Dynamical Protection of Multipartite Entanglement from Decoherence:

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Sponsors: EU, ISF

Synopsis

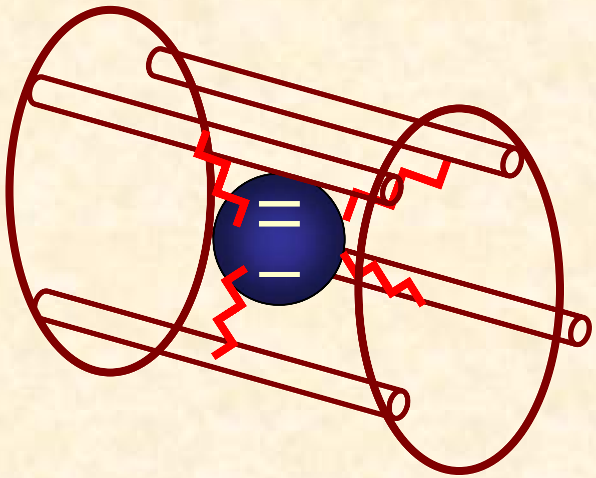
- Multipartite decoherence causes disentanglement:
  - Error correction: high overhead & threshold
  - Dynamical decoupling [Zanardi, Lidar, Viola] : All particles must be subject to very strong and fast pulses, averaging out environment
  - Decoherence-free subspace (DFS): formation impeded by asymmetric coupling to a bath.
- We propose universal dynamical control for suppressing multipartite decoherence: relaxed requirements
- It may allow high-fidelity gate & storage operations in realistic scenarios without error correction

Single Particle Decoherence

1) Finite T + noisy fields
Ion trap

Keller et al.
Nature **431**,
1075 (2004)

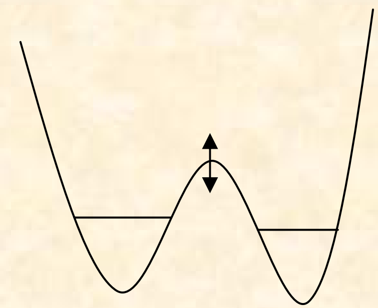
Häffner et al.
Nature **438**
643 (2005)



3) Josephson Junction analogs

Barone, Kofman ,GK, PRL (2004)

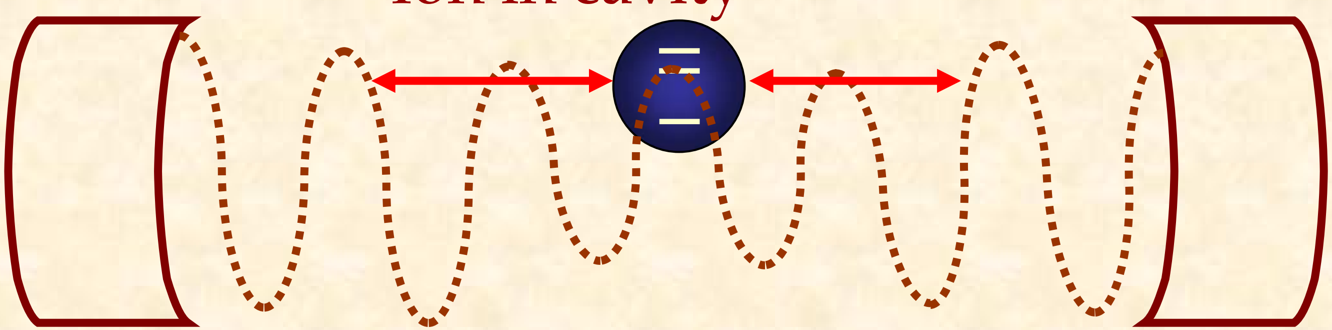
4) BEC in double-well potential



Schumm et al.
Nature Physics
1, 57 (2005)

2) Finite T + radiative decay
Ion in cavity

Kreuter et al.
PRL **92**
203002 (2004)



Control of Single Particle Decoherence

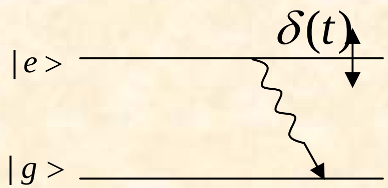
A. G. Kofman and G. Kurizki,
Nature **405**, 546 (2000), PRL **87**, 270405 (2001)
Facchi & Pascazio, Prog. In Opt. 42, 147 (2001)

A. G. Kofman and G. Kurizki PRL **93**,130406(2004)

Amplitude Noise (AN): population change

Off-diagonal $H_{\text{int}} = \hat{B}\hat{\sigma}_x$

Off-resonant modulation (AC Stark shifts)

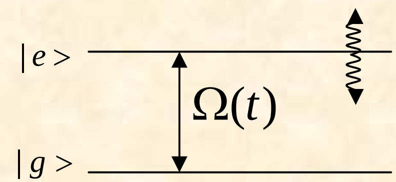


$$\varepsilon(t) = e^{i\int_0^t \delta(\tau)d\tau}$$

Phase noise (PN): dephasing

Diagonal $H_{\text{int}} = \hat{B}\hat{\sigma}_z$

Resonant modulation (flipping)



