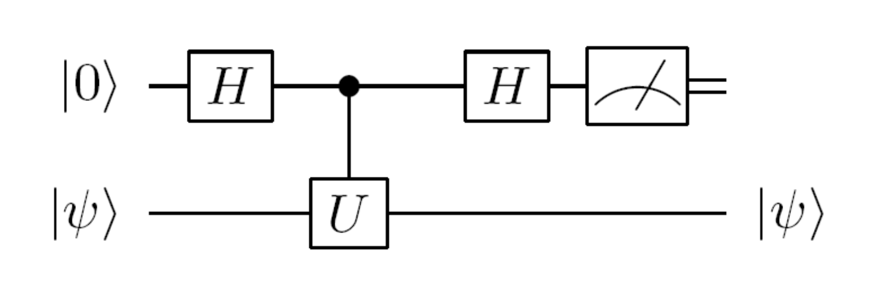
Obtain ϕ with n bits precision (accuracy ~ $2\pi\ 2^{-n}$),
where n is the number of ancilla qubits.

Experiment in 3-qubit NMR-system, Û 1-dim + 2 ancillary qubits (accuracy $\pi/2$)

Jae-Seung Lee, Jaehyun Kim, Yongwook Cheong, and Soonchil Lee

PHYSICAL REVIEW A **66**, 042316 (2002)

A single ancillary qubit: the Naive Approach



$$P_0 = \cos^2(\pi\phi)$$

Repeat N times ⇒ accuracy ~ $N^{-1/2}$
n bits accuracy ⇒ $N \sim 2^{2n}$

Example: $\phi = 0.01101001010100000...$

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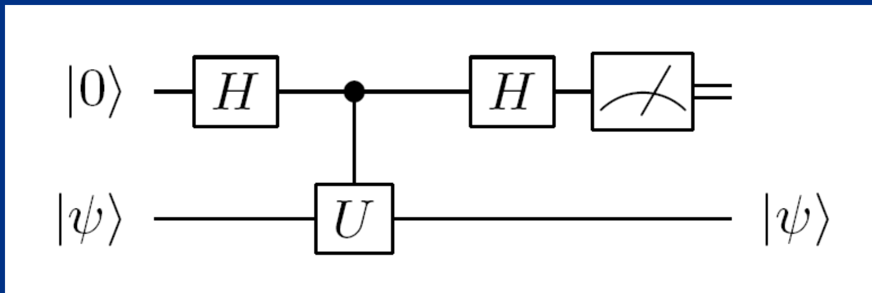

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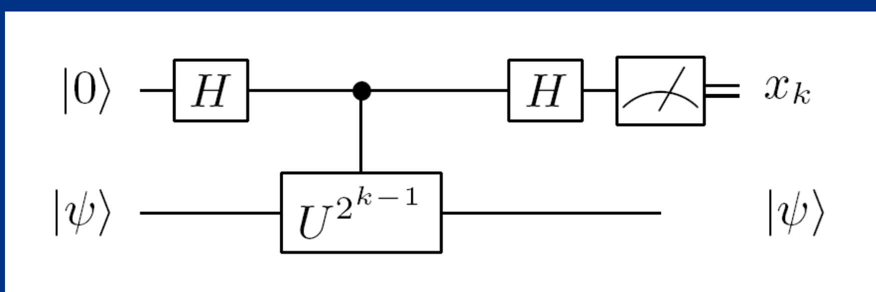
Following Kitaev (1996)

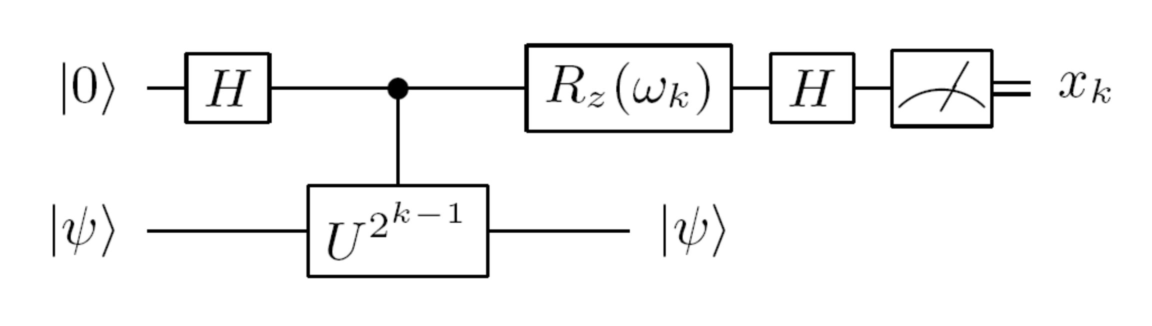


Accuracy $\sim 2^{-(n+2)}$ with probability $1-\varepsilon$, using $O(n \log n/\varepsilon)$ repetitions.

Classical post-processing needed.

