Quantum manipulation of information by hyper-entangled two photon states

M. Barbieri, G. Vallone, E. Pomarico, F. De Martini, P. Mataloni

Quantum Optics Group, Dipartimento di Fisica
Università di Roma "La Sapienza"

http://quantumoptics.phys.uniroma1.it

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- Entangled photon pairs generated by Spontaneous Parametric Down Conversion (SPDC). Qubits generally entangled in a single degree of freedom (limited to a 2x2 Hilbert space)

Increase the information content by enlarging the Hilbert space

Possible approaches:

1) Multiphoton entanglement (4, 5 photons, *)
2) d-level (d>2) quantum systems (*qu-dits*)
3) Two photon hyper-entanglement

* Hefei group: 6 photons, quant-ph/0609130

Outline

- Polarization-momentum hyperentangled photon states

- Nonlocality tests, AVN test of quantum mechanics

- Growing with size quantum nonlocality

- Bell state analysis for quantum communication protocols

- Two photon linear cluster states

- Perspectives

Two photon hyperentanglement

$$|\Xi_{ab}\rangle = |Bell_1\rangle \otimes |Bell_2\rangle .... \otimes |Bell_N\rangle$$

N different degrees of freedom, with $|Bell_i\rangle$ corresponding to one of the Bell states:

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|00\rangle \pm |11\rangle\right]$$

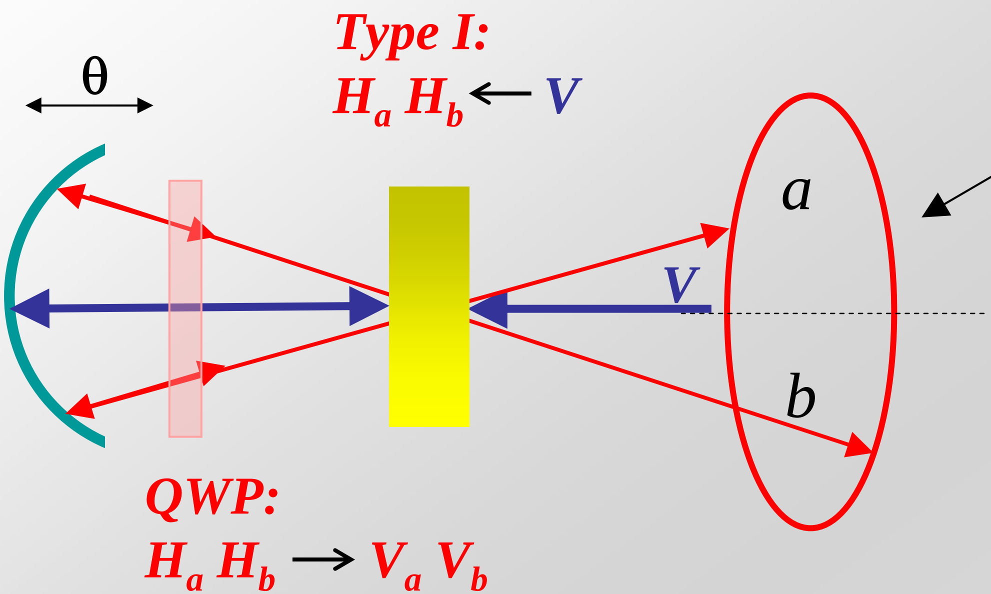
$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|01\rangle \pm |10\rangle\right]$$

Motivations:

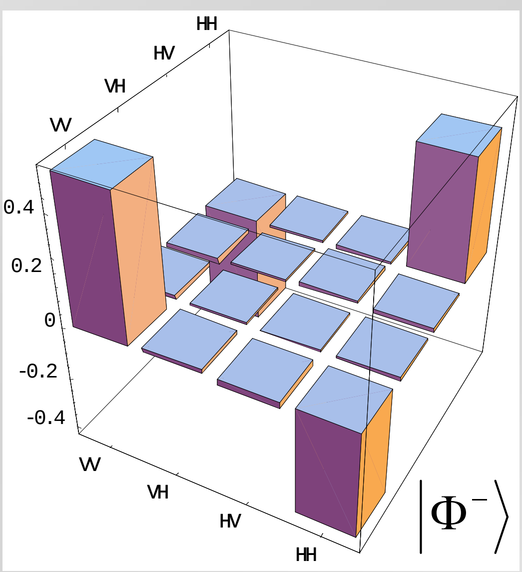
By expanding the Hilbert space to 2^N x 2^N dimensions, nonlocality tests which are more resistant to noise and present more significant deviations from classical bounds can be performed