Decay rate: (T=0)

$$R(t)=2\pi\int_{-\infty}^{\infty} d\omega\, G_0(\omega)F_t(\omega-\omega_a)$$

Dephasing rate:

$$R(t)=2\pi\int_{-\infty}^{\infty} d\omega\, G_r(\omega)F_t(\omega-\omega_a)$$

Reservoir coupling spectrum

$$G_0(\omega)\to\rho(\omega)|\mu(\omega)|^2$$

Spectral density of dephasing

$$G_r(\omega)=(2\pi)^{-1}\int d\omega\, \underbrace{\overline{\delta_r(t)\delta_r(0)}}_{\Phi(t)}\, e^{i\omega t}$$

Spectral intensity of modulation

$$F_t(\omega)=|\varepsilon_t(\omega)|^2$$

Spectral intensity of flipping

$$F_t(\omega)=|\varepsilon_t(\omega)|^2$$

Control of Single Particle Decoherence

A. G. Kofman and G. Kurizki, Nature **405**, 546 (2000), PRL **87**, 270405 (2001), PRL **93**,130406(2004)

AN + Off-resonant modulation

Bath coupling spectrum

e.g. Cutoff

$$G(\omega) = \frac{\sqrt{\omega}}{\omega + \Gamma}\theta(\omega - \omega_0)$$

Impulsive phase modulation

Repetitive Weak Pulses

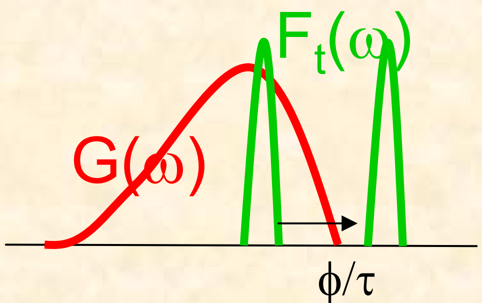
$\varepsilon(t)=e^{i[t/\tau]\phi}$, $\phi \ll \pi$

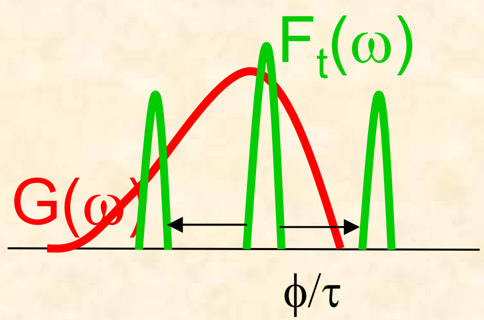
single shifted peak

Repetitive π Pulses

$\varepsilon(t)=e^{i[t/\tau]\pi}$

two symmetric peaks





PN + Resonant modulation

Dephasing spectrum

e.g. Lorentzian

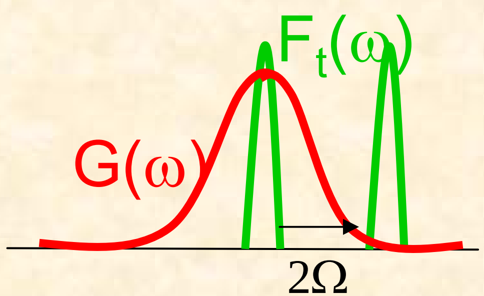
$$\Phi(\tau) = \frac{\gamma}{t_c}\exp(-\tau / t_c)$$

$$G(\omega) = \gamma \frac{1/t_c^{\,2}}{1/t_c^{\,2} + \omega^2}$$

Modulation

$$\varepsilon(t) = e^{2i\int_0^t d\tau \Omega(\tau)}$$

CW: $F_t(\omega) = \delta(\omega - 2\Omega)$



Dynamic decoupling.

Viola & Lloyd PRA **58** 2733 (1998)

Shiokawa & Lidar PRA **69** 030302(R) (2004)

Vitali & Tombesi PRA **65** 012305 (2001)

Agarwal, Scully, Walther PRA **63**, 044101 (2001)

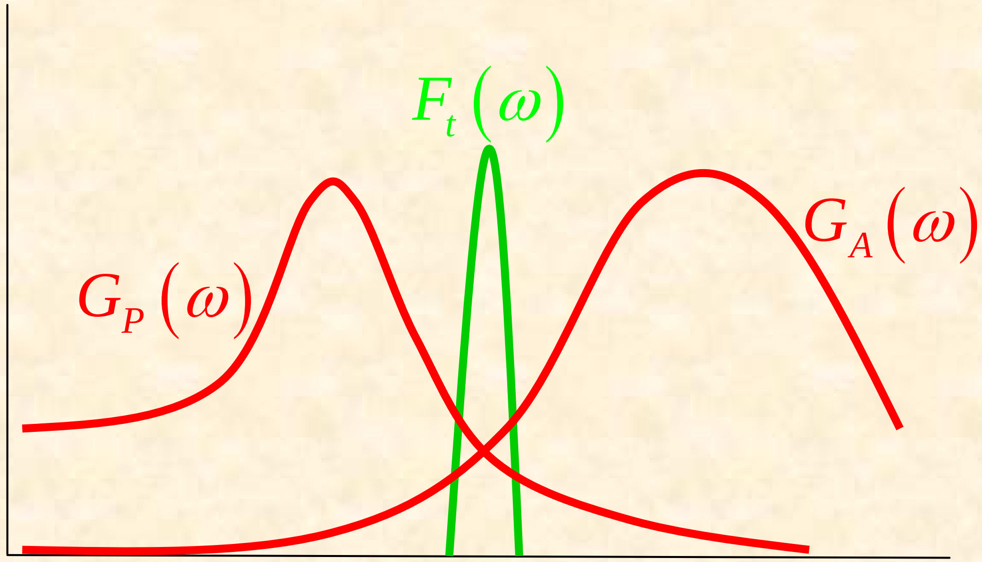
Combined Amplitude & Phase Noise

Gordon, Kofman & GK

Exact PN, 2nd order AN

Double convolution,
Off-resonant modulation:

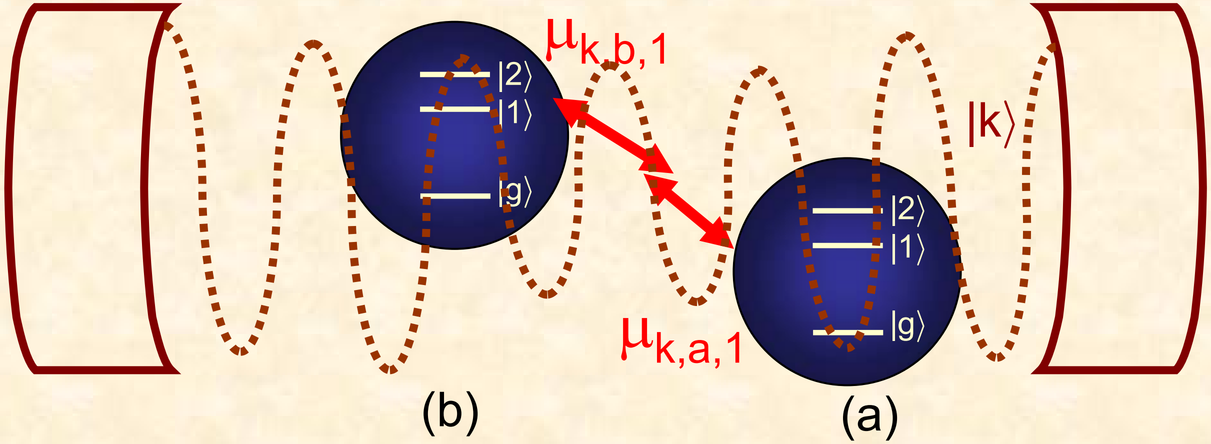
$$R(t) = 2\pi \int d\omega \int d\omega' G_A(\omega) G_P(\omega') F_t(\omega - \omega')$$



No decoupling for finite shifts!

Realistic Scenario: Asymmetric Particle-Bath coupling

Gordon, Kofman GK Opt. Comm. 264, 398 (2006); quant-ph/0608175
Gordon & Kurizki, PRL 97, 110503 (2006), , quant-ph/0608174



- Particles:
  - **Ions**
  - Cold atoms
- Bath:
  - **Cavity modes**
  - Vibrational modes
- Bath-particle coupling: Position dependent. (& cross-decoherence)
- Can we restore symmetric coupling

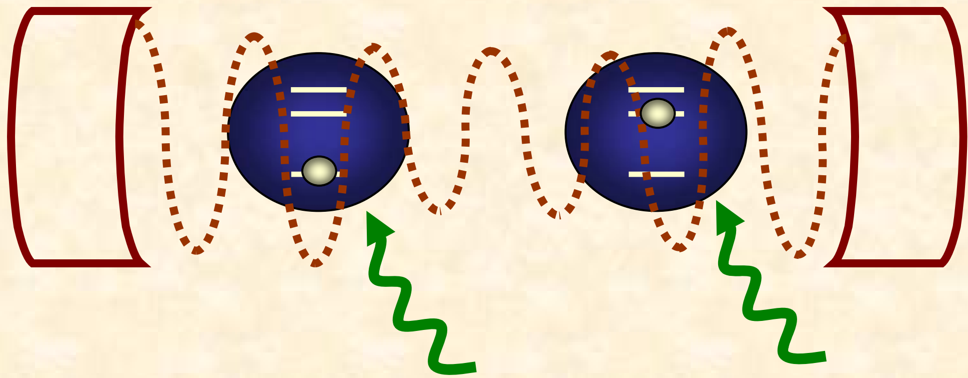
➔Decoherence free subspace?

Viola et al. PRL **85**, 3520 (2000);
Wu & Lidar, PRL **88**, 207902 (2002)
DD assumptions: fast, strong, frequent

Dynamically Modified Decoherence Matrix Elements

Gordon, Kofman GK Opt. Comm. 264, 398 (2006); quant-ph/0608175
Gordon & Kurizki, PRL 97, 110503 (2006), , quant-ph/0608174

$$\begin{pmatrix} \alpha_{a,1}(t) \\ \alpha_{a,2}(t) \\ \alpha_{b,1}(t) \\ \alpha_{b,2}(t) \end{pmatrix} = T_+ e^{-\begin{pmatrix} J_{aa,11} & J_{aa,12} & J_{ab,11} & J_{ab,12} \\ J_{aa,21} & J_{aa,22} & J_{ab,21} & J_{ab,22} \\ J_{ba,11} & J_{ba,12} & J_{bb,11} & J_{bb,12} \\ J_{ba,21} & J_{ba,22} & J_{bb,21} & J_{bb,22} \end{pmatrix}} \begin{pmatrix} \alpha_{a,1}(0) \\ \alpha_{a,2}(0) \\ \alpha_{b,1}(0) \\ \alpha_{b,2}(0) \end{pmatrix}$$



