

Quantum Decoherence and Gravitational Waves

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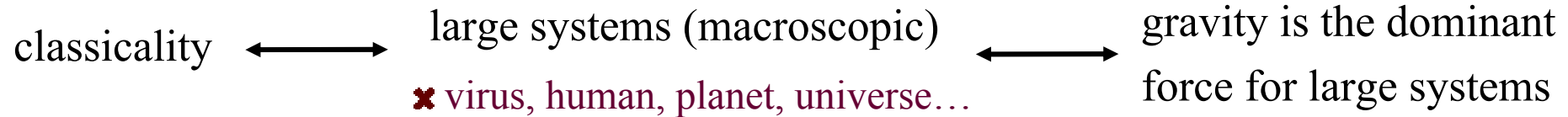
Informations available on
<http://www.spectro.jussieu.fr/Vacuum>

→ **Quantum fluctuations and relativity**

PhD thesis (in French)

<http://tel.ccsd.cnrs.fr>

Gravitation and Decoherence



FEYNMAN :
 appearance of classicality due to gravitation

R.P. Feynman, *Feynman Lectures on Gravitation* (1963)


Qualitative argument of Feynman based on Planck scales :

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \simeq 5 \times 10^{-44} \text{ s} \quad l_P = \sqrt{\frac{\hbar G}{c^3}} \simeq 10^{-35} \text{ m} \quad m_P = \sqrt{\frac{\hbar c}{G}} \simeq 22 \mu\text{g}$$

✖ The Planck mass is accessible !

Gravitation leads to a decoherence effect for $m \geq m_P \iff \ell_C = \frac{\hbar}{mc} \leq l_P$

Gravitational decoherence scenarios

 Penrose mechanism : quantum state reduction.

- ✘ A quantum superposition inherently possess a time uncertainty due spacetime curvature which in return leads to an energy uncertainty ΔE .
- ✘ ΔE reduces to the Newtonian self energy of the superposition.
- ✘ Decoherence time $\tau = \hbar/\Delta E$.

R. Penrose, Gen. Rel. Grav. **28** (1996), 581

 GEID (Gravitational Environnement Induced Decoherence)

- ✘ A quantum superposition interacts with a fluctuating gravitational environnement.
- ✘ Many gravitational fluctuations have been studied → I focus on gravitational waves.
- ✘ Use of Feynman-Vernon functional influence.

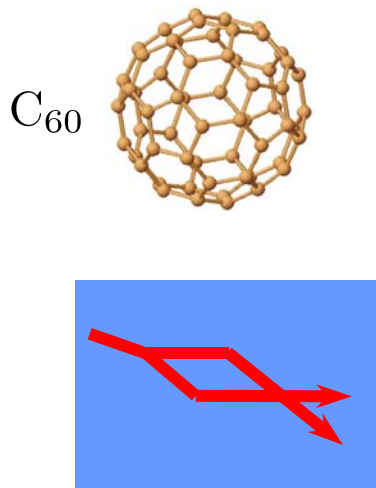
Gravity induced quantum/classical transition

Quantum Domain

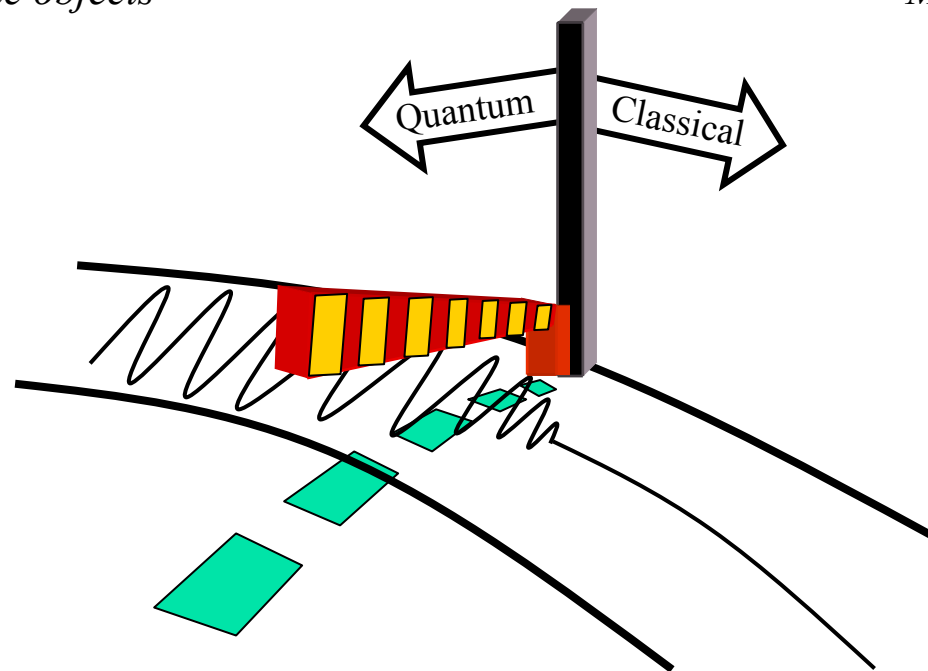
Classical Domain

Microscopic objects

Macroscopic objects



interferences



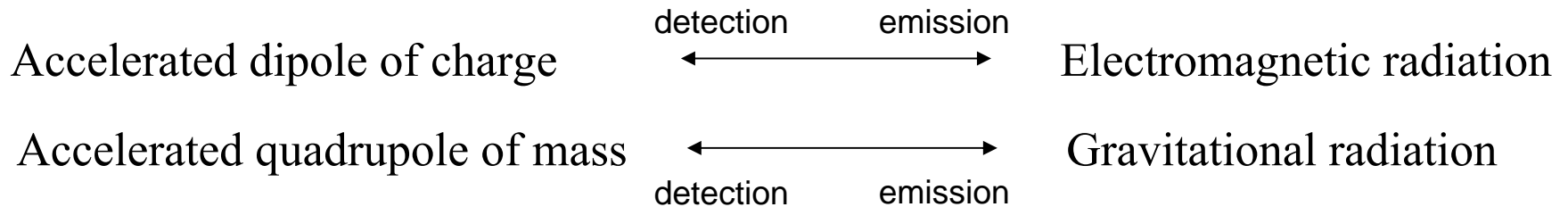
No interferences

- ✘ Where is the borderline ?
- ✘ Is it observable ?