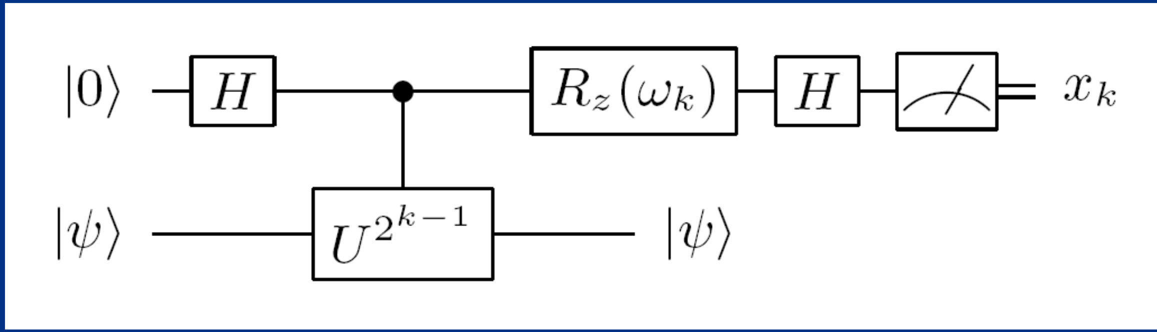
$1 \leq k \leq n$
Relies on an efficient implementation of U^{2^k}.

An iterated phase estimation algorithm with feed-back



$1 \leq k \leq n$, Example: $\phi = 0.\phi_1\phi_2\phi_3\phi_4\phi_5 0000\ldots$ and $n=5$.

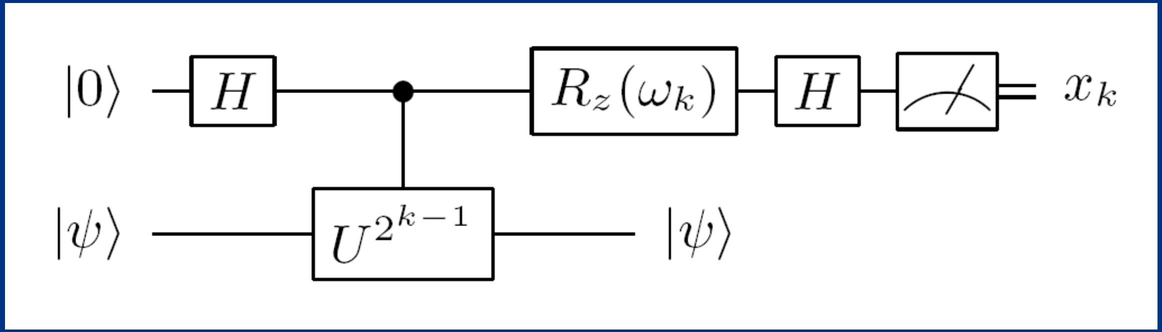
An iterated phase estimation algorithm with feed-back



$1 \leq k \leq n$, Example: $\phi = 0.\phi_1 \phi_2 \phi_3 \phi_4 \phi_5 0000\ldots$ and $n=5$.

Start from $k=5$: $P_0=\cos^2(\pi \phi_5/2)$, so $x_5=\phi_5$, deterministically.

An iterated phase estimation algorithm with feed-back



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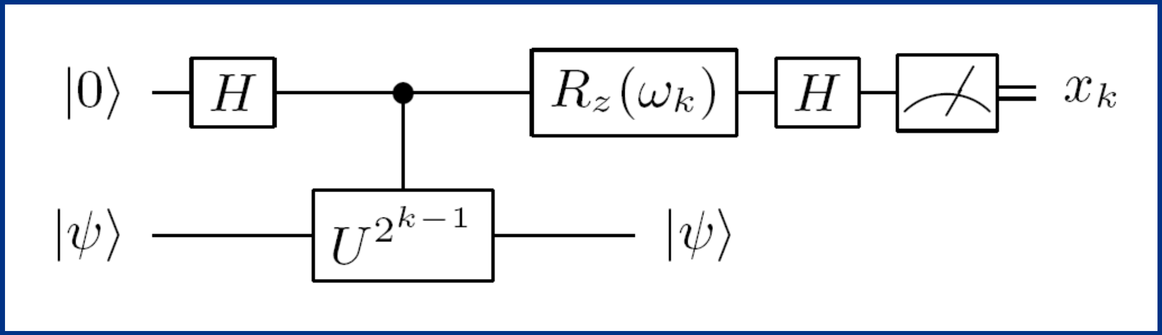
Start from k=5: $P_0=\cos^2(\pi\phi_5/2)$, so $x_5=\phi_5$, deterministically.

For k=4 the phase before feed-back: $2\pi\,(0.\phi_4\phi_5)$

Use $\omega_4=-2\pi\,(0.0x_5)$ to achieve $P_0=\cos^2(\pi\phi_4/2)$, so $x_4=\phi_4$, deterministically.

