QMFPA - Bertinoro - 2006

Robustness of non ideal holonomic gates under parametric noise

C. L., P. Aniello, M. Napolitano and G. Florio

quant-ph/0611079

INFN - sezioni di Napoli & Bari

Università di Napoli “Federico II“

Università di Bari

summary

- ✓ intro to Holonomic Quantum Computation
- ✓ on the Adiabatic Limit
- ✓ the noisy case
- ✓ Holonomic Gates at finite time

intro to Holonomic Gates

three ingredients (at least) are needed to define an HG:

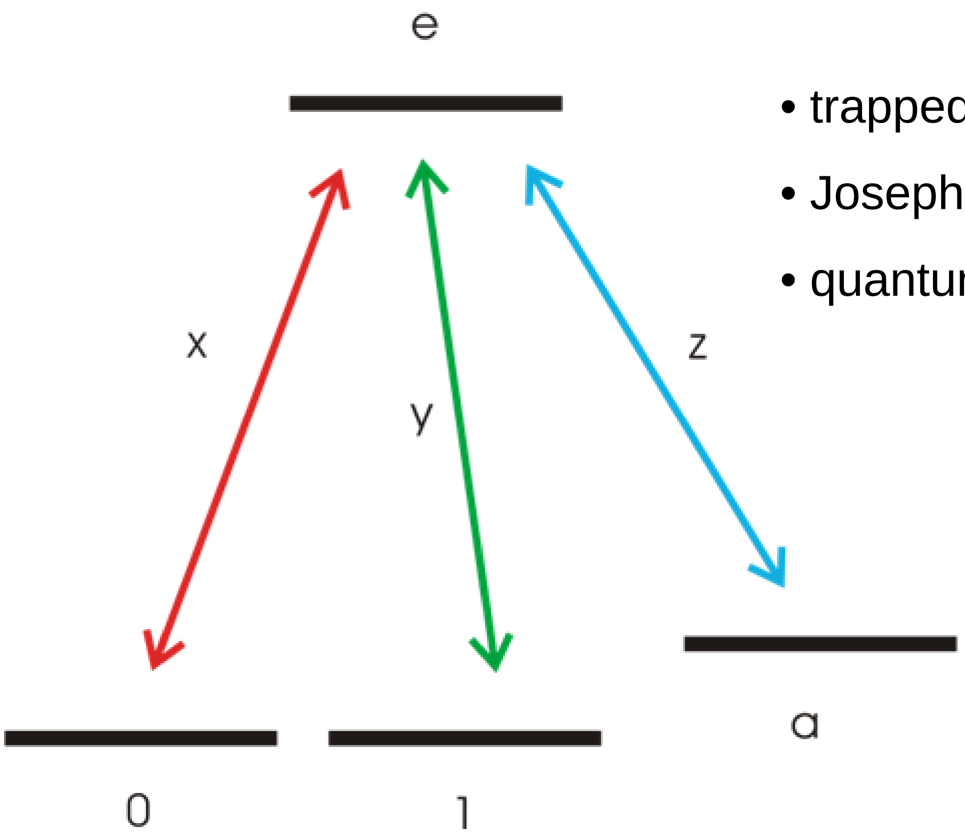
[**Zanardi, Rasetti PLA 1999**]

1. Hamiltonian depending on a set of (controllable) parameters (x,y,z,...)

2. an eigenspace depending smoothly on the parameters (x,y,z...)

3. a „*balanced*“ working time: short enough (fast gates + decoherence) and long enough (Adiabatic Limit)

description of the system (1)



- trapped ions [**Duan *et al.* Science 2001**]
- Josephson Junctions [**Faoro *et al.* PRL 2003**]
- quantum dots [**Solinas *et al.* PRA 2003**]

computational space

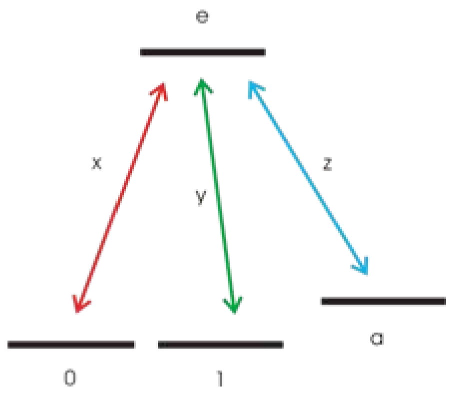
$$\sigma = \{\underbrace{0, 0}, \Omega, -\Omega\}$$

$$\Omega = \sqrt{x^2 + y^2 + z^2}$$

$$H(x, y, z) = x|0\rangle\langle e| + y|1\rangle\langle e| + z|a\rangle\langle e| + h.c.$$

description of the system (2)

$$\sigma = \{0, 0, \Omega, -\Omega\} \qquad \Omega = \sqrt{x^2 + y^2 + z^2}$$



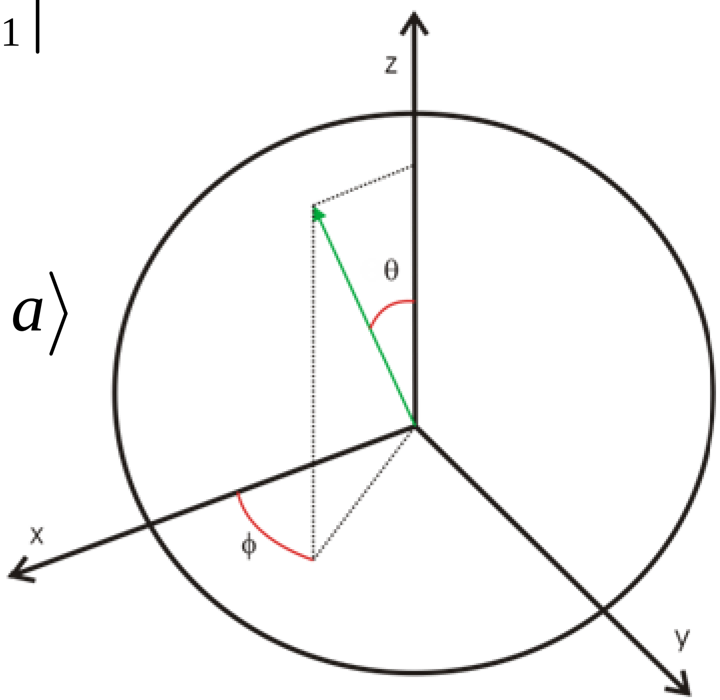
computational space: $S(x,y,z) = \{\psi_0, \psi_1\}$