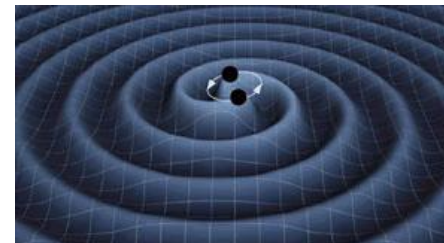
The gravitational environment

Gravitational waves (GW) are freely propagating solutions of linearized Einstein equations (Einstein 1918).

✘ Analog to electromagnetic radiation.



Effect on freely falling particles



- ✘ GW propagate at the speed of light.**
- ✘ GW have 2 transverse polarizations.**
- ✘ Perturbation of the metric**

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad h_{\mu\nu} \ll 1$$

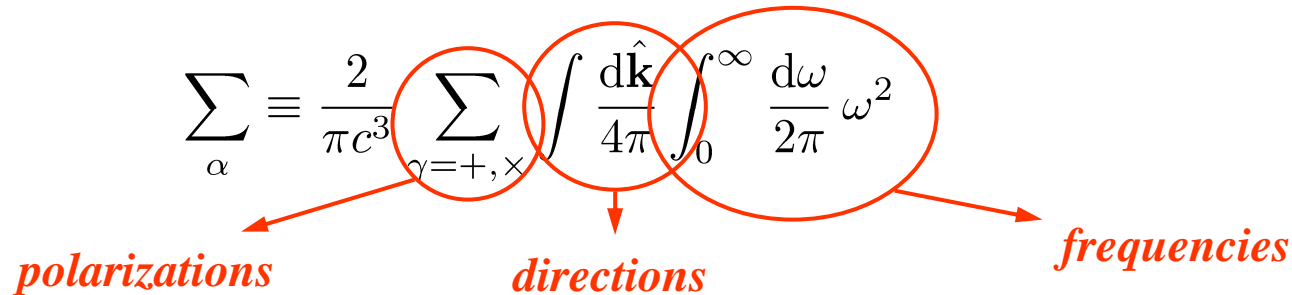
Gravitational waves background

GW backgrounds ! collection of harmonic oscillators

$$h_{ij}(t) = \sum_{\alpha} \lambda_{\alpha,ij} h_{\alpha}(t)$$

$$S_{\mathcal{E}} = \frac{c^2}{32\pi G} \sum_{\alpha} \int_{t_i}^{t_f} \frac{1}{2} \left(\dot{h}_{\alpha}^2(t) + \omega^2 h_{\alpha}^2(t) \right) dt$$

\times h_{α} are normal variables of the field and can be quantized ! \hat{h}_{ij}




Classical metric perturbations are characterized by a noise spectrum $S_h[\omega]$:

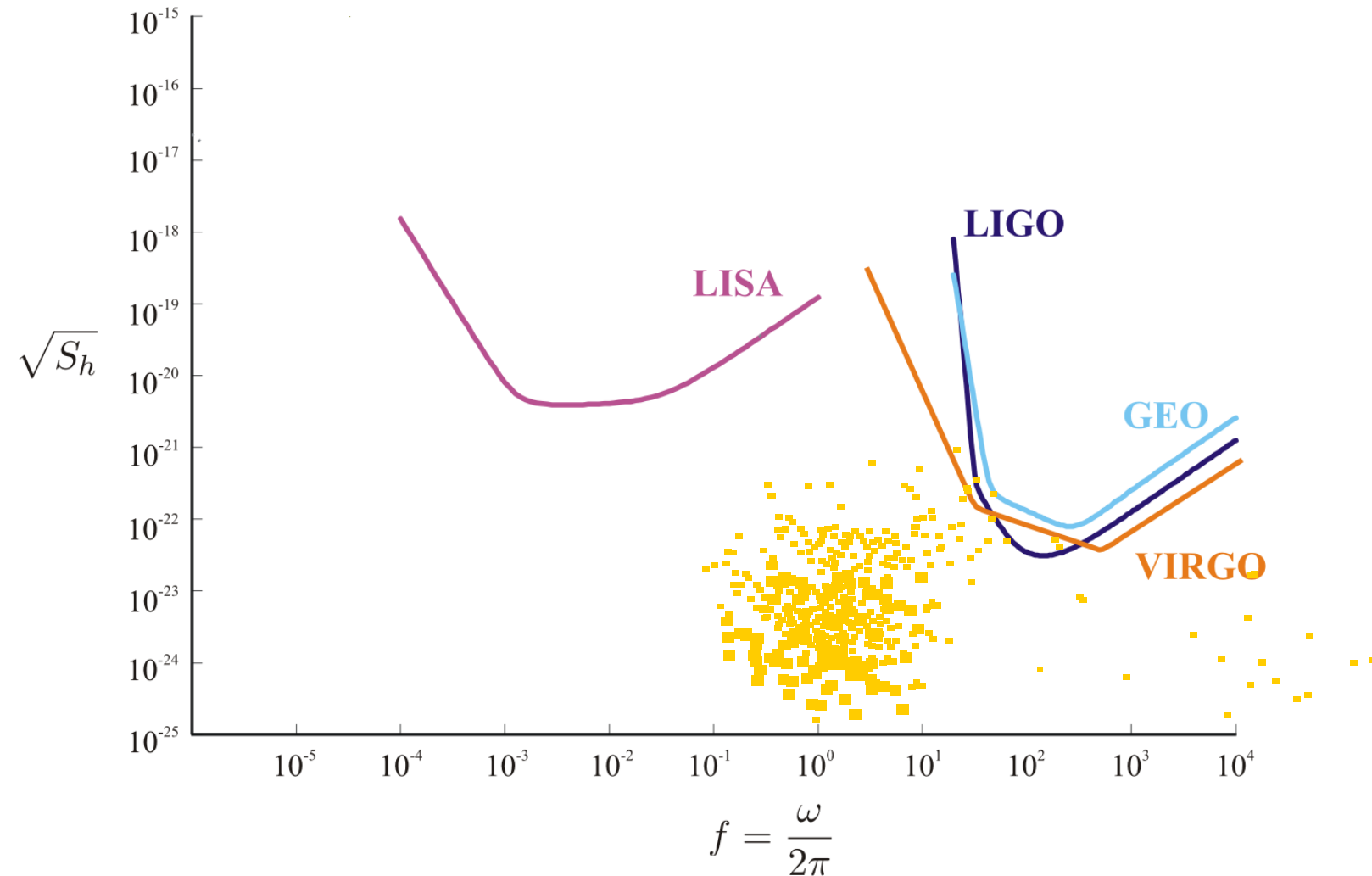
$$\langle h_{ij}(t) h_{kl}(0) \rangle = \delta_{ijkl} \int \frac{d\omega}{2\pi} S_h[\omega] e^{-i\omega t}$$