Diagonal elements: Individual particle decoherence

$$J_{jj,nn}(t) = 2\pi \int_{-\infty}^{\infty} d\omega G_{jj,nn}(\omega) |\varepsilon_{t,j,n}(\omega - \omega_{j,n})|^2$$

Dynamically Modified Decoherence Matrix Elements

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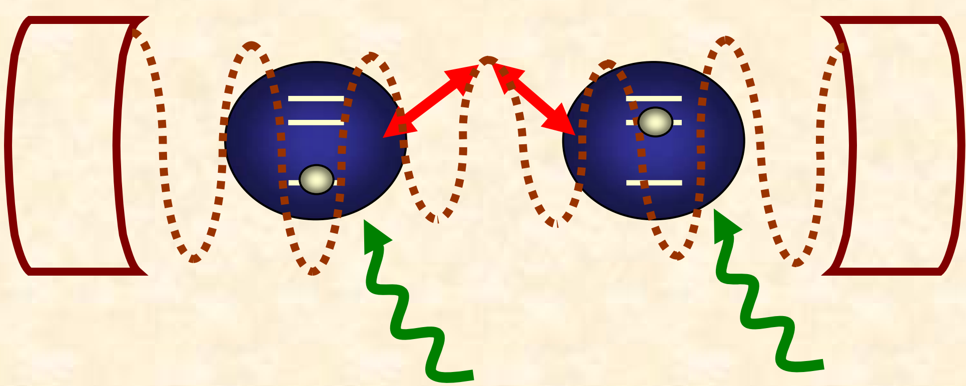
$$\begin{pmatrix} \alpha_{a,1}(t) \\ \alpha_{a,2}(t) \\ \alpha_{b,1}(t) \\ \alpha_{b,2}(t) \end{pmatrix} = T_+ e^{-\begin{pmatrix} J_{aa,11} & J_{aa,12} & J_{ab,11} & J_{ab,12} \\ J_{aa,21} & J_{aa,22} & J_{ab,21} & J_{ab,22} \\ J_{ba,11} & J_{ba,12} & J_{bb,11} & J_{bb,12} \\ J_{ba,21} & J_{ba,22} & J_{bb,21} & J_{bb,22} \end{pmatrix}} \begin{pmatrix} \alpha_{a,1}(0) \\ \alpha_{a,2}(0) \\ \alpha_{b,1}(0) \\ \alpha_{b,2}(0) \end{pmatrix}$$



Off-diagonal elements: Cross-decoherence

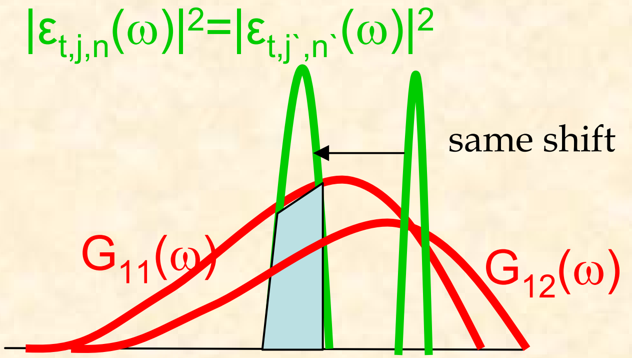
$$J_{jj',nn'}(t) = 2\pi \int_{-\infty}^{\infty} d\omega G_{jj',nn'}(\omega) \varepsilon^*_{t,j,n}(\omega - \omega_{j,n}) \varepsilon_{t,j',n'}(\omega - \omega_{j',n'})$$

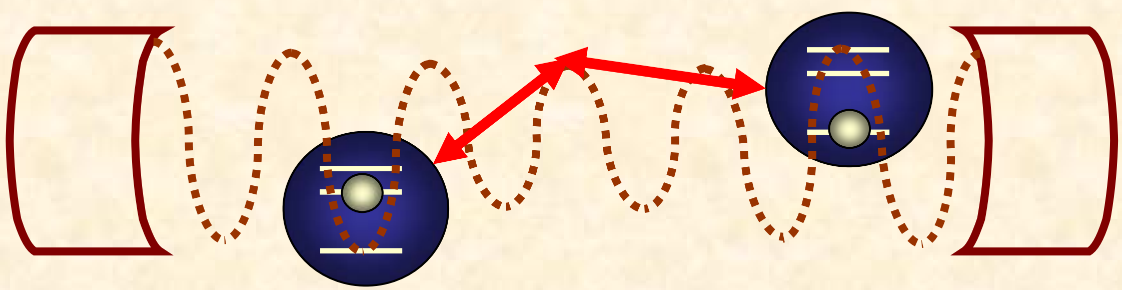
Distance dependent: how to control?

Global Modulation

$\varepsilon_{j,n}(t)=\varepsilon(t) \quad \forall\, j,n$

$J_{jj`,nn`}(t)=2\pi\int_{-\infty}^{\infty} d\omega\; G_{jj`,nn`}(\omega)\varepsilon^{*}_{t}(\omega-\omega_{j,n})\varepsilon_{t}(\omega-\omega_{j',n'})$





- N(N+1)/2 elements
  uncontrollable by single $\varepsilon_t(\omega)$!