$$P = |\psi_0\rangle\langle\psi_0| + |\psi_1\rangle\langle\psi_1|$$

$$x, y, z \to \Omega, \vartheta, \varphi$$

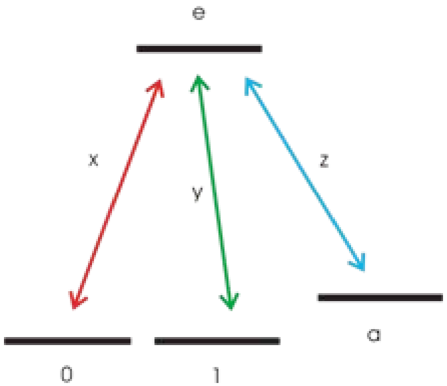
$$|\psi_0\rangle = \cos\vartheta\left(\cos\varphi|0\rangle + \sin\varphi|1\rangle\right) - \sin\vartheta|a\rangle$$

$$|\psi_1\rangle = -\sin\varphi|0\rangle + \cos\varphi|1\rangle$$



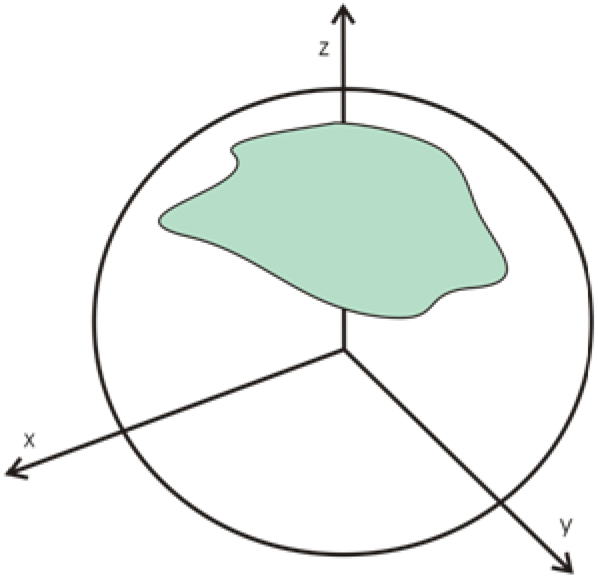
At the north pole: $S = \{|0\rangle, |1\rangle\}$

description of the system (3)



a loop around the north pole

$$\big(x(\tau), y(\tau), z(\tau)\big) =$$
$$\big(x(0), y(0), z(0)\big) = (0,0,1)$$

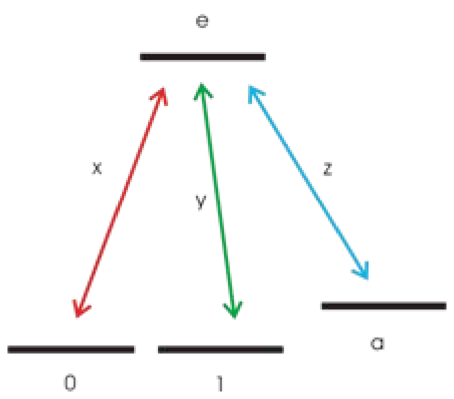


$$\psi(0) \in S(0,0,1)$$

$$\frac{d\psi(t)}{dt} = -iH\big(x(t), y(t), z(t)\big)\psi(t) \qquad t \in [0,\tau]$$

description of the system (4)

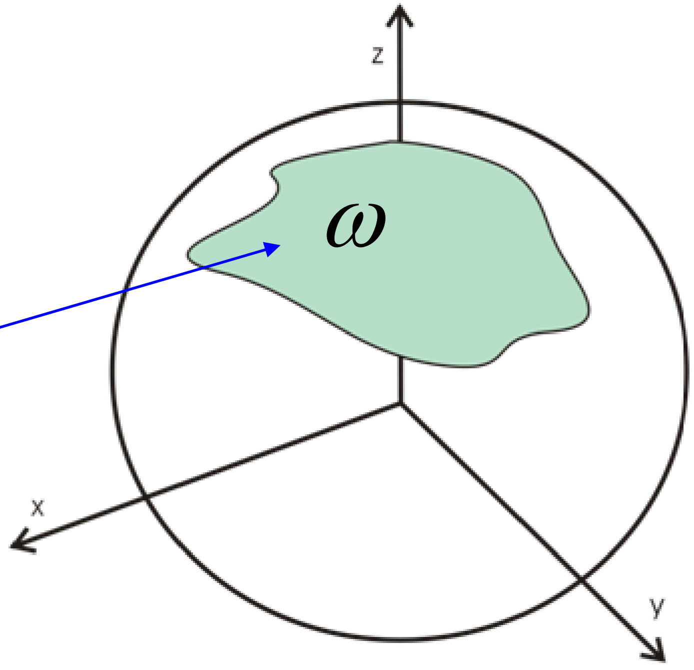
a loop around the north pole



$$S \rightarrow S \qquad\qquad P \rightarrow P$$

$$\psi = \alpha\left|0\right\rangle + \beta\left|1\right\rangle \rightarrow \psi' = U\psi$$

$$U = e^{-i\sigma_y \omega}$$



...but only in the adiabalic limit !

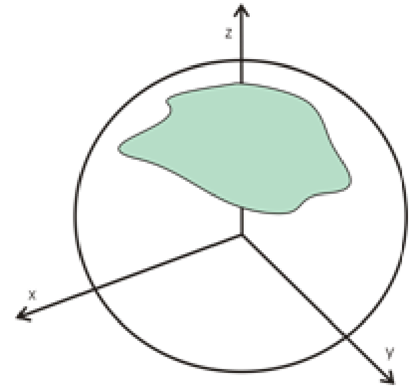
the Adiabatic Limit (1)

we take the point of view of the proof of the adiabatic thereom by Kato
[**Kato J.Phys.Soc.Jap. 1951**]

Dynamical Transformation	Adiabatic Transformation
Schrödinger equation	a proper set of differential equation
V_τ	U

$$\Omega\tau \to \infty \qquad \boxed{\| V_\tau - U \| \to 0}$$

the Adiabatic Limit (2)