$$S_h[\omega] = \frac{16G}{5c^5} k_B T_{\text{gw}}[\omega] = \frac{16G}{5c^5} n_{\text{gw}}[\omega] \hbar \omega$$

- \times no blackbody for gravitational radiation
-) frequency dependent noise temperature
- \times GW detectors need $n_{\text{gw}} \sim 10^{31}$ gravitons at 1kHz

Gravitational waves background

 Resolved binaries pulsars



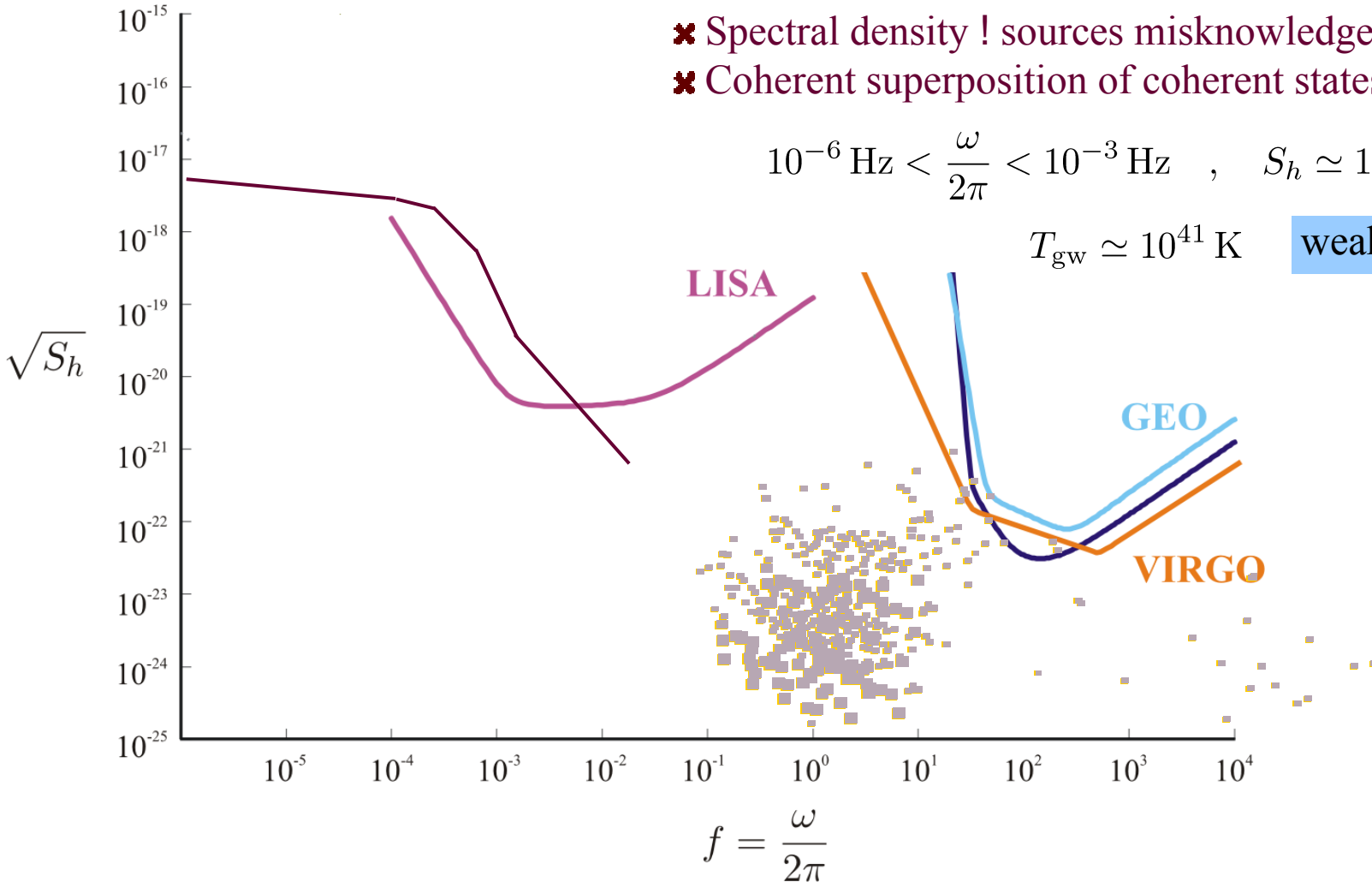
Gravitational waves background

Binary confusion background

- ✖ Spectral density ! sources misknowledge.
- ✖ Coherent superposition of coherent states.