- Cross-coupling,
  different couplings to bath: persist

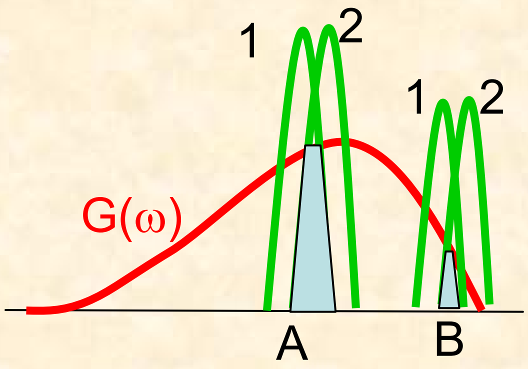
Independent Trapping by Local Modulation

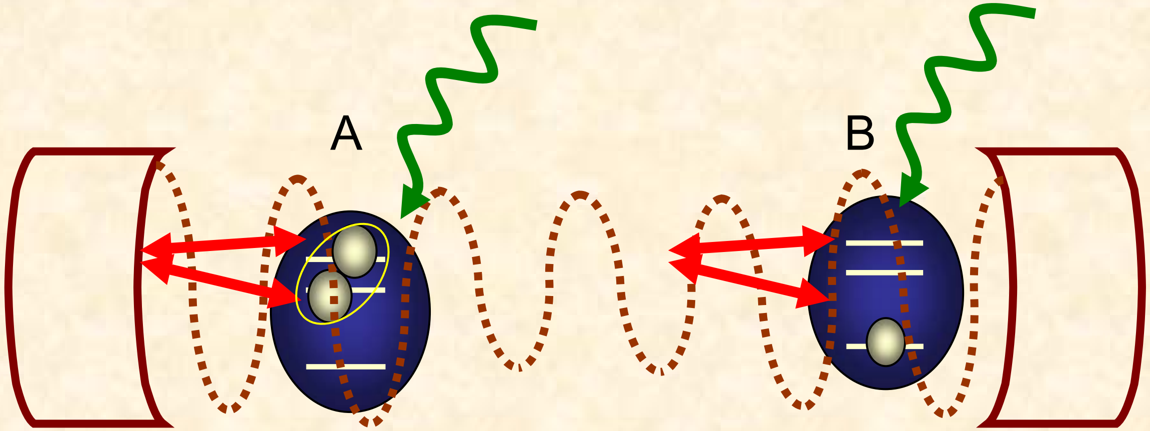
Gordon & Kurizki, PRL 97, 110503 (2006), , quant-ph/0608174

- For N three-level particles
- Anti-symmetric state of each particle $|-\rangle_j = 1/\sqrt{2}(|1\rangle_j - |2\rangle_j)$
  = Intra-particle Decoherence Free Subspace
- **Equating intraparticle** decoherence rates (impose des. interf.)
- **Eliminating interparticle** cross-decoherence

modulation need not lead to decoupling

(works even for SE – broad G)