$$t \in [0, \tau] \qquad t \to s \equiv t/\tau \qquad s \in [0,1]$$

Dynamical Transformation

$$\frac{d\psi}{ds} = -i\tau H\left(x(s), y(s), z(s)\right)\psi$$

$$\frac{dV_\tau}{ds} = -i\tau H V_\tau$$

$$\psi = V_\tau \psi$$

Adiabatic Transformation

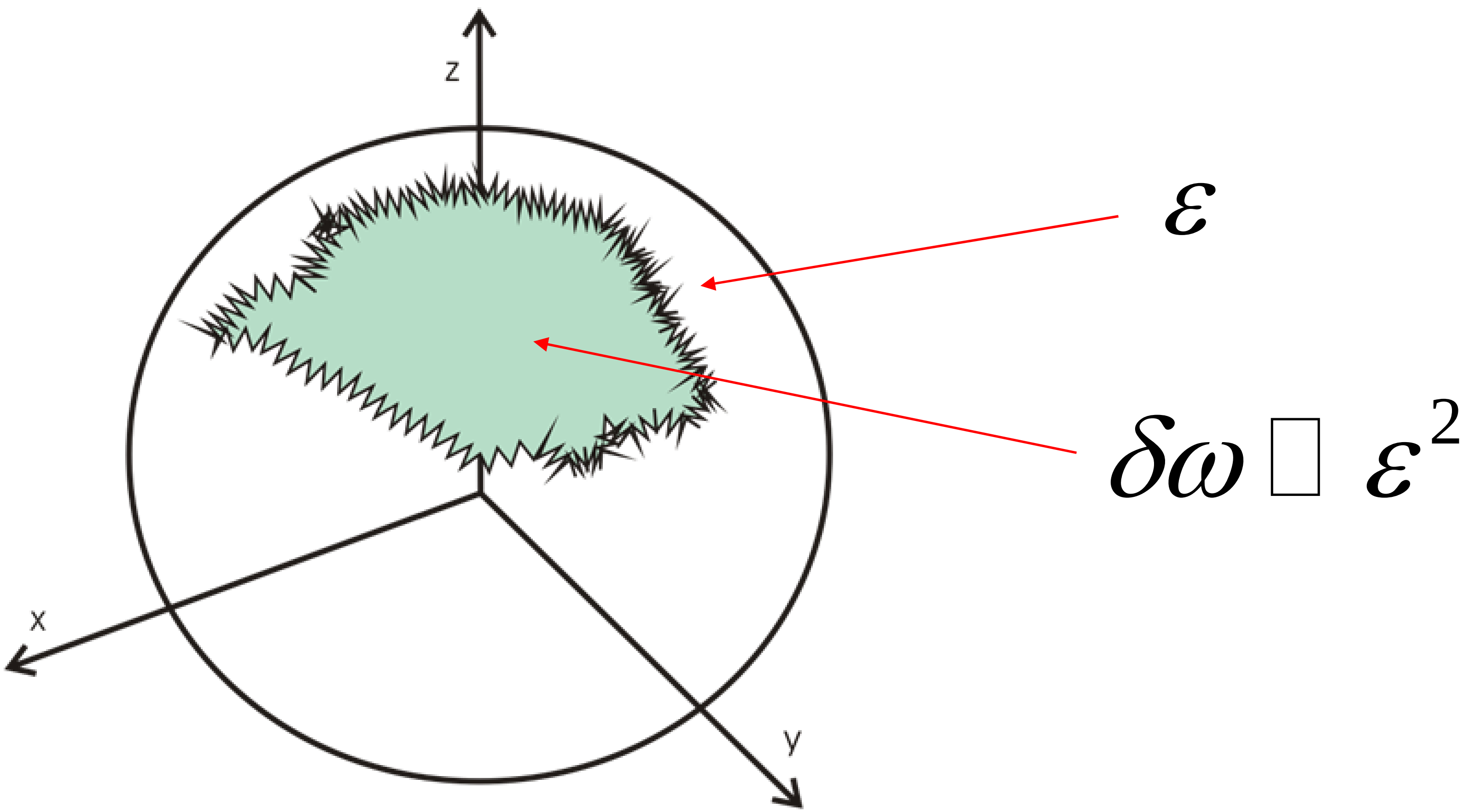
$$\frac{dU}{ds} = iA(s)U(s)$$

$$iA \equiv \left[\frac{dP}{ds}, P\right]$$

$$\psi = U\psi$$

the noisy case (1)

- in the AL the transformation at the end of the loop depends only on the area of the loop
- H.G. are considered to be robust against parametric noise....



the noisy case (2)

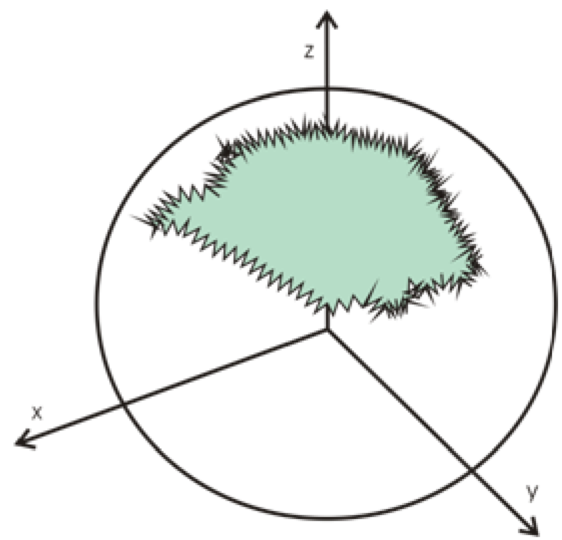
...this *qualitative* argument was studied *quantitatively* in [**Solinas *et al.* PRA 2004**]...

1. before they take the adiabatic limit $\Omega\tau \square 1$
2. after they add a noise in the loop

three regimes were found:

$$\frac{\tau}{\tau_{corr}} \square 1 \qquad \frac{\tau}{\tau_{corr}} \square 1$$ → ***high fidelity***