$$10^{-6} \text{ Hz} < \frac{\omega}{2\pi} < 10^{-3} \text{ Hz} \quad , \quad S_h \simeq 10^{-34} \text{ s}^{-1}$$

$$T_{\text{gw}} \simeq 10^{41} \text{ K} \quad \text{weak coupling}$$



Gravitational waves background

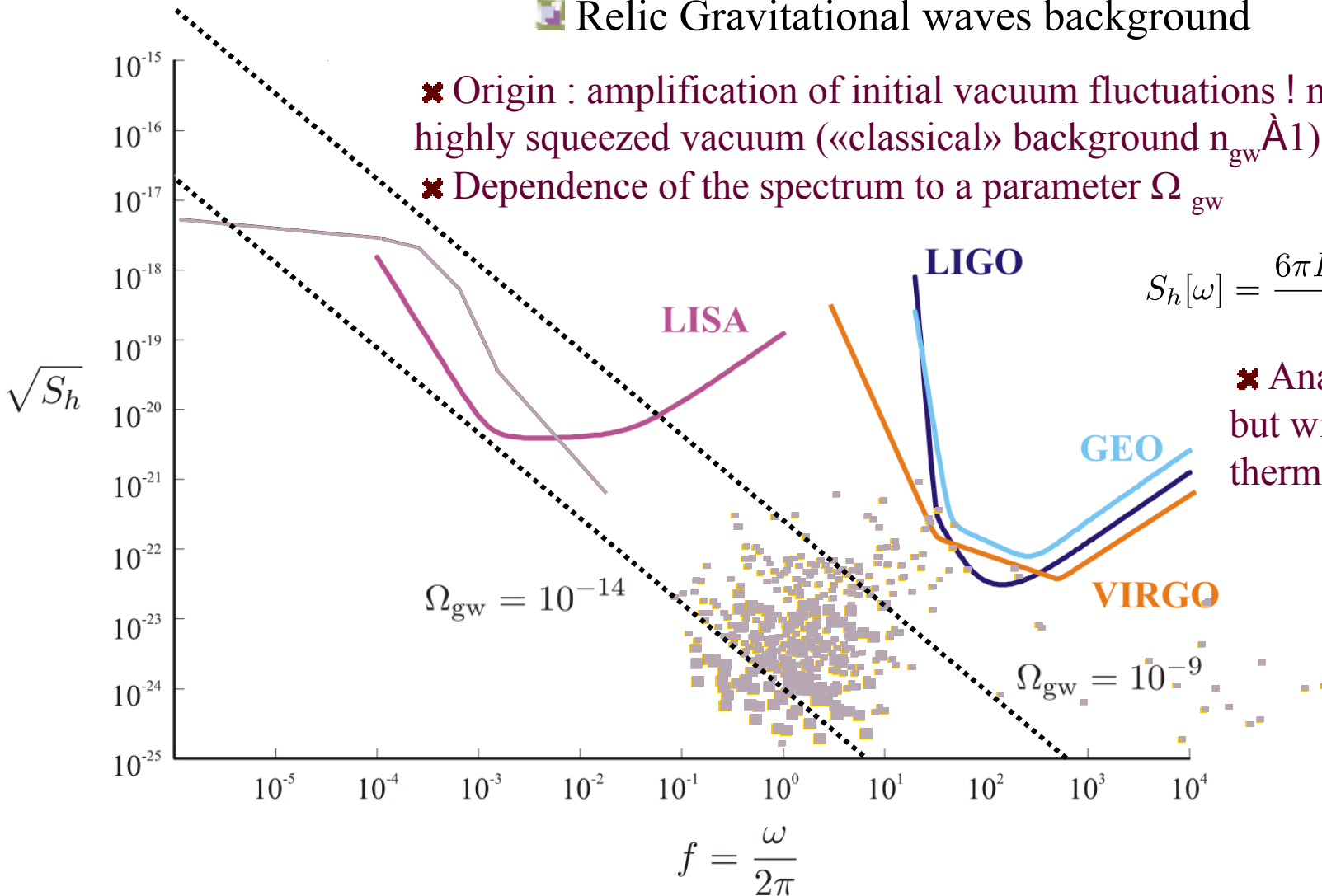
Relic Gravitational waves background

✘ Origin : amplification of initial vacuum fluctuations ! nowadays a highly squeezed vacuum («classical» background $n_{\text{gw}} \sim 1$).

✘ Dependence of the spectrum to a parameter Ω_{gw}

$$S_h[\omega] = \frac{6\pi H_0^2 \Omega_{\text{gw}}}{\omega^3}$$

✘ Analog to CMB but with a non thermal spectrum.



Coupling to gravitational waves

Linear coupling of matter to gravitation : $\mathcal{L}_{S/\varepsilon} = \frac{1}{2} h_{\mu\nu} T^{\mu\nu}$

✘ $T^{\mu\nu}$ energy momentum tensor

Quadrupole coupling in the long wavelength approximation : $\lambda_{\text{gw}} \geq 3000 \text{ km}$

$$S_{S/\varepsilon} = \frac{1}{4} \int dt h_{ij}(t) \ddot{Q}^{ij}(t) = \frac{1}{4} \sum_{\alpha} \lambda_{\alpha,ij} \int dt h_{\alpha}(t) \ddot{Q}^{ij}(t)$$

$$Q^{ij}(t) = \frac{1}{c^2} \int d^3\vec{x} \left(x^i x^j - \frac{1}{3} \delta^{ij} x^k x_k \right) T^{00}(t, \vec{x})$$

✘ equivalent to the dipole approximation of electromagnetism

✘ expression in the TT gauge

Decoherence in interferometers

B. Lamine, *PhD Thesis (2004)*

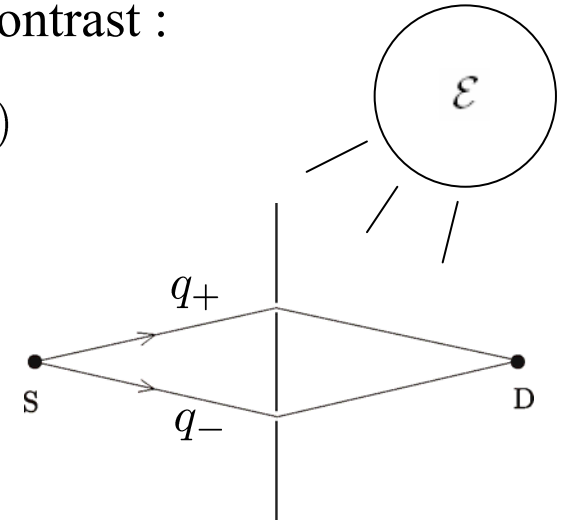
Coherence of an interferometer characterized by its contrast :

$$\mathcal{I} = |\mathcal{A}_+ + \mathcal{A}_-|^2 = |\mathcal{A}_+|^2 + |\mathcal{A}_-|^2 + 2\text{Re}(\mathcal{A}_+\mathcal{A}_-^*)$$