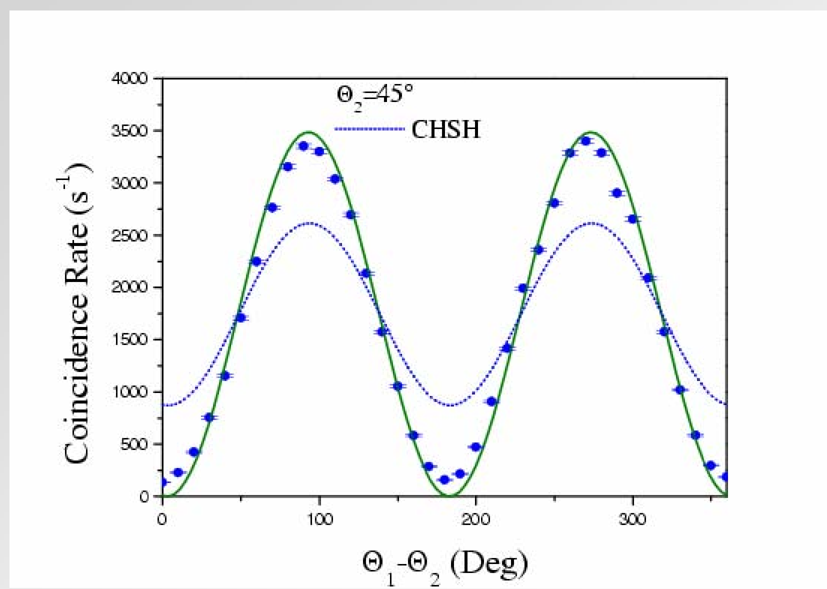
N = 2: hyperentanglement allows to codify two qubits on the same physical carrier, allows complete Bell State Analysis → no need for multiparticle states

1) Polarization qubits

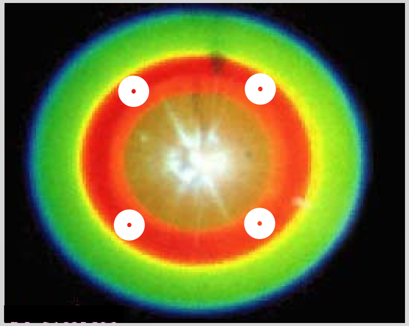


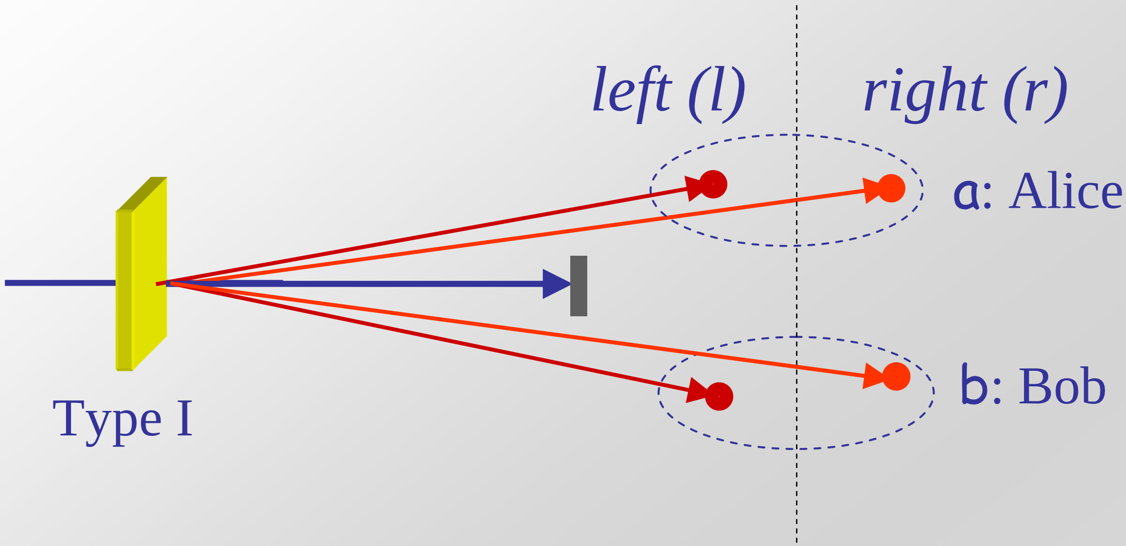
$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|H_a H_b\rangle \pm |V_a V_b\rangle\right)$$