intermediate values → ***low fidelity***

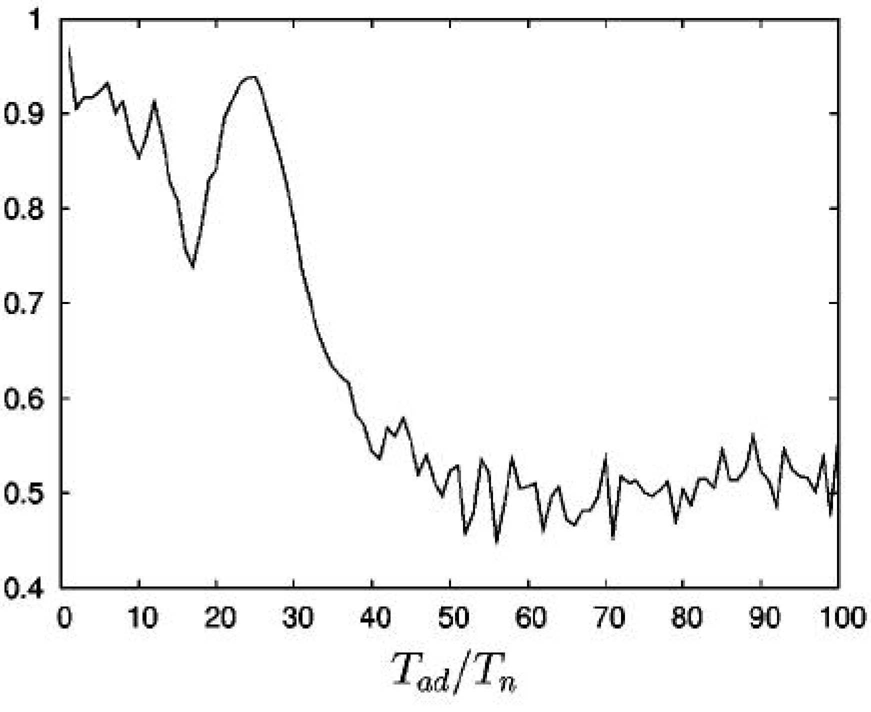


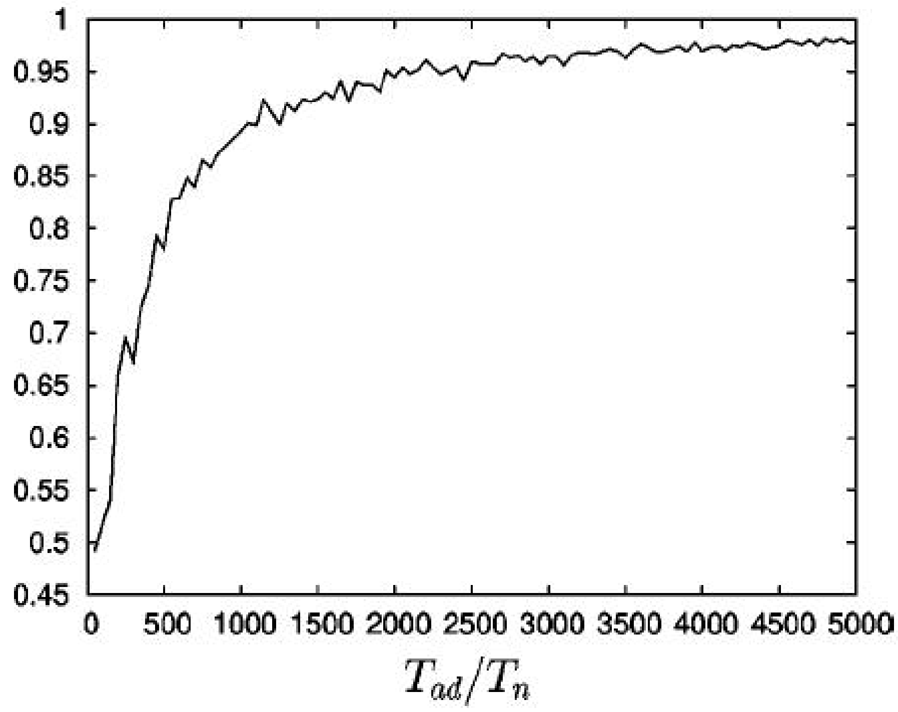
the noisy case (3)

[**Solinas *et al*. PRA 2004**]

$$\Omega\tau \approx 150$$

$$\varepsilon = 0.1\Omega$$





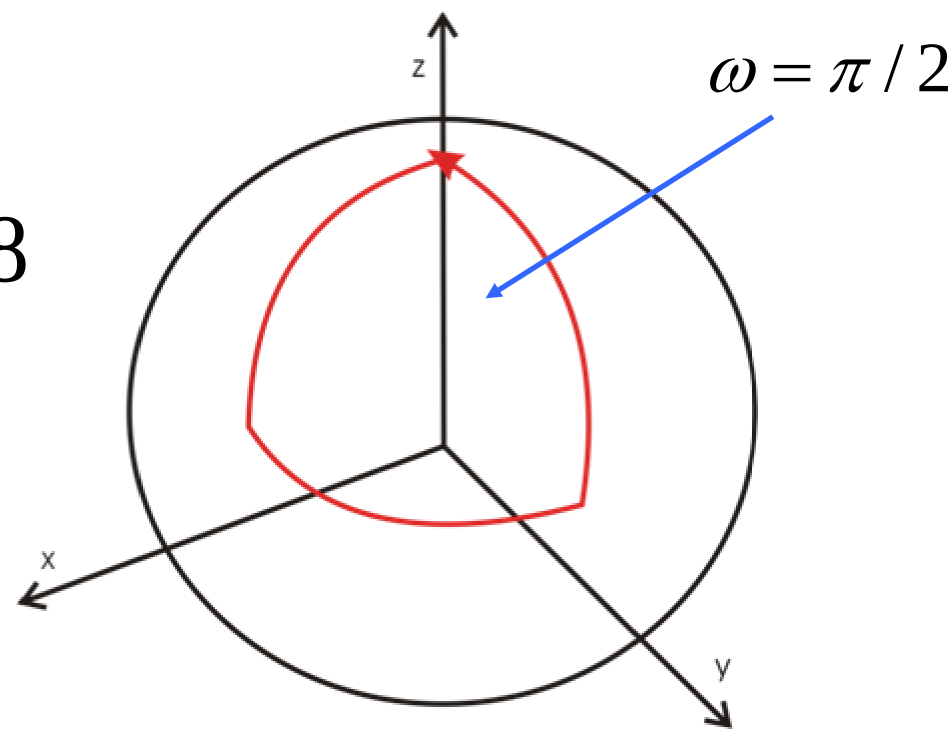
finite time gates (1)

[**Florio *et al.* PRA 2006**] **considered a special class of loops which „*mimic*“ the adiabatic dynamics far before the adiabatic limit is reached....**