An iterated phase estimation algorithm with feed-back



$1 \leq k \leq n$, Example: $\phi = 0.\phi_1\phi_2\phi_3\phi_4\phi_5 0000\ldots$ and $n=5$.

Start from $k=5$: $P_0=\cos^2(\pi\phi_5/2)$, so $x_5=\phi_5$, deterministically.

For $k=4$ the phase before feed-back: $2\pi\,(0.\phi_4\phi_5)$

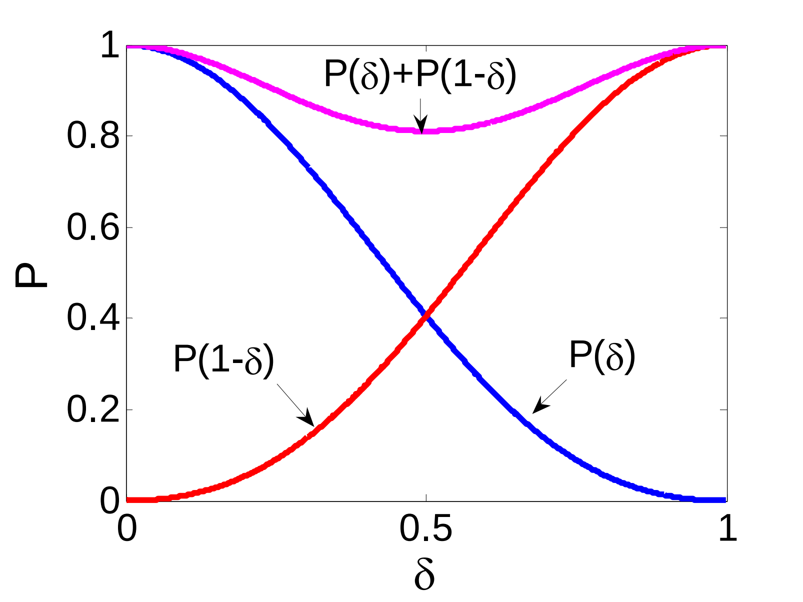
Use $\omega_4=-2\pi\,(0.0x_5)$ to achieve $P_0=\cos^2(\pi\phi_4/2)$, so $x_4=\phi_4$, deterministically.

Iteratively, using feed-back $\omega_k=-2\pi\,(0.0x_{k+1}x_{k+2}\ldots x_n)$, *all m digits are extracted deterministically*!

General phase

For general $\phi = 0.\phi_1 \phi_{2\ldots} \phi_{n-1} \phi_n + \delta\, 2^{n-1}$, $0 \leq \delta < 1$
the success probability is $P(\delta)+P(1-\delta)$, where

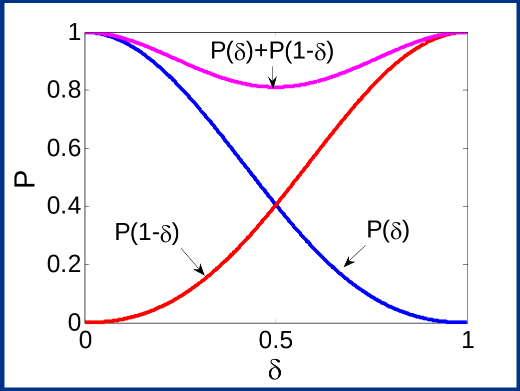
$$P(\delta) = \prod_{k=1}^{m} \cos^2(\pi\, 2^{k-m-1}\delta) = \frac{\sin^2(\pi\,\delta)}{2^{2m}\sin^2(\pi 2^{-m}\delta)}$$



(cmp. QFT)
Lower bound is $8/\pi^2 > 0.81$

General phase

For general $\phi = 0.\phi_1 \phi_{2\ldots} \phi_{n-1} \phi_n + \delta\, 2^{n-1}$, $0 \leq \delta < 1$ the success probability is $P(\delta)+P(1-\delta)$, where

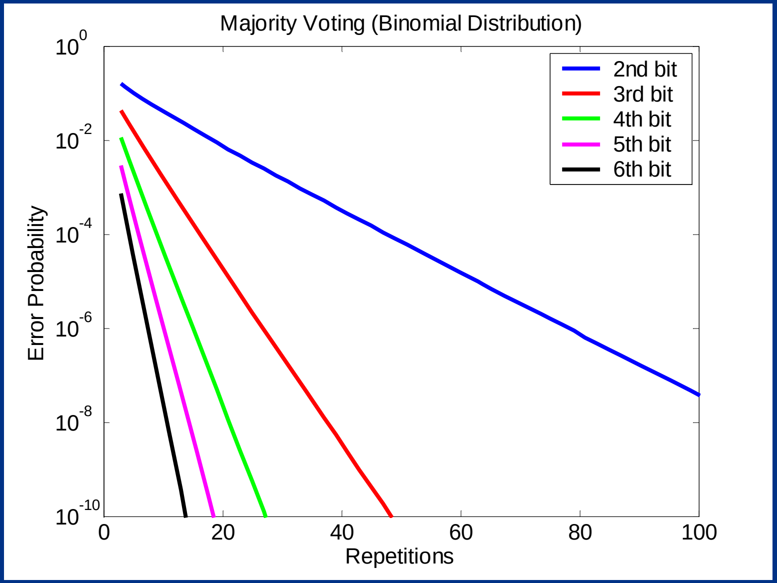


$$P(\delta) = \prod_{k=1}^{m} \cos^2(\pi\, 2^{k-m-1}\delta) = \frac{\sin^2(\pi\,\delta)}{2^{2m}\sin^2(\pi 2^{-m}\delta)}$$