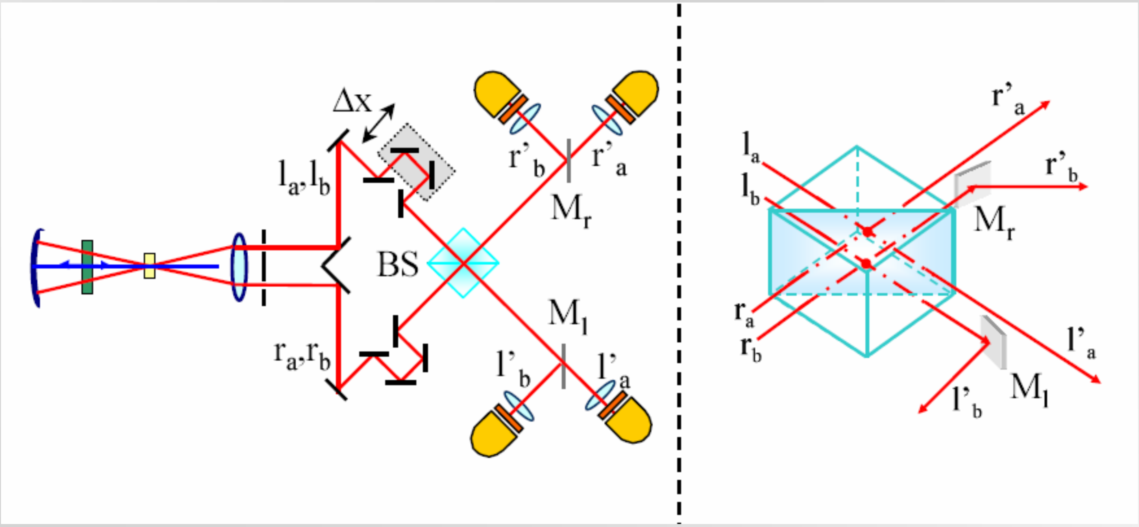


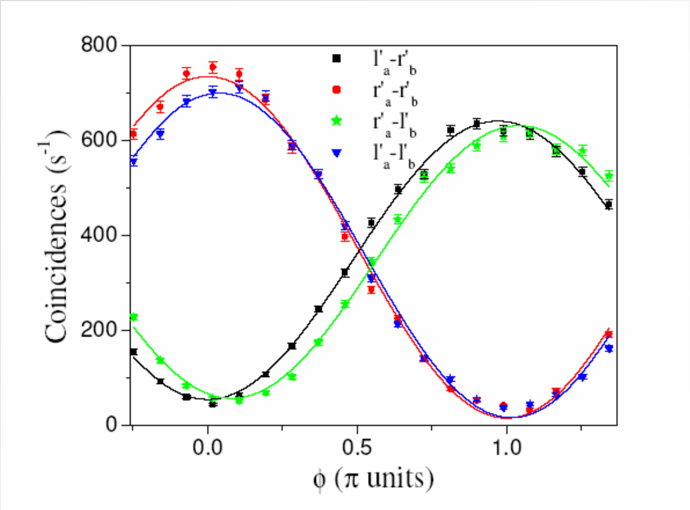
2) linear momentum qubits



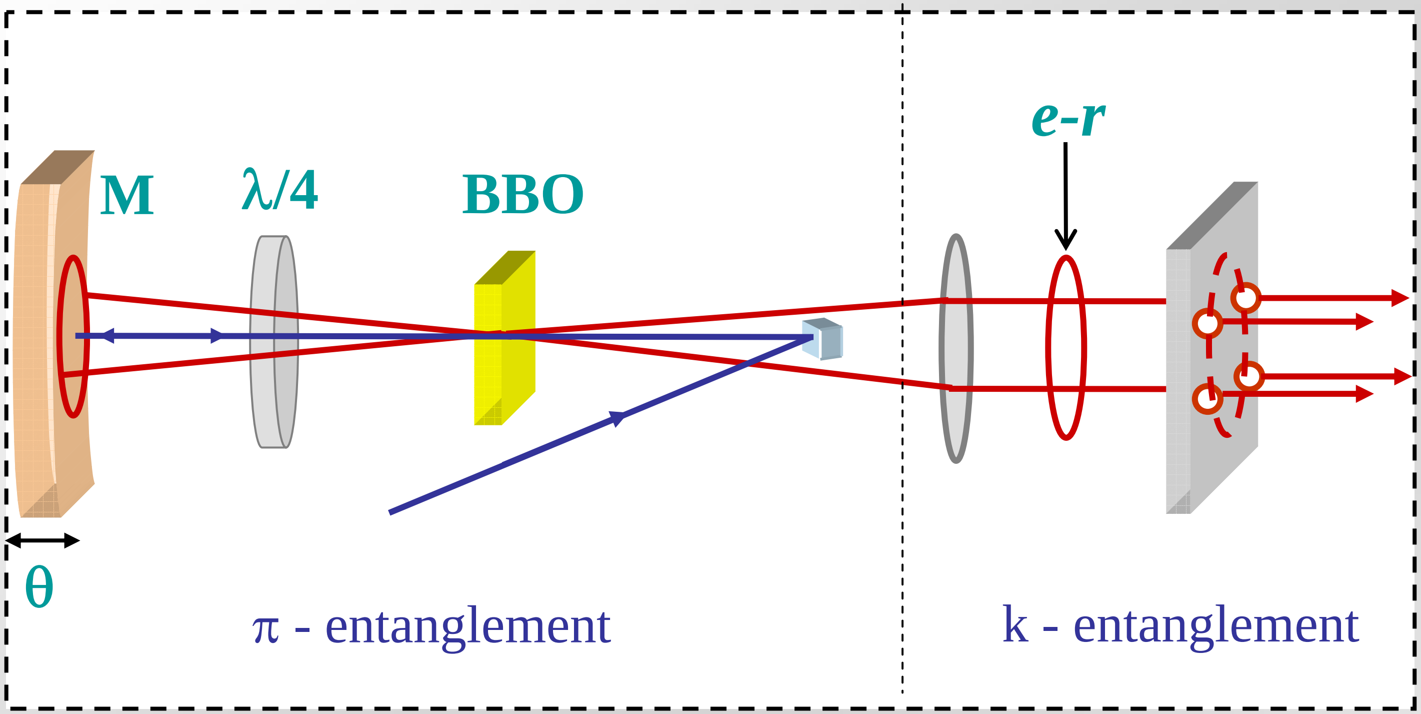


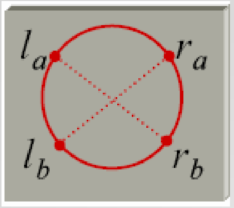
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|l_a r_b\rangle \pm |r_a l_b\rangle\right)$$