...in operational time τ

$$\Omega\tau \approx 18$$

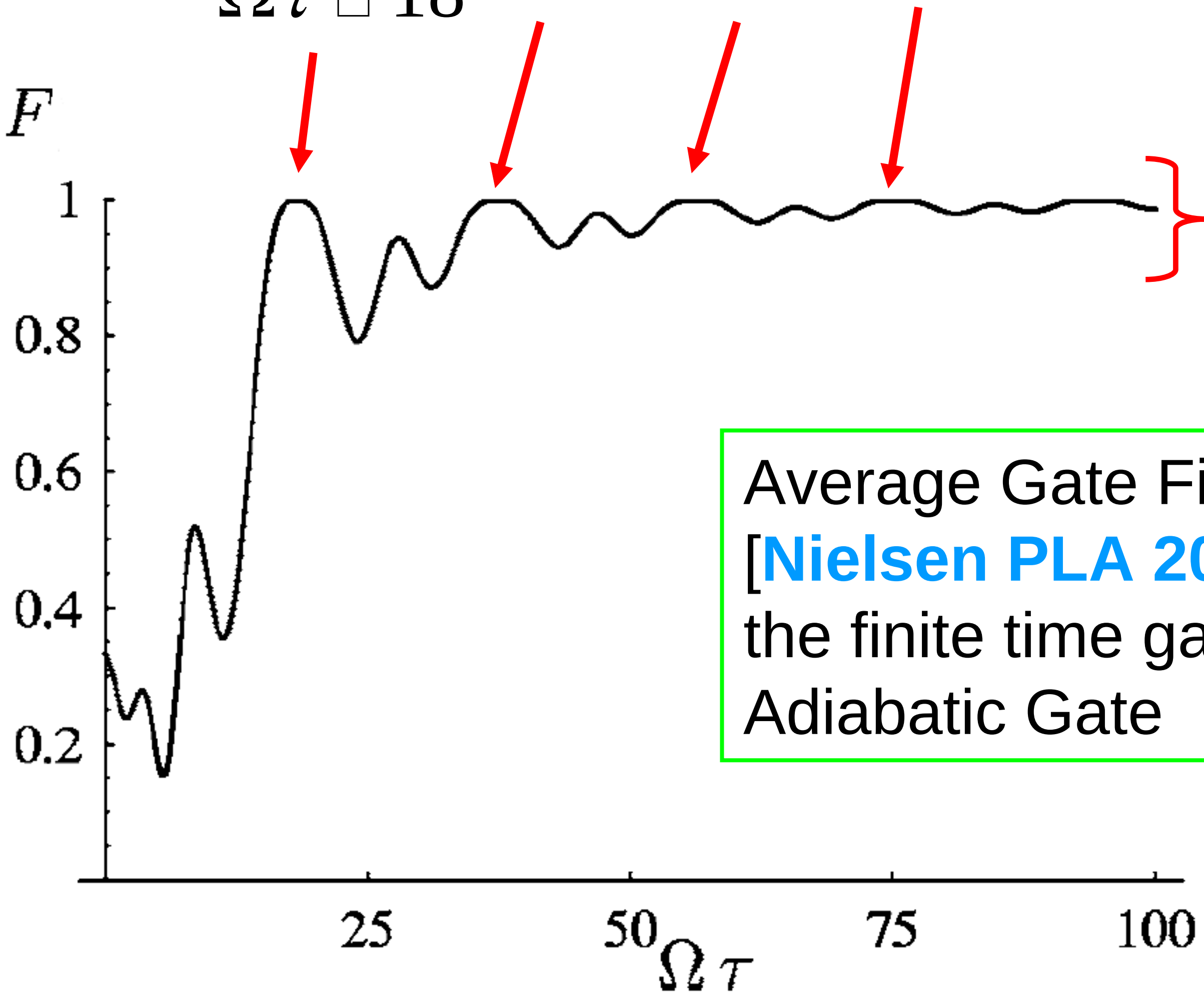
$$\boxed{U = -i\sigma_y}$$



finite time gates (2)

[**Florio *et al.* PRA 2006**]

$\Omega\tau \approx 18$



approaching AL

Average Gate Fidelity [**Nielsen PLA 2002**] between the finite time gate and the Adiabatic Gate

questions

This gate mimics the Adiabatic Limit....

...but in what sense???

1. it can be argued that it is more robust against decoherence [**Trullo *et al.* Las.Phys. 2006**]

2. what happen in presence of parametric noise?
   does the robustness argument still hold???

parametric perturbations

- ✓ we would like to understand the dynamics in the operational time τ with perturbations in the control parameters...

- ✓ some models of perturbations in the control parameters were considered...

- ✓ the first idea was that the response of system should depend on the perturbation typical frequency... so we considered a *monochromatic* perturbation...

monochromatic perturbation (1)

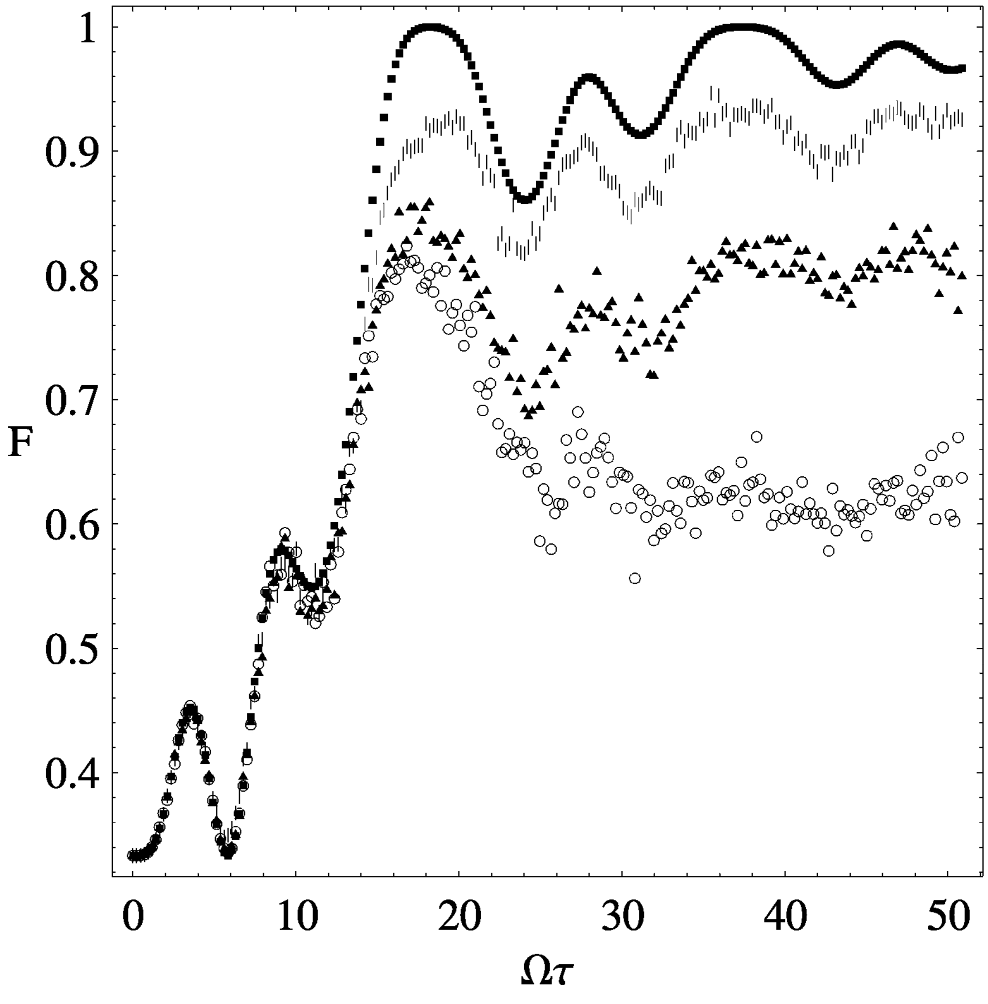
$$\begin{cases} x(s) + \varepsilon \sin\left(\eta\tau s + \phi_x\right) \\ y(s) + \varepsilon \sin\left(\eta\tau s + \phi_y\right) \\ z(s) + \varepsilon \sin\left(\eta\tau s + \phi_z\right) \end{cases}$$