(cmp. QFT)

Lower bound is $8/\pi^2 > 0.81$

The least significant bits contribute most to the error probability. The contribution decays exponentially.



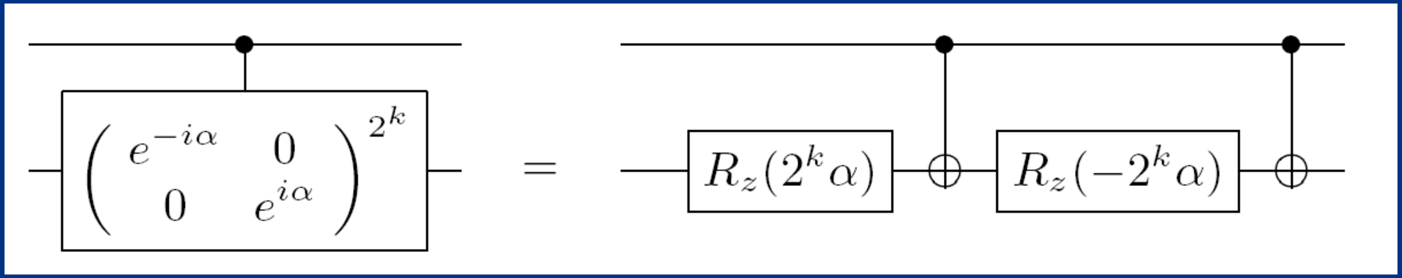
$\log(1/\varepsilon)$ repeated measurements of the $\log(1/\varepsilon)$ least significant bits increase the success probability to $1-\varepsilon$, *independently* of n.

2-qubit benchmark circuits

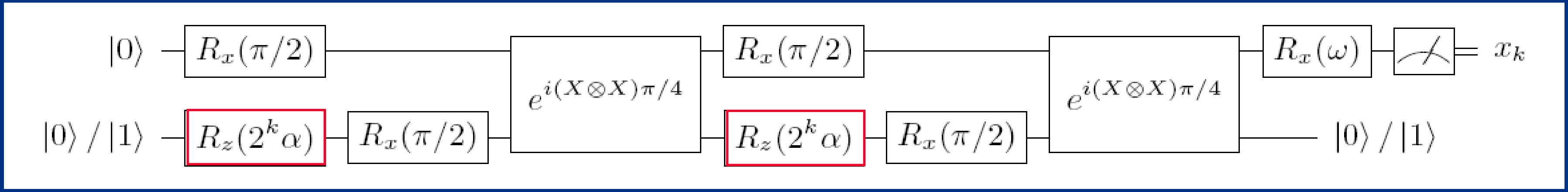
$$\hat{U} = \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

Simple to prepare the eigenstate |0>.

An efficient implementation of U^{2^k}:

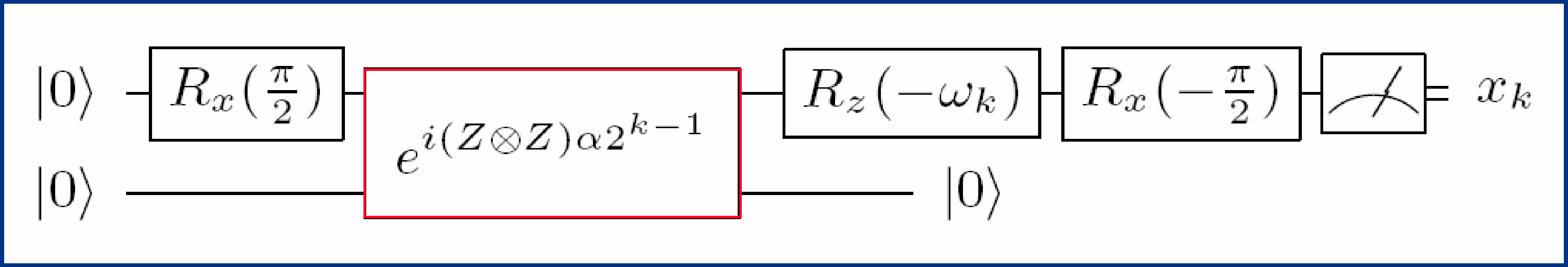


One realization of the whole circuit for X-X two-qubit coupling:



2-qubit circuit and fault tolerance

The whole circuit if the natural two-qubit coupling is ZZ:



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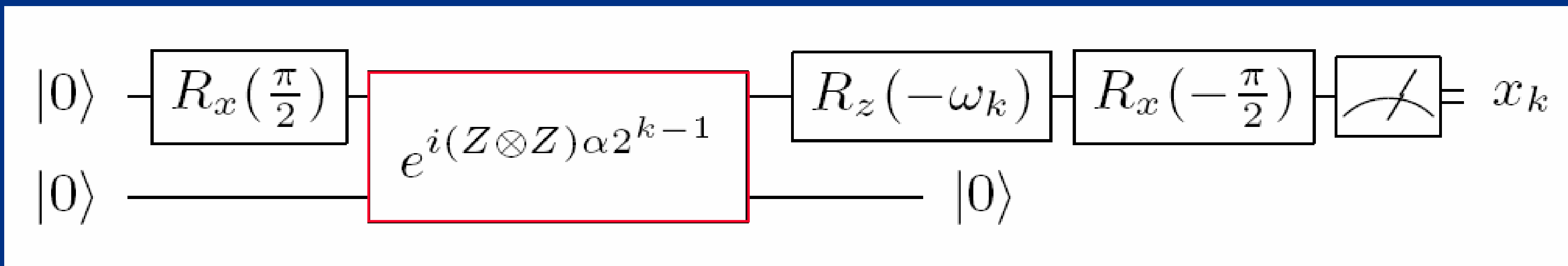
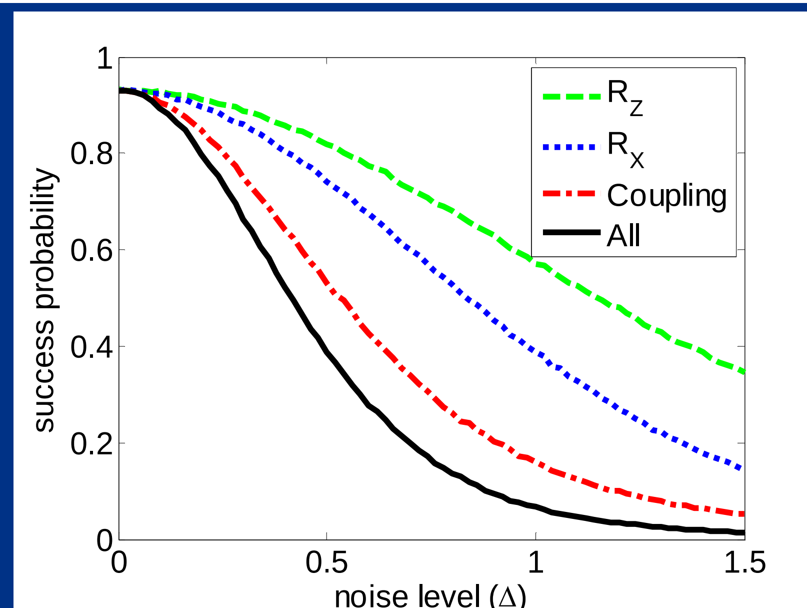


Simulate gate errors:

$$\varphi_i = \lambda_i T_i$$

$$\varphi_i \rightarrow \varphi_i(1+\delta\lambda_i/\lambda_i)$$

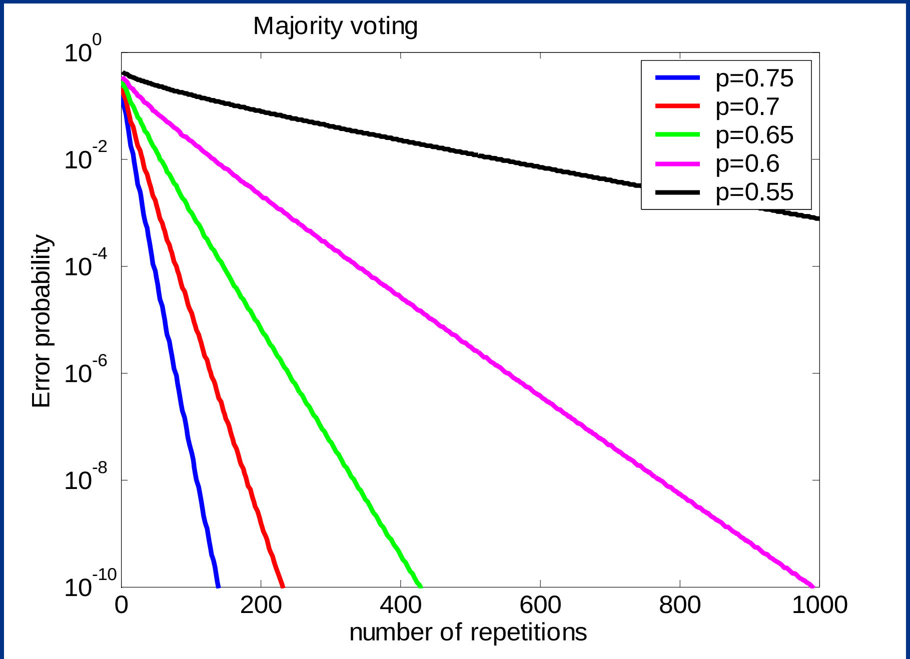
$$-\Delta/2 < \delta\lambda_i/\lambda_i < \Delta/2$$