3) hyperentangled photons

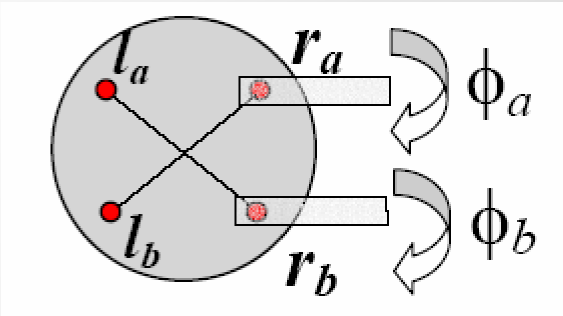


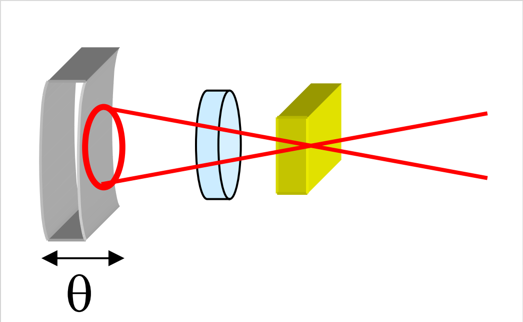


$$|\Pi\rangle = \frac{1}{\sqrt{2}}\left[|H_aH_b\rangle + e^{i\theta}|V_aV_b\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|l_ar_b\rangle + e^{i\phi}|r_al_b\rangle\right]$$

2 photons → 4 qubits

Manipulate hyperentangled states





$\phi = 0 \longrightarrow \pi$

$\theta = 0 \longrightarrow \pi$

Half wave plate) on modes l_a , l_b:

$$\left|\Pi^{\pm\pm}\right\rangle = \left|\Phi^{\pm}\right\rangle \otimes \left|\psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left[\left|H_a H_b\right\rangle \pm \left|V_a V_b\right\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[\left|l_a r_b\right\rangle \pm \left|r_a l_b\right\rangle\right]$$

$$\left|\Xi^{\pm\pm}\right\rangle = \left|\Psi^{\pm}\right\rangle \otimes \left|\psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left[\left|H_a V_b\right\rangle \pm \left|V_a H_b\right\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[\left|l_a r_b\right\rangle \pm \left|r_a l_b\right\rangle\right]$$

$$\left|\Pi^{++}\right\rangle \Rightarrow \left|\Xi^{++}\right\rangle$$
$$\left|\Pi^{-+}\right\rangle \Rightarrow \left|\Xi^{--}\right\rangle$$
$$\left|\Pi^{+-}\right\rangle \Rightarrow \left|\Xi^{+-}\right\rangle$$
$$\left|\Pi^{--}\right\rangle \Rightarrow \left|\Xi^{-+}\right\rangle$$

Nonlocality tests with hyperentangled states

Separate Bell-CHSH inequality tests:

$$\left|S_\pi\right| = \left|P(\theta_1,\theta_2) - P(\theta_1,\theta'_2) + P(\theta'_1,\theta_2) + P(\theta'_1,\theta'_2)\right|$$

$$P(\theta_1,\theta_2) = \frac{\left[C(\theta_1,\theta_2) + C(\theta_1^\perp,\theta_2^\perp) - C(\theta_1,\theta_2^\perp) - C(\theta_1^\perp,\theta_2)\right]}{\left[C(\theta_1,\theta_2) + C(\theta_1^\perp,\theta_2^\perp) + C(\theta_1,\theta_2^\perp) + C(\theta_1^\perp,\theta_2)\right]}$$

$$\theta_1 = 0, \theta'_1 = \pi/4,$$
$$\theta_2 = \pi/8, \theta'_2 = 3\pi/8$$

$$\left|S_k\right| = \left|E(\phi_a,\phi_b) - E(\phi_a,\phi^*_b) + E(\phi^*_a,\phi_b) + E(\phi^*_a,\phi^*_b)\right|$$

$$E(\phi_a,\phi_b) = \frac{\left[C(a'_1,b'_1) + C(a'_2,b'_2) - C(a'_1,b'_2) - C(a'_2,b'_1)\right]}{\left[C(a'_1,b'_1) + C(a'_2,b'_2) + C(a'_1,b'_2) + C(a'_2,b'_1)\right]}$$

$$\phi_a = 0, \phi^*_a = \pi/2,$$
$$\phi_b = \pi/4, \phi^*_b = 3\pi/4$$

→

Polarization: 130-σ violation
Linear momentum: 102-σ violation

“All-versus nothing” (AVN) proof of Quantum Mechanics

Different approach: one observes that assigning to certain Pauli observables a value which pre-exists to the measurement leads to logical incosistancy