ideal case:
the loop is
independent
on τ

noisy case the loop
itself depends on τ

1. solve the Schöredinger equation with noise
2. compare with the Adiabatic Transformation in the ideal case

monochromatic perturbation (2)



no perturbation

$\varepsilon = 0.1\Omega$

$\eta = 0.10\Omega$

$\varepsilon = 0.1\Omega$

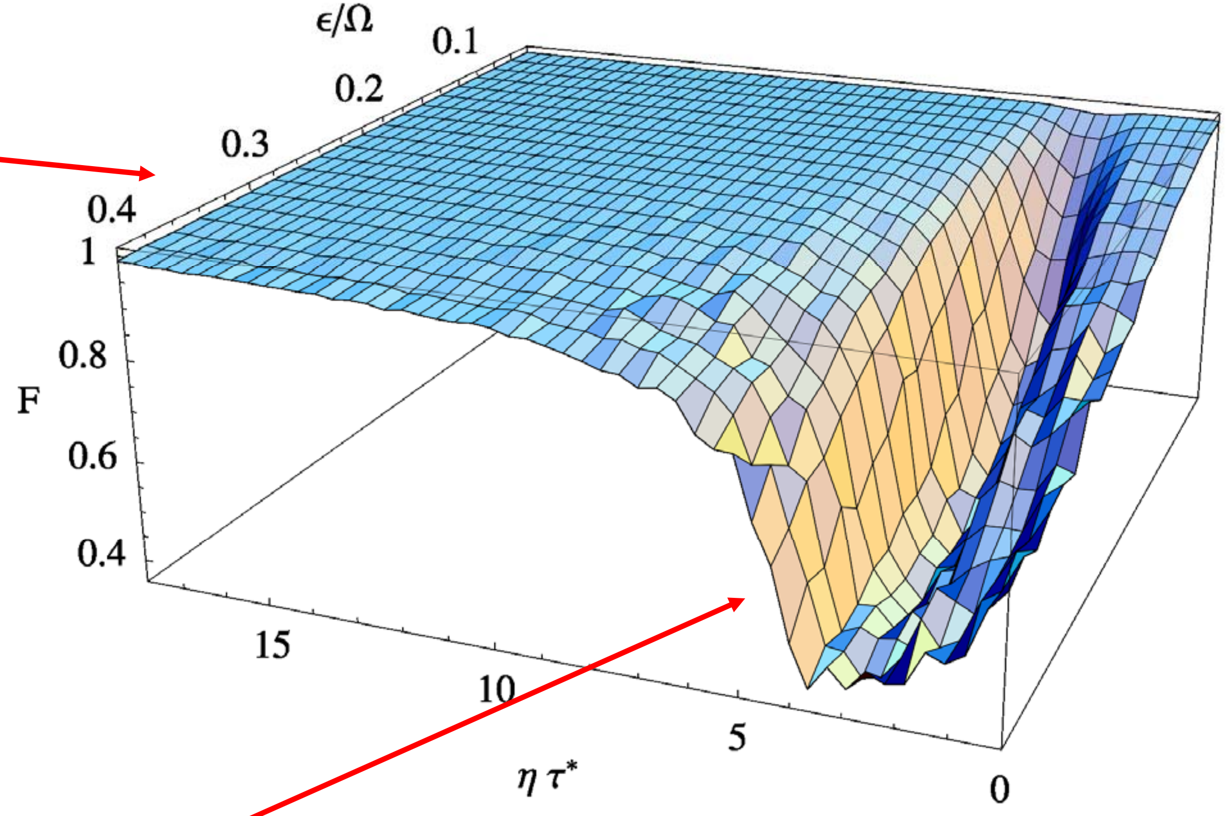
$\eta = 0.19\Omega$

$\varepsilon = 0.1\Omega$

$\eta = 0.18\Omega$

monochromatic perturbation (3)

high fidelity



at the first optimal operational time

breakdown of fidelity

noise models (1)

noise on the sphere

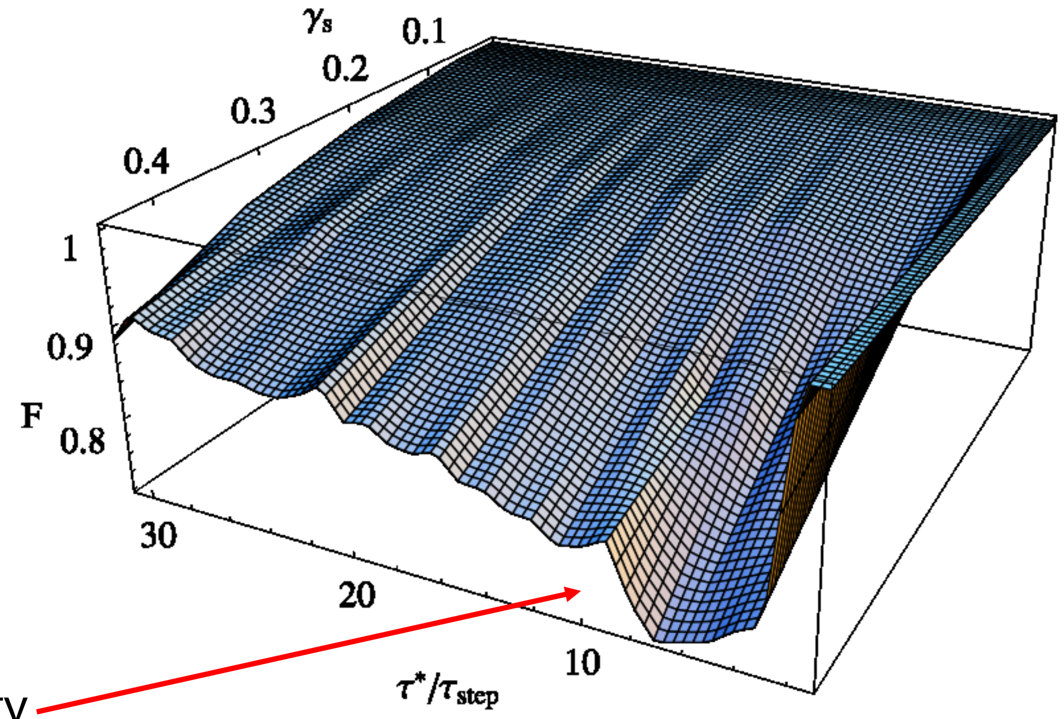
random step in the angular variables ϑ, φ

the spectrum is preserved

analytical solution

at the first optimal operational time

τ_{step}



smaller fidelity

noise models (2)