In presence of an environnement :

$$\mathcal{I} = |\mathcal{A}_+|^2 + |\mathcal{A}_-|^2 + 2\text{Re}(\mathcal{A}_+\mathcal{A}_-^* \mathcal{F}[q_+, q_-])$$

$$\langle \mathcal{E}_+ | \mathcal{E}_+ \rangle = \langle \mathcal{E}_- | \mathcal{E}_- \rangle = 1 \quad \mathcal{F}[q_+, q_-] = \langle \mathcal{E}_+ | \mathcal{E}_- \rangle$$



$\mathcal{F}[q_+(t), q_-(t)]$: **Feynman-Vernon influence functional** ! coherence between two paths $q_+(t)$ and $q_-(t)$.

After temporal averaging on the integration time :

$$\mathcal{V} = |\overline{\mathcal{F}[q_+, q_-]}| = |\overline{\langle \mathcal{E}_+ | \mathcal{E}_- \rangle}| \quad \times \text{decoherence observable.}$$

Influence functional

The influence functional can be written as an exponential of linear and two quadratic kernels:

$$\mathcal{F}[q_+, q_-] = \exp \left(\frac{i}{4\hbar} \int_{t_i}^{t_f} (\ddot{Q}^{ij}[q_+(t)] - \ddot{Q}^{ij}[q_-(t)]) \langle \hat{h}_{ij}(t) \rangle dt \right) \leftarrow i\Delta\varphi_{\text{lin}}$$

$$\times \exp \left(-\frac{1}{4\hbar} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} ds (\ddot{Q}^{ij}[q_+(t)] - \ddot{Q}^{ij}[q_-(t)]) \sigma_{ijkl}(t-s) (\ddot{Q}^{kl}[q_+(s)] - \ddot{Q}^{kl}[q_-(s)]) \right) \leftarrow -\frac{1}{2} \Delta\varphi_{\text{noise}}^2$$

$$\times \exp \left(-\frac{1}{4\hbar} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} ds (\ddot{Q}^{ij}[q_+(t)] + \ddot{Q}^{ij}[q_-(t)]) \xi_{ijkl}(t-s) (\ddot{Q}^{kl}[q_+(s)] - \ddot{Q}^{kl}[q_-(s)]) \right) \leftarrow -\frac{i}{2} \Delta\varphi_{\text{diss}}^2$$

With the following kernels :

$$\sigma_{ijkl}(\tau) = \frac{1}{4\hbar} \left(\langle \hat{h}_{ij}(\tau) \cdot \hat{h}_{kl}(0) \rangle - \langle \hat{h}_{ij}(\tau) \rangle \langle \hat{h}_{kl}(0) \rangle \right)$$

✖ Noise kernel.

$$\xi_{ijkl} = \frac{1}{4\hbar} \langle [\hat{h}_{ij}(\tau), \hat{h}_{kl}(0)] \rangle = i\delta_{ijkl} \int_0^\infty d\omega \frac{2G\omega}{5\pi c^5} \sin(\omega\tau)$$

✖ Dissipation kernel (imaginary)
(spontaneous emission of GW).

Gravitational decoherence

Double limit of the gravitational waves environment :

- ✘ low dissipation (weak coupling) : $\xi_{ijkl}(\tau) \simeq 0$
- ✘ high noise (large effective temperature) : $\sigma_{ijkl}(\tau)$ is a classical correlation function.

This limit is formally equivalent to $G \rightarrow 0$ and $T_{\text{gw}} \rightarrow 1$ with a finite product.

$$\mathcal{F}[q_+, q_-] = \exp \left(i\Delta\varphi_{\text{lin}} - \frac{1}{2}\Delta\varphi_{\text{noise}}^2 \right)$$

Relic gravitational waves :

$$\langle \hat{h}_{ij}(t) \rangle = 0 \quad \cancel{\Delta\varphi_{\text{lin}}}$$

- ✘ « Quantum » decoherence by a squeezed vacuum.

Binary confusion background : $\langle \hat{h}_{ij}(\tau) \cdot \hat{h}_{kl}(0) \rangle - \langle \hat{h}_{ij}(\tau) \rangle \langle \hat{h}_{kl}(0) \rangle \simeq 0 \quad \cancel{\Delta\varphi_{\text{noise}}^2}$

- ✘ « Classical » decoherence due to the fluctuations of the linear term when averaging over the integration time.