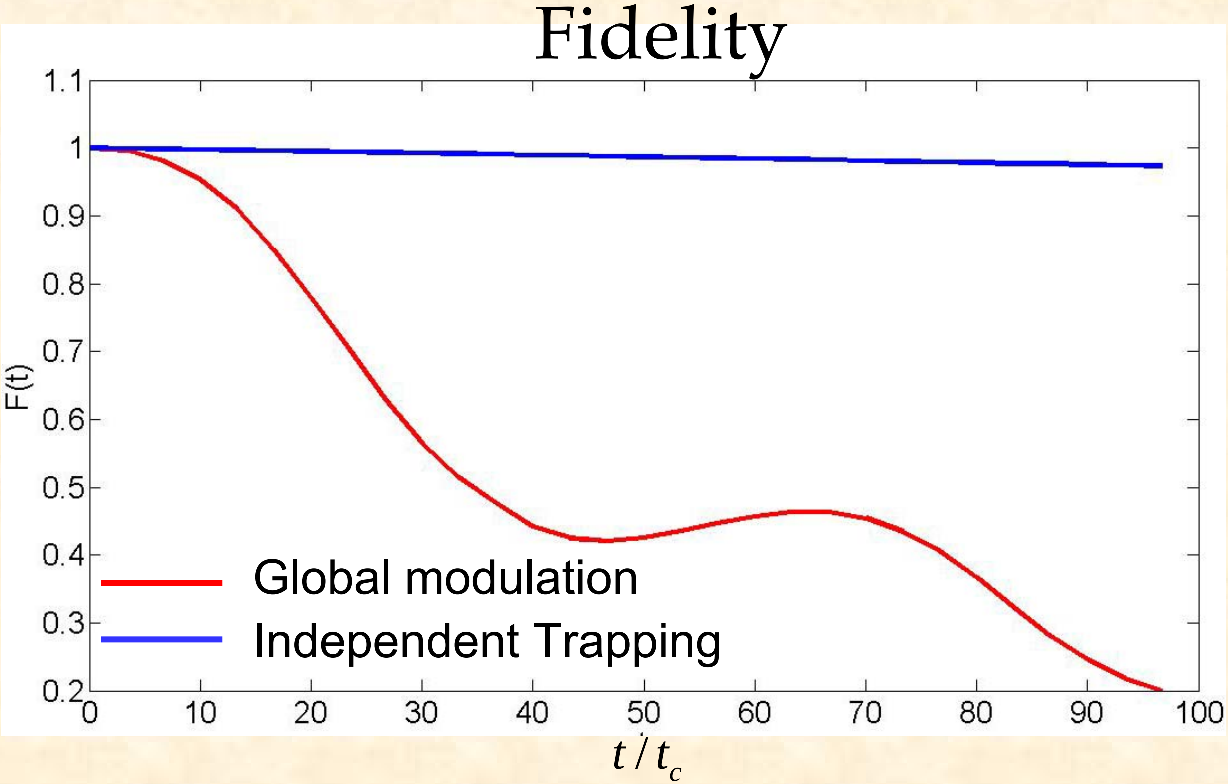


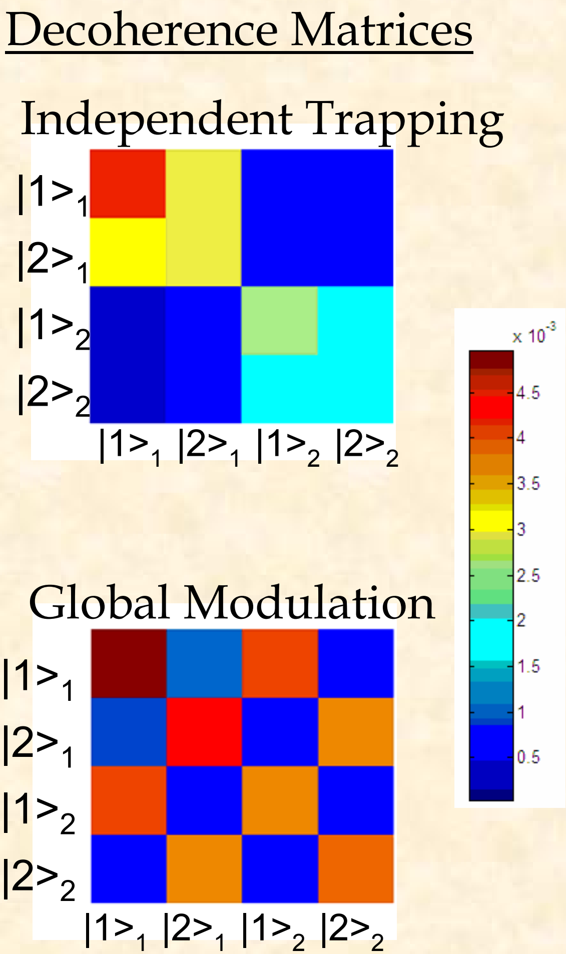


N decoherence-free qubits

Numerical Example: Independent Trapping

Godon & Kurizki, PRL 97, 110503 (2006), , quant-ph/0608174





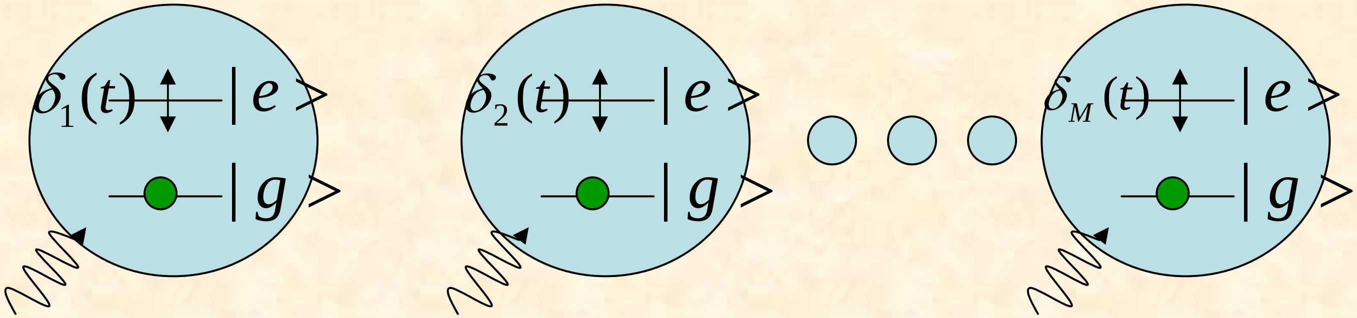
$|\Psi(0)\rangle = 1/\sqrt{2}(|-\rangle_a|g\rangle_b \pm |g\rangle_a|-\rangle_b)$

$|-\rangle_j = 1/\sqrt{2}(|1\rangle - |2\rangle)_j$

Singly excited N-particle MES may be always protected by 2N local fields !

Multi-qubit dephasing control

Gordon et al. Opt. Comm. **264**, 398 (2006), quant-ph/0608175



Not an ensemble of independent qubits

$\Omega_1(t)e^{i\omega_0 t}$ $\Omega_2(t)e^{i\omega_0 t}$ $\Omega_M(t)e^{i\omega_0 t}$

Each qubit undergoes (different) proper dephasing

Each qubit independently driven (addressability)

Cross-correlation Function

$$\overline{\delta_i(t)\delta_j(t-\tau)} = \Phi_{ij}(\tau)$$

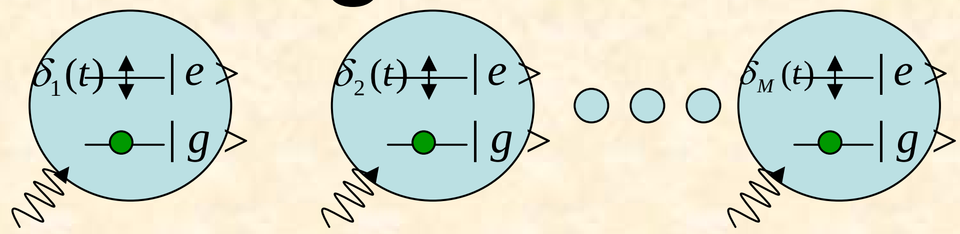
$$\varepsilon_i(t) = \exp\left(2i\int_0^t dt'\Omega_i(t')\right)$$

$$\Phi_{ij}(\tau) = \frac{2\gamma}{t_{c,i}+t_{c,j}} \frac{\exp\left(-\tau/2t_{c,i}-\tau/2t_{c,j}\right)}{1+|r_i-r_j|/r_c}$$