noise outside the sphere

random steps in the coupling constants x, y, z

the spectrum is not preserved

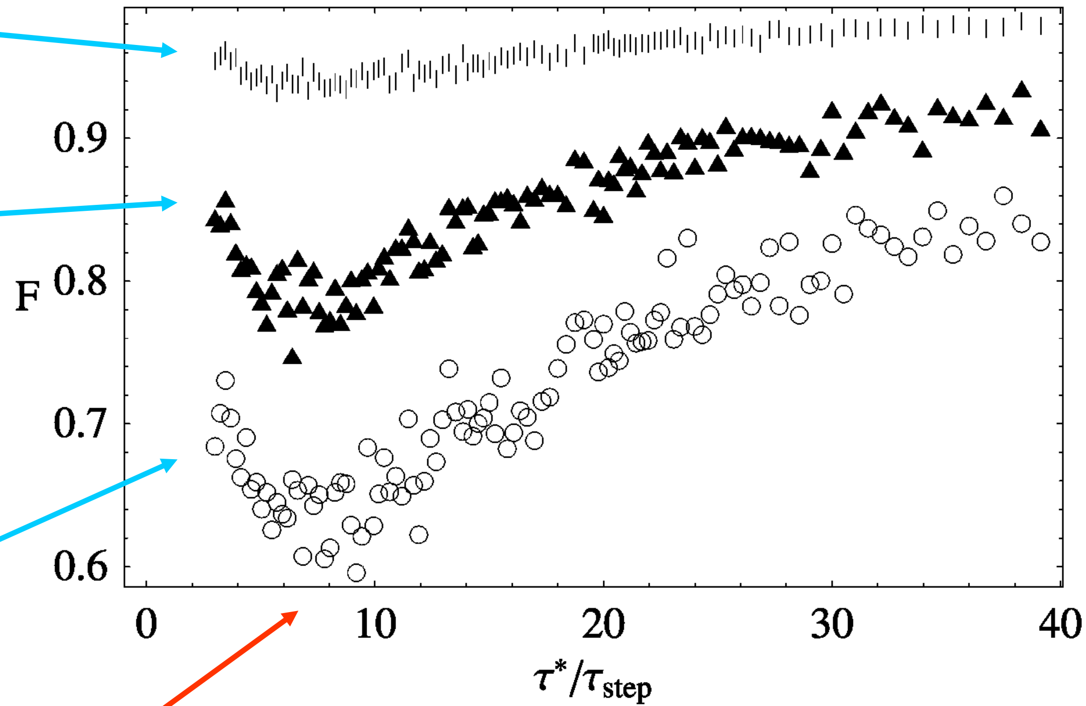
$\varepsilon = 0.1\Omega$

$\varepsilon = 0.2\Omega$

at the first optimal operational time

τ_{step}

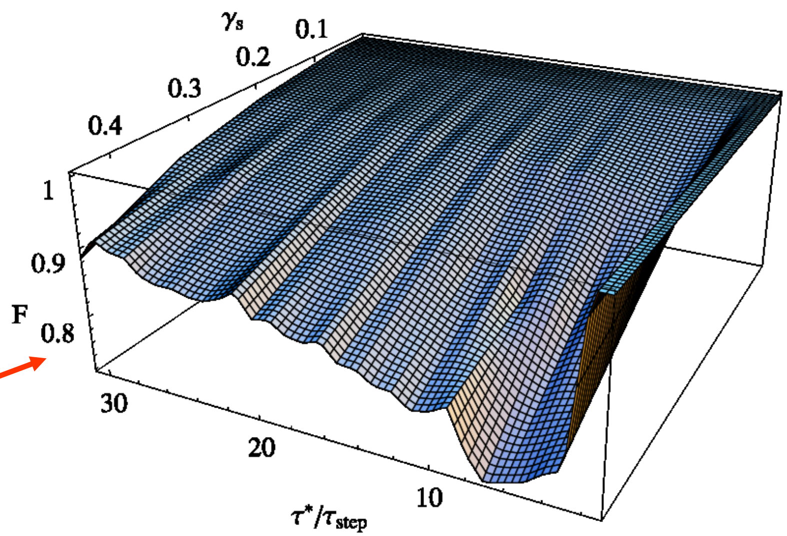
$\varepsilon = 0.3\Omega$



fidelity decreases

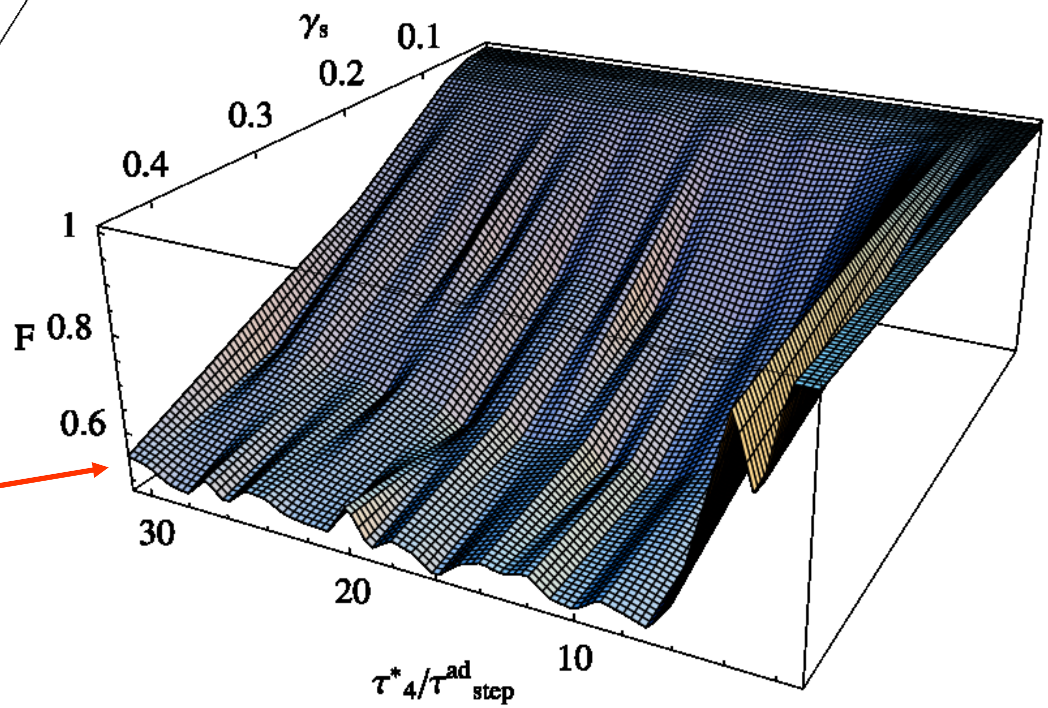
relation with longer times (1)