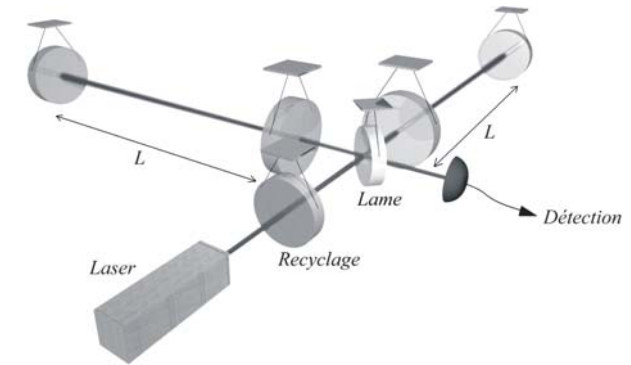
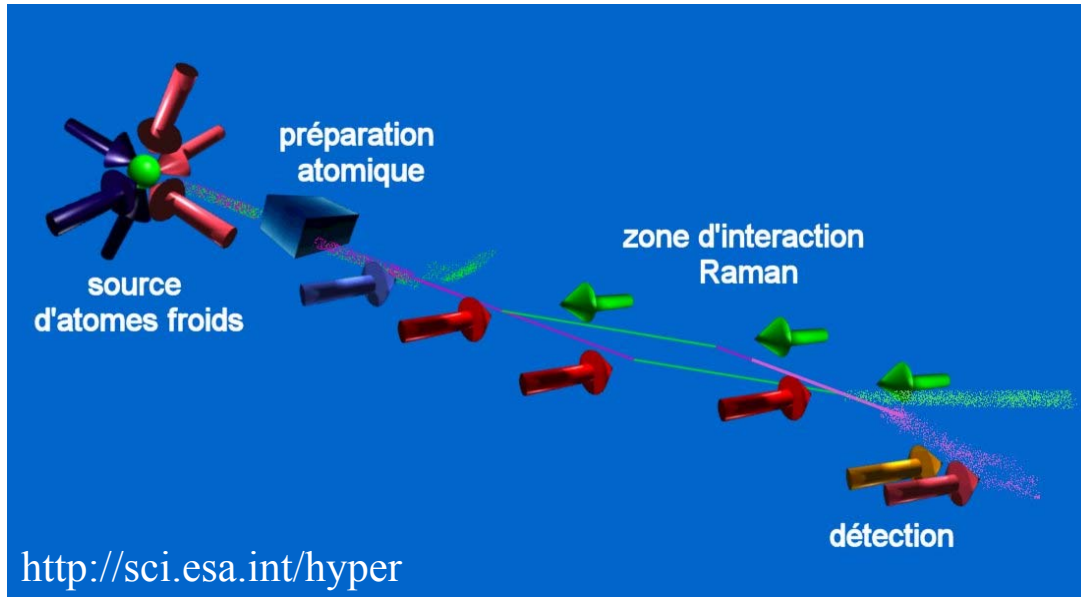
$$\mathcal{V} = \left| \overline{\mathcal{F}[q_+, q_-]} \right| = \exp \left(-\frac{1}{2} \Delta\varphi^2 \right)$$

$$\Delta\varphi^2 = \overline{\Delta\varphi_{\text{noise}}^2} + \overline{\Delta\varphi_{\text{lin}}^2}$$

Application to atomic interferometers

S. Reynaud & *al.*, Gen. Rel. Grav. **36** (2004), 2271

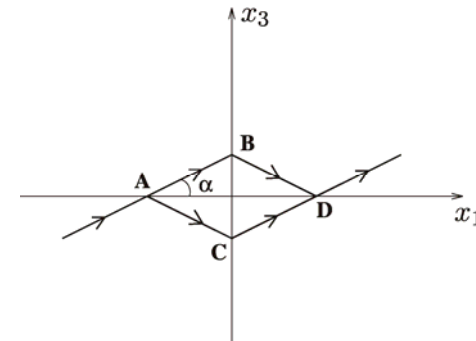
The lasers used for stimulated Raman transition provide nearly freely falling mirrors and beam splitters for atoms.



✘ It behaves as a GW detector.

Dephasing in the eikonal approximation

$$\Delta\varphi_{\text{lin}} = \frac{1}{2\hbar} \oint h_{\mu\nu} \frac{P^\mu P^\nu}{m} dt$$



Discussion of a simplified model

B. Lamine & *al*, Eur. Phys. J. **D20** (2002), 165

Assume a white noise model S_h

$[\omega] = C^{te}$ for the gravitational background

$$\Delta\varphi^2 = (2\Omega_{at} \sin(2\alpha))^2 S_h 2\tau_{at}$$

$$\Omega_{at} = \frac{m_{at} v_{at}^2}{2\hbar}$$

✘ Brownian diffusion of the phase.

✘ For photons, replace Ω_{at} by the frequency Ω_{phot} of photon.

Relevant parameters for the estimation of decoherence :

- ✘ kinetic energy (in comoving frame) of the probe (not its mass energy !),
- ✘ geometry (aperture $\sin(2\alpha)$),
- ✘ level of the noise spectrum (S_h),
- ✘ exposition time of the probe to the GW environment

the comparison of mass with Planck mass enters here, but only as one of the parameters

Discussion

B. Lamine & *al.*, Phys. Rev. Lett. **96** (2006), 050405

☞ All existing interferometers verify $\Delta\varphi^2 \ll 1$

☞ At present time, gravitational decoherence is unobservable :

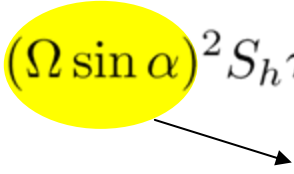
✘ too slow for microscopic probes,

✘ too fast for macroscopic probes (example of the center of mass motion of the Moon).

S. Reynaud & *al.*, Europhys. Lett. 53 (2001), 135

☞ Would it be possible to design experiments allowing one to explore the transition on « mesoscopic » objects ?