m=10, no repetitions

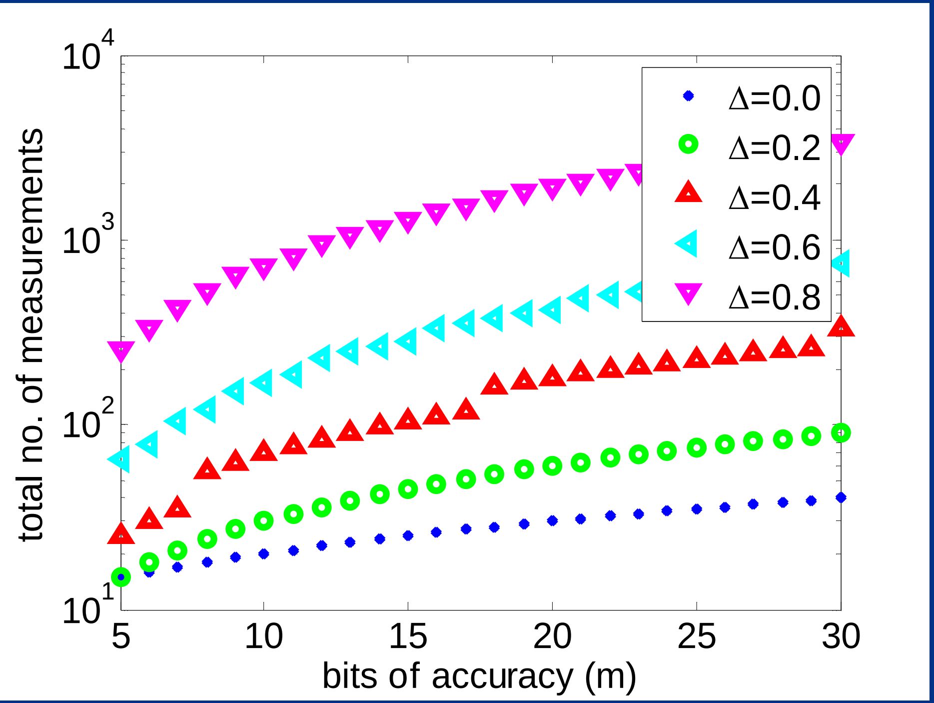
Repeated measurements

- Errors due to finite remainder δ and gate errors.
- Both can be fought by repeating each iteration.
- Heuristic for approximating the optimal number of repetitions.



Repeated measurements

- Errors due to finite remainder δ and gate errors.
- Both can be fought by repeating each iteration.
- Heuristic for approximating the optimal number of repetitions.
- 10 digits with 95% probability for error level Δ=0.8 using ~70 repetitions per bit. Independent errors.
- Still sensitive to static or slowly fluctuating errors.