→ Depends on distance between qubits

Multi-qubit dephasing control

Gordon et al. Opt. Comm. **264**, 398 (2006), quant-ph/0608175



$\Omega_1(t)e^{i\omega_0 t}$ $\Omega_2(t)e^{i\omega_0 t}$ $\Omega_M(t)e^{i\omega_0 t}$

For each qubit: $|g>,|e>\Rightarrow|\pm>$

Ensemble average, density matrix:

$$\overline{\rho}(t)=\rho(0)-\frac{1}{4}\int_0^t dt'\int_0^{t'} dt''\overline{\left[W_{off}(t'),\left[W_{off}(t''),\rho(0)\right]\right]}$$

$W_{off}(t)$

2^M x 2^M matrix

n^{th} qubit-flip → $\delta_n(t)\varepsilon_n(t)$

Example: <u>Two qubits</u>

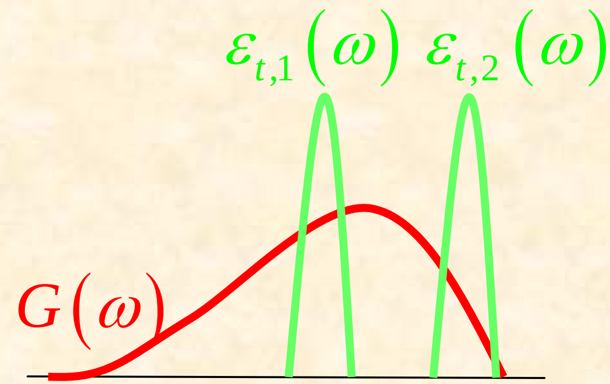
$$\begin{array}{c} \\ |--> \\ |-+> \\ |+-> \\ |++> \end{array}\begin{array}{cccc} |--> & |-+> & |+-> & |++> \\ \end{array}$$

$$\begin{array}{c} |--> \\ |-+> \\ |+-> \\ |++> \end{array}\begin{pmatrix} 0 & \delta_2\varepsilon_2 & \delta_1\varepsilon_1 & 0 \\ \delta_2\varepsilon_2^* & 0 & 0 & \delta_1\varepsilon_1 \\ \delta_1\varepsilon_1^* & 0 & 0 & \delta_2\varepsilon_2 \\ 0 & \delta_1\varepsilon_1^* & \delta_2\varepsilon_2^* & 0 \end{pmatrix}$$

Two-qubit dephasing control

Gordon et al. Opt. Comm. **264**, 398 (2006),
quant-ph/0608175

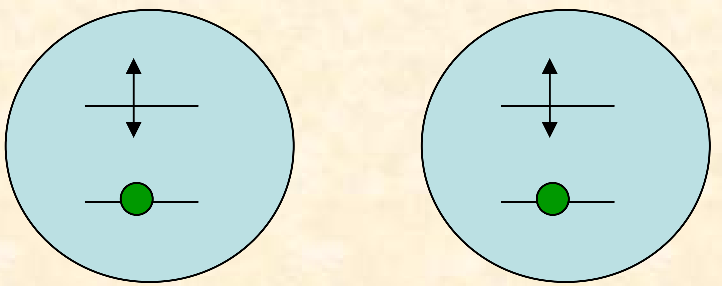
Pairwise DFS: M fields suffice



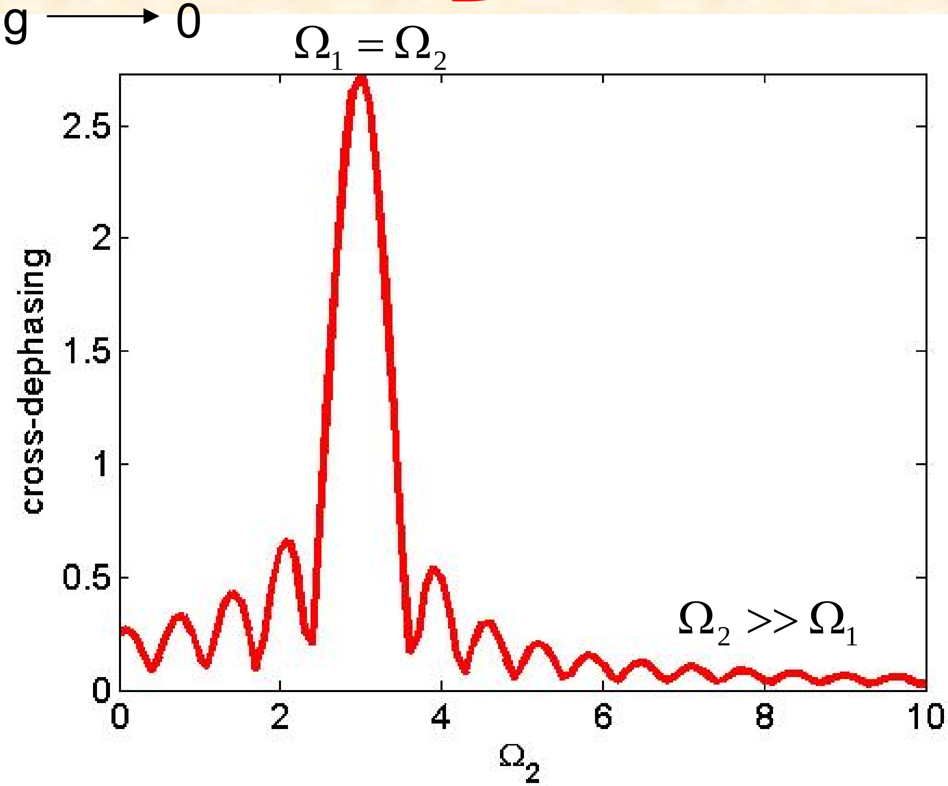
$$J_{jj'}(t) = \int d\omega G_{jj'}(\omega)\,\varepsilon_{t,j}(\omega)\,\varepsilon_{t,j'}(\omega)$$