noise on the sphere



first optimal operational time

$$\Omega \tau \approx 18$$

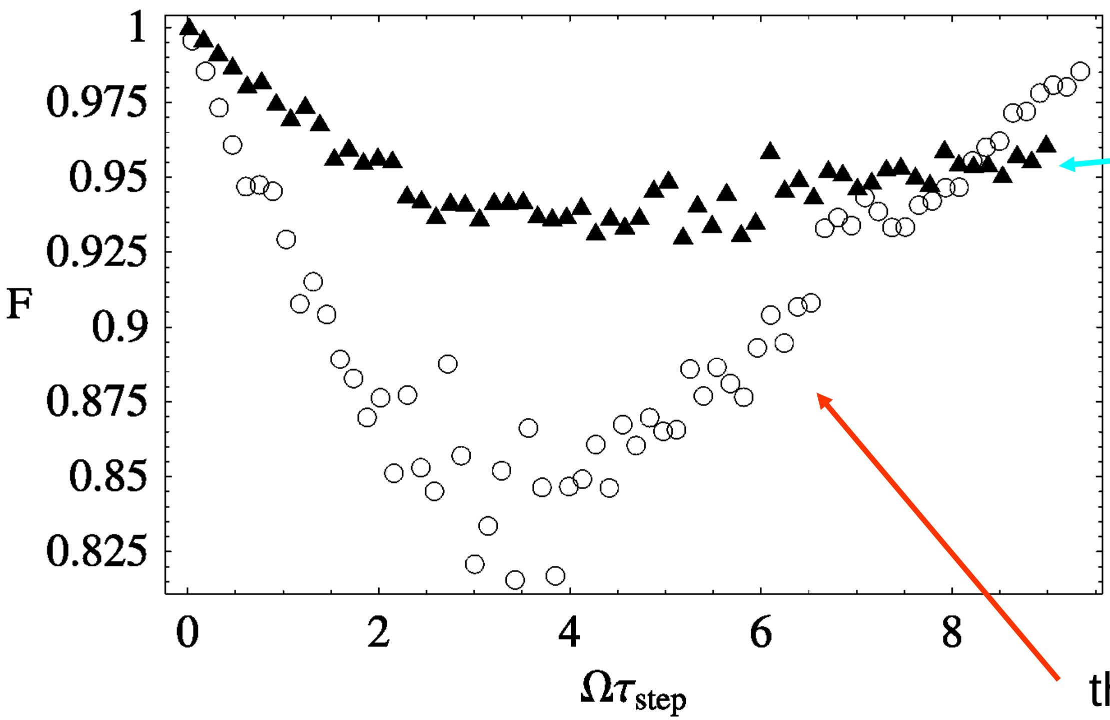


fourth optimal operational time

$$\Omega \tau \approx 75$$

relation with longer times (2)

noise outside the sphere



first optimal operational time

$\Omega\tau \approx 18$

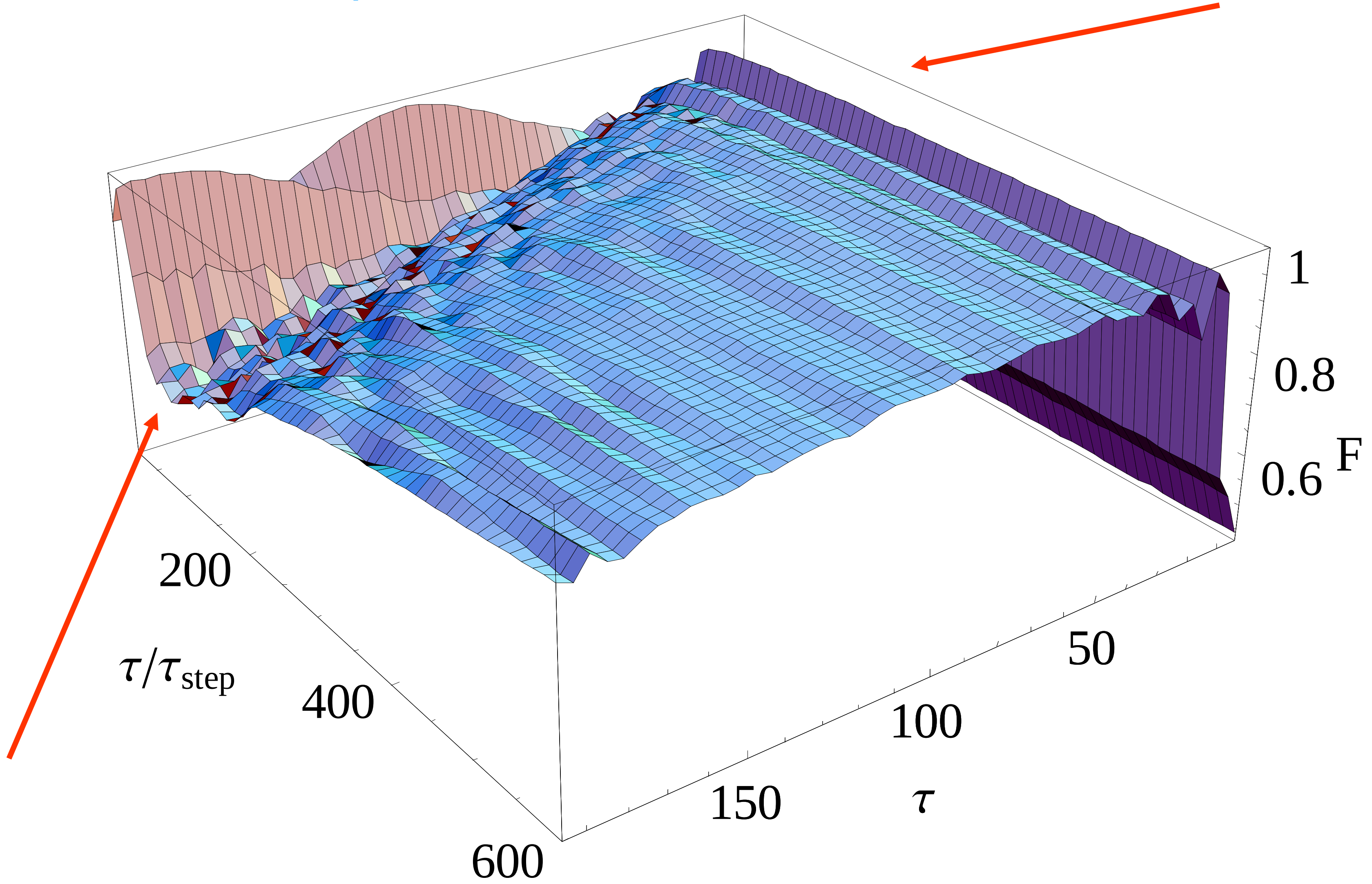
$$\varepsilon = 0.1\Omega$$

third optimal operational time

$\Omega\tau \approx 56$

relation with longer times (2)

noise outside the sphere



conclusions

- ✓ robustness argument applies only in the adiabatic limit
- ✓ the nature of these shorter-time Holonomic Gates is not completely clear
- ✓ surprise: they can be more robust against parametric noise (and decoherence)