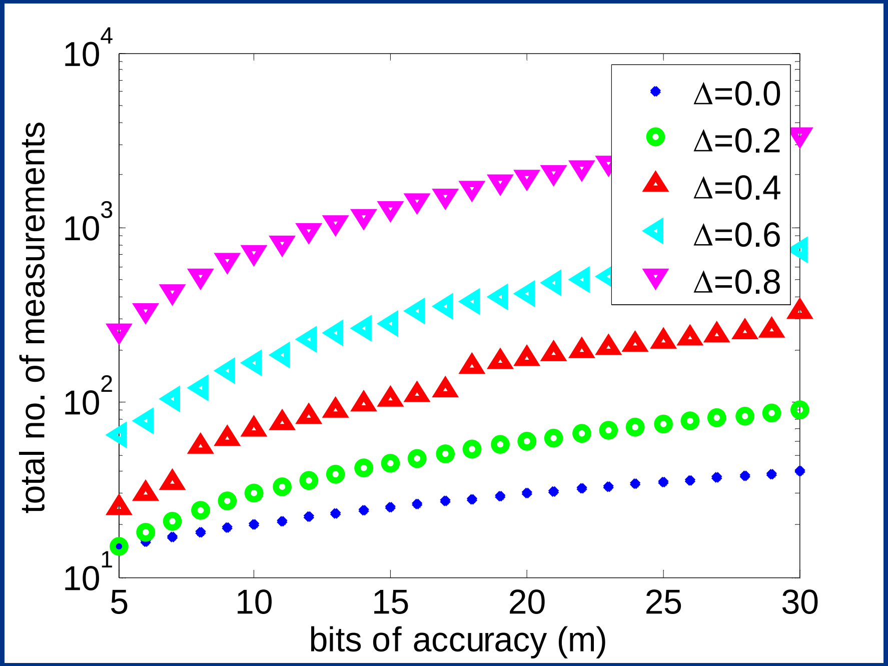


Fight errors with repetitions

Error probability ε<0.05

MC2

Summary



Fight errors with repetitions

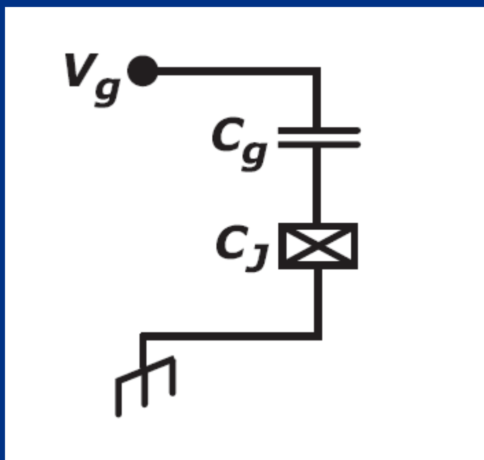
Error probability $\varepsilon<0.05$

- Superconducting qubits have reached the two-qubit stage
- Phase estimation is an important algorithm.
- Arbitrary accuracy with one ancillary qubit and one measurement per bit is possible
- High accuracy also with low (realistic) gate fidelities, using repeated measurements
- Useful as benchmark

M. Dobšíçek, G. Johansson, V. S. Shumeiko, G. Wendin, *quant-ph/0610214* (2006)

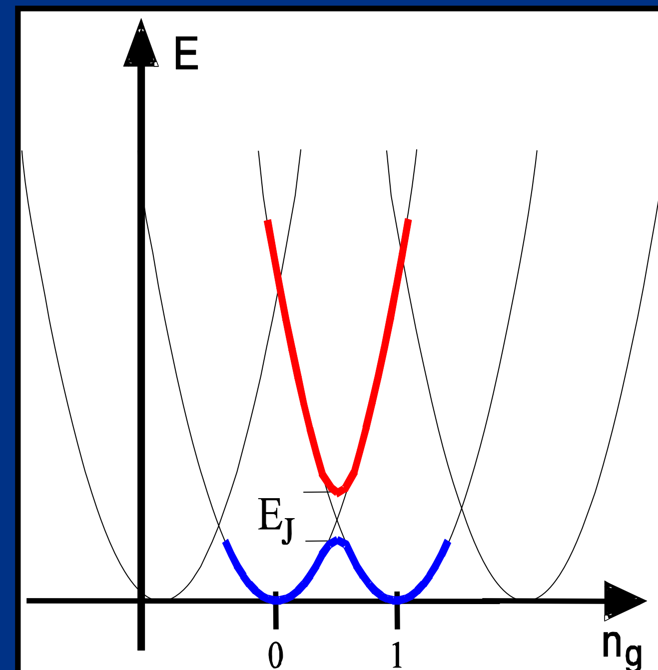
MC2

The Cooper-pair box – A Superconducting Charge Qubit



$E_J << E_C$

|n> is the state with n (extra) Cooper-pairs on the island

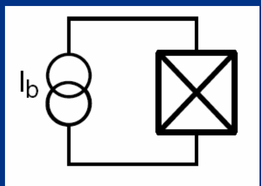


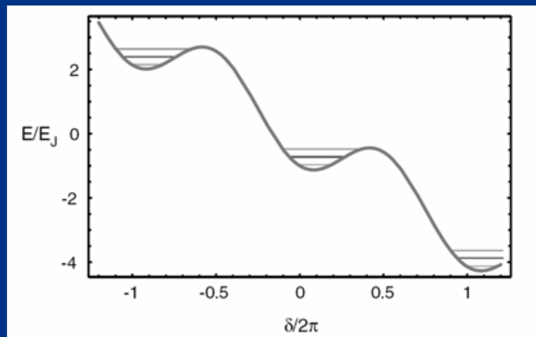
$$H = \sum_n \frac{4e^2(n-n_g)^2}{2(C_J+C_g)} |n><n| - \frac{1}{2} E_J \left(|n><n+1| + |n+1><n| \right)$$

$$n_g = \frac{V_g C_g}{2e}$$

M. Büttiker, PRB (1985)
A. Shnirman, G. Schön, and Z. Hermon, PRL (1997)
V. Bouchiat et al., Phys. Scr. (1998)

Phase Qubit





Rabi Oscillations in a Large Josephson-Junction Qubit

John M. Martinis, S. Nam, and J. Aumentado* C. Urbina†

PHYSICAL REVIEW LETTERS 9 SEPTEMBER 2002

T_2= 80 ns, Q=460

Coherent Temporal Oscillations of Macroscopic Quantum States in a Josephson Junction

Yang Yu,[1] Siyuan Han,[1*] Xi Chu,[2†] Shih-I Chu,[2] Zhen Wang[3]