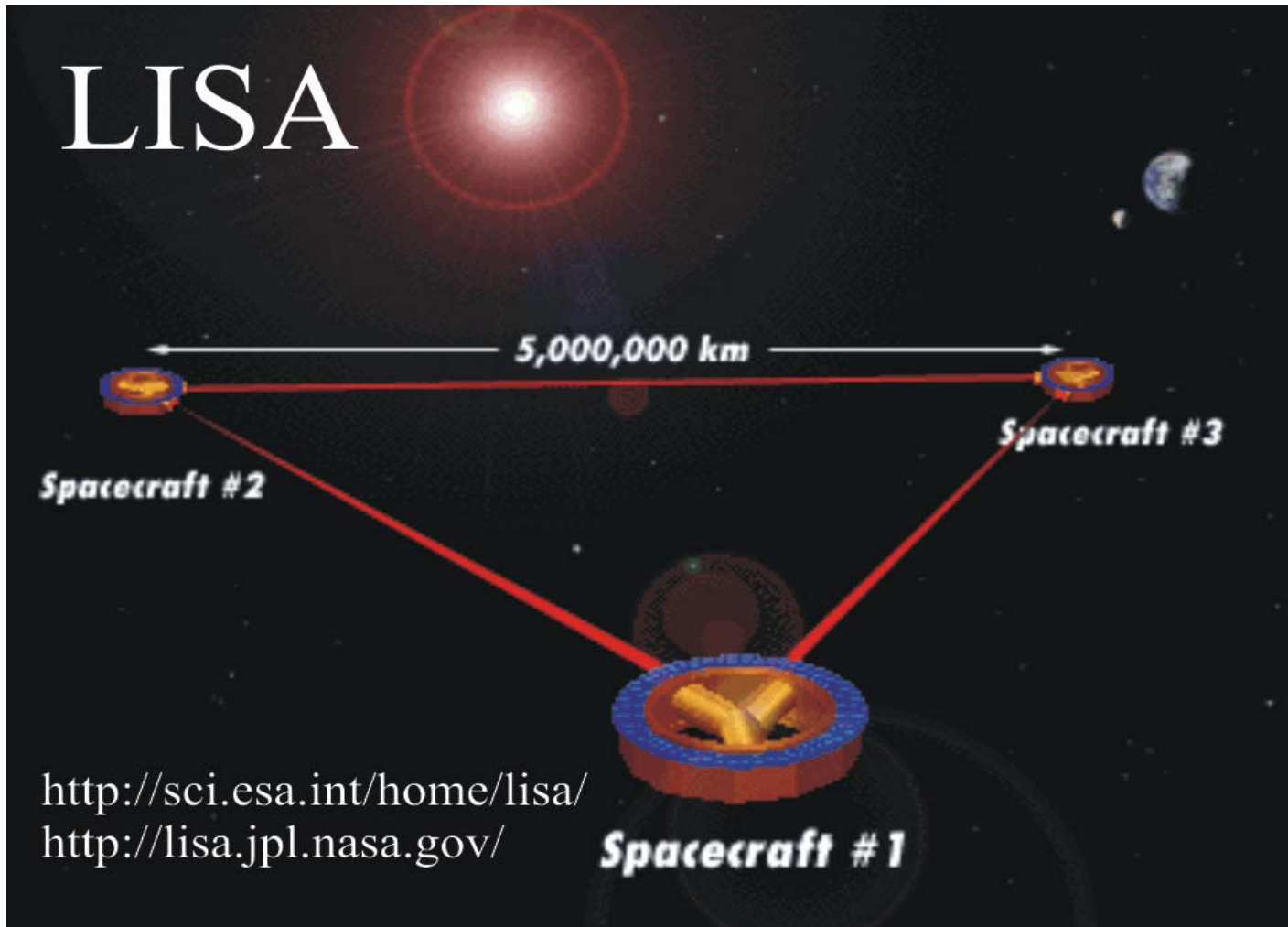
$$\Delta\varphi^2 \simeq (\Omega \sin \alpha)^2 S_h \tau \sim 1$$