For different local modulations

$\varepsilon_1(t) \neq \varepsilon_2(t)$ Cross-dephasing $\longrightarrow$ 0



Individual local modulations decouple the qubits (eliminate cross dephasing)



Two-qubit dephasing control

Gordon et al. Opt. Comm. **264**, 398 (2006),
quant-ph/0608175

Initial entangled state:

$$|\psi_{\pm}(0)>=\frac{1}{\sqrt{2}}\left(|ge>\pm|eg>\right)$$ triplet\singlet

Fidelity

$$F_{\pm}(t)=1-\frac{1}{2}\left[\underbrace{\left(J_{11}(t)+J_{22}(t)\right)}_{\text{Dephasing of Individual qubits}}\pm\underbrace{\left(J_{12}(t)+J_{21}(t)\right)}_{\text{Cross dephasing}}\right]$$

Dephasing of
Individual qubits

Cross dephasing

Modulation Scheme 1:

Equate dephasing and cross-dephasing rates $J_{jj'}(t)=J(t)$

$F_{-}(t)=1$ ➔**Singlet**
Decoherence free subspace (DFS)

Viola et al. PRL **85**, 3520 (2000);
Wu & Lidar, PRL **88**, 207902 (2002)

Modulation Scheme 2:

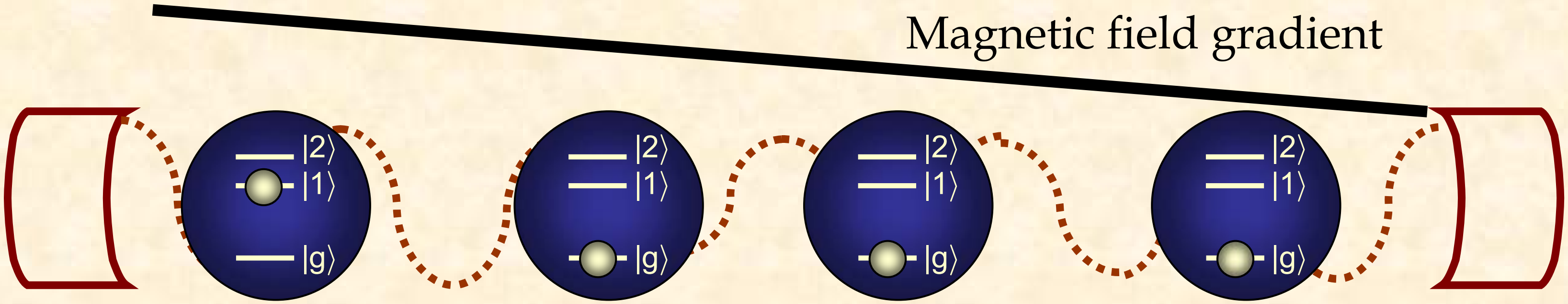
Eliminate cross-dephasing $J_{jj'}(t)=0 \quad j\neq j'$

$F_{\pm}(t)=F(t)$ ➔Equal fidelity for all Bell-states

Advantageous in quantum communications

Suggested Experimental Setup: Multiple $^{40}Ca^+$ Ions in Cavity

Kreuter et al. PRL **92** 203002 (2004);Barton et al. PRA **62** 032503 (2000)
D. Schrader et al., Phys. Rev. Lett. **93**, 150501 (2004).



Spatial addressability → Spectral addressability
N=10 ions → 10 control fields
50 gate fields

Three level system:

$|g\rangle = 4^2S_{1/2}$
$|1\rangle = 3^2D_{3/2}$
$|2\rangle = 3^2D_{5/2}$
$\tau_1/\tau_2 = 1.026$

Multiple ions in cavity:

Position in cavity: τ_a-$\tau_b \approx 15\%$

Two ions in each cavity
+ Local modulations
$1/\sqrt{2}(|g\rangle_a|1\rangle_b - |1\rangle_a|g\rangle_b)$ = DFS

Conclusions

- Universal approach to dynamical decoherence control allows to guess the optimal control fields for different (amplitude and phase) noises, in multipartite systems
- Local modulations (spectral addressability) can
  - Create a multipartite decoherence-free system despite asymmetric and/or cross-couplings to a bath
  - Enhance gate fidelity under local dephasing by specific control fields for each qubit pair
- Tradeoff: ancilla → 2N or $N^2/2$ control fields

Know thine enemy