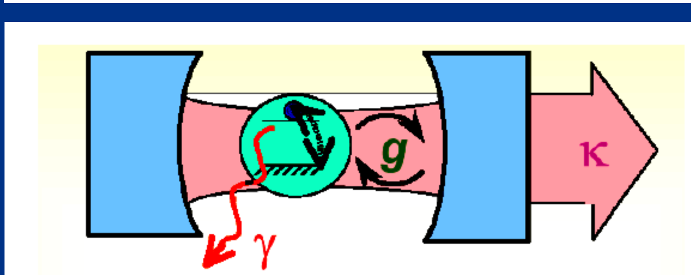
SCIENCE VOL 296 3 MAY 2002

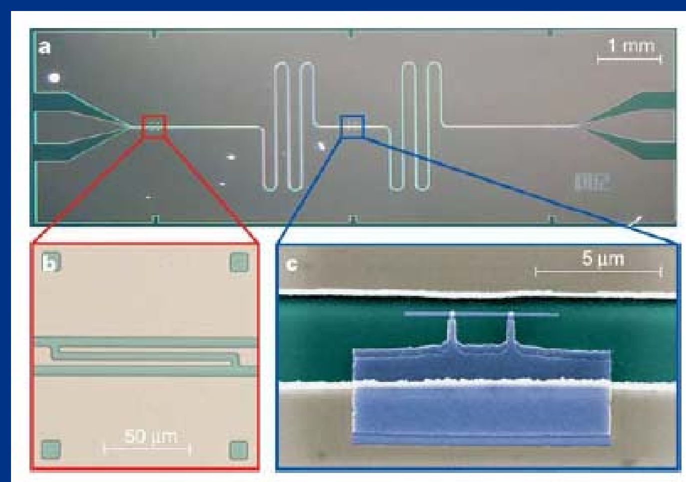
T_2=5 μs, Q=78400 (?)

Charge Qubit in a Cavity

Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics

A. Wallraff[1], D. I. Schuster[1], A. Blais[1], L. Frunzio[1], R.- S. Huang[1,2], J. Majer[1], S. Kumar[1], S. M. Girvin[1] & R. J. Schoelkopf[1]





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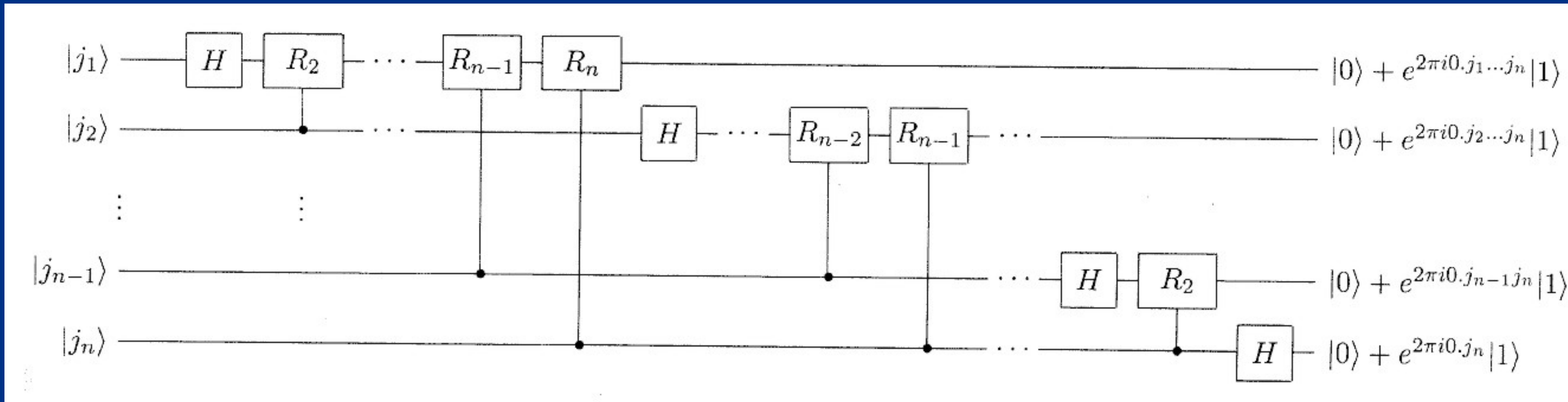
T_2=500 ns, Q=6500

MC2


$|j_1\rangle$
H
R_2
R_{n-1}
R_n
$|0\rangle + e^{2\pi i 0.j_1 \ldots j_n}|1\rangle$
$|j_2\rangle$
H
R_{n-2}
R_{n-1}
$|0\rangle + e^{2\pi i 0.j_2 \ldots j_n}|1\rangle$
$|j_{n-1}\rangle$
H
R_2
$|0\rangle + e^{2\pi i 0.j_{n-1} j_n}|1\rangle$
$|j_n\rangle$
H
$|0\rangle + e^{2\pi i 0.j_n}|1\rangle$