$$\frac{\Delta E_k}{\hbar}$$

✘ high energy splitting

Optical interferometer



$$\tau_{\text{phot}} \simeq 10 \text{ s}$$

$$\Omega_{\text{phot}} \simeq 10^{15} \text{ rad.s}^{-1}$$

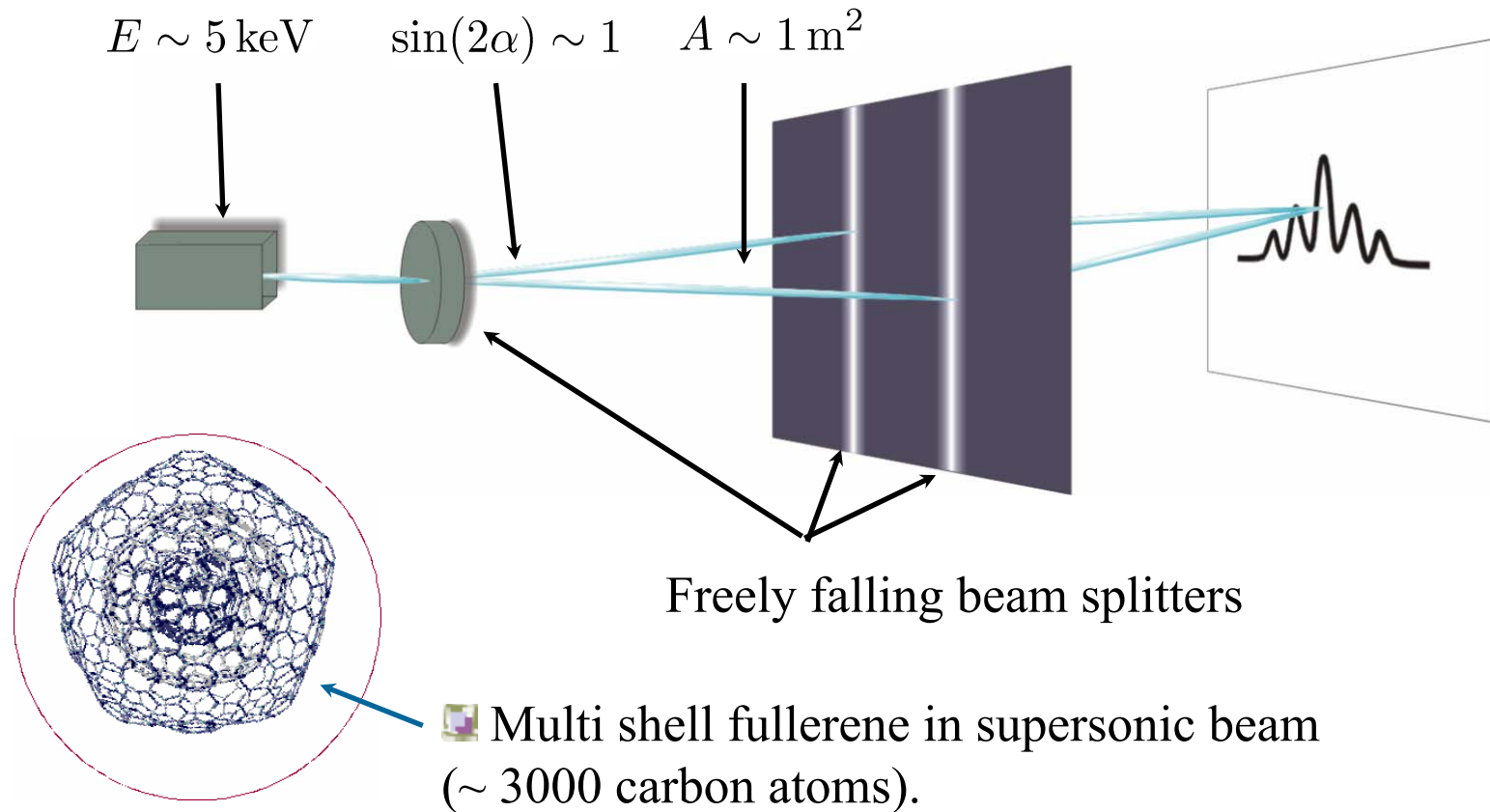
$$\Delta\varphi^2 \ll 1$$

**LISA is
microscopic !**

Good news for the
fringes...

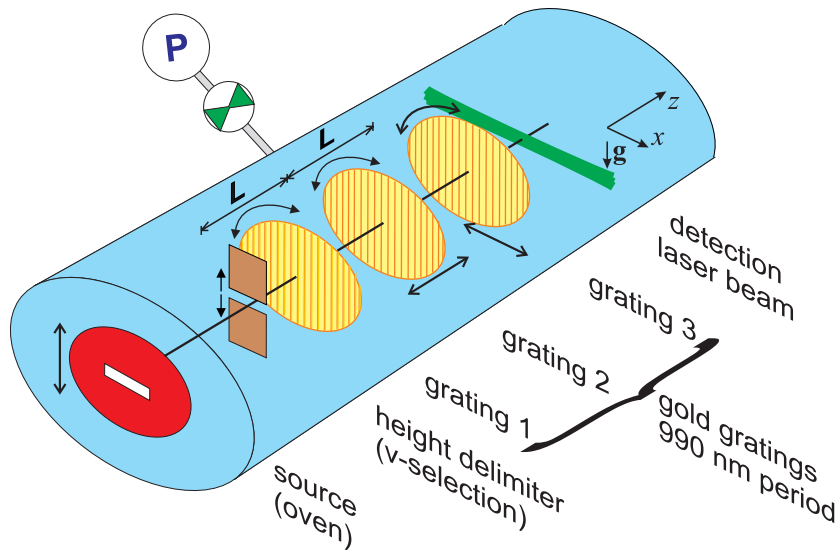
Though experiment

 $\Delta\varphi^2 \sim 1$ would need :

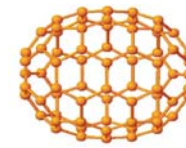


State of the art

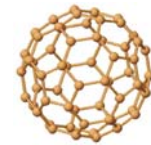
Zeilinger & al., *Phys. Rev. Lett.* **88** (2002), 100404



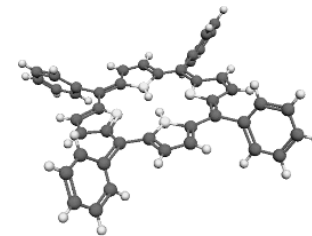
Interferences with large molecules.



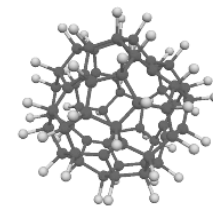
C_{70}



C_{60}



$C_{44}H_{30}N_4$



$C_{60}F_{48}$

Nevertheless, small separation angle, small kinetic energy and not freely falling beam splitters.

$$E \sim 100 \text{ meV}$$

$$\Delta\varphi^2 \ll 1$$

Future ?

Improving state of the art experiment à la Zeilinger with larger and hotter molecules ?

Going to space : increase of time of flight.

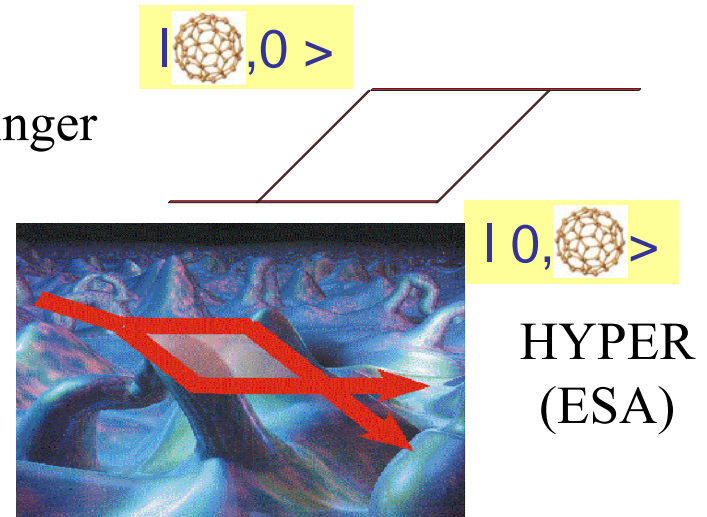
Using Bose-Einstein condensate ? (N^2 amplification effect of decoherence)

Looking for decoherence effects in condensed matter physics ?

Schrödinger cats using micro-mechanical systems ?

S. Bose & al., Phys. Rev. A **59** (1999), 3204

Any idea is welcome !



Conclusions

Gravitational waves set an ultimate frontier to quantum coherence

! Feynman intuition

GW can be revealed by means of decoherence, in the same way than Brownian motion revealed the atomic nature of matter before we could observe atoms.

I.C. Percival, *Phys. World* **10** (1997) 48

Gravitational waves astronomy : the next revolution in astrophysic ?

visible



Infra-red