Generalization of the GHZ argument to the case of two observers. Test performed over *all* the quantum particles involved in the experiment

Polarization observables

$$Z_i \equiv \sigma_{Z_i} = |H_i\rangle\langle H_i| - |V_i\rangle\langle V_i|$$

$$X_i \equiv \sigma_{X_i} = |H_i\rangle\langle V_i| + |V_i\rangle\langle H_i|$$

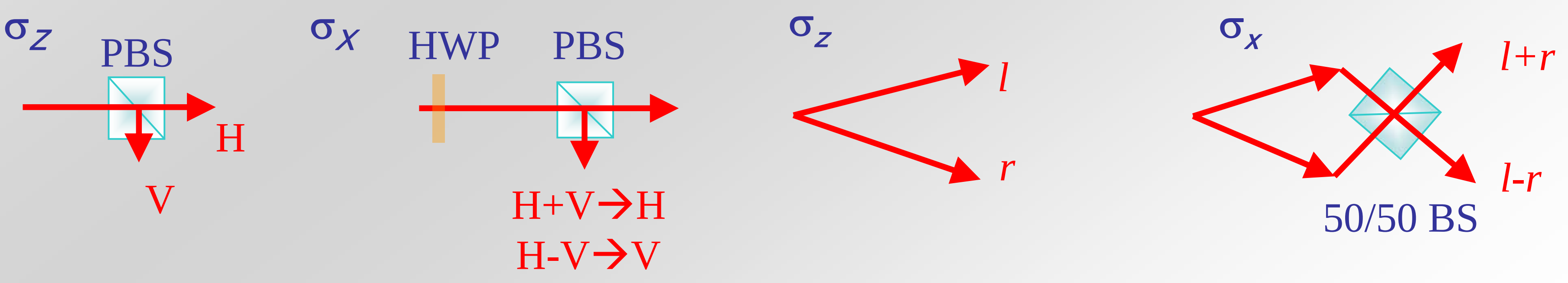
Momentum observables

$$z_i \equiv \sigma_{z_i} = |l_i\rangle\langle l_i| - |r_i\rangle\langle r_i|$$

$$x_i \equiv \sigma_{x_i} = |l_i\rangle\langle r_i| + |r_i\rangle\langle l_i|$$

$$i = a, b$$

easily performed by linear optics



Double singlet state: $$\left|\Xi^{--}\right\rangle = \frac{1}{\sqrt{2}}\left[\left|H_a V_b\right\rangle - \left|V_a H_b\right\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[\left|l_a r_b\right\rangle - \left|r_a l_b\right\rangle\right]$$

Quantum Mechanics:

$$Z_a \bullet Z_b \left|\Xi^{--}\right\rangle = -\left|\Xi^{--}\right\rangle \quad X_a \bullet X_b \left|\Xi^{--}\right\rangle = -\left|\Xi^{--}\right\rangle$$
$$z_a \bullet z_b \left|\Xi^{--}\right\rangle = -\left|\Xi^{--}\right\rangle \quad x_a \bullet x_b \left|\Xi^{--}\right\rangle = -\left|\Xi^{--}\right\rangle$$
$$Z_a z_a \bullet Z_b \bullet z_b \left|\Xi^{--}\right\rangle = \left|\Xi^{--}\right\rangle$$
$$X_a x_a \bullet X_b \bullet x_b \left|\Xi^{--}\right\rangle = \left|\Xi^{--}\right\rangle$$
$$X_a \bullet z_a \bullet X_b z_b \left|\Xi^{--}\right\rangle = \left|\Xi^{--}\right\rangle$$
$$Z_a \bullet x_a \bullet Z_b x_b \left|\Xi^{--}\right\rangle = \left|\Xi^{--}\right\rangle$$
$$Z_a z_a \bullet X_a x_a \bullet X_b z_b \bullet Z_b x_b \left|\Xi^{--}\right\rangle = -\left|\Xi^{--}\right\rangle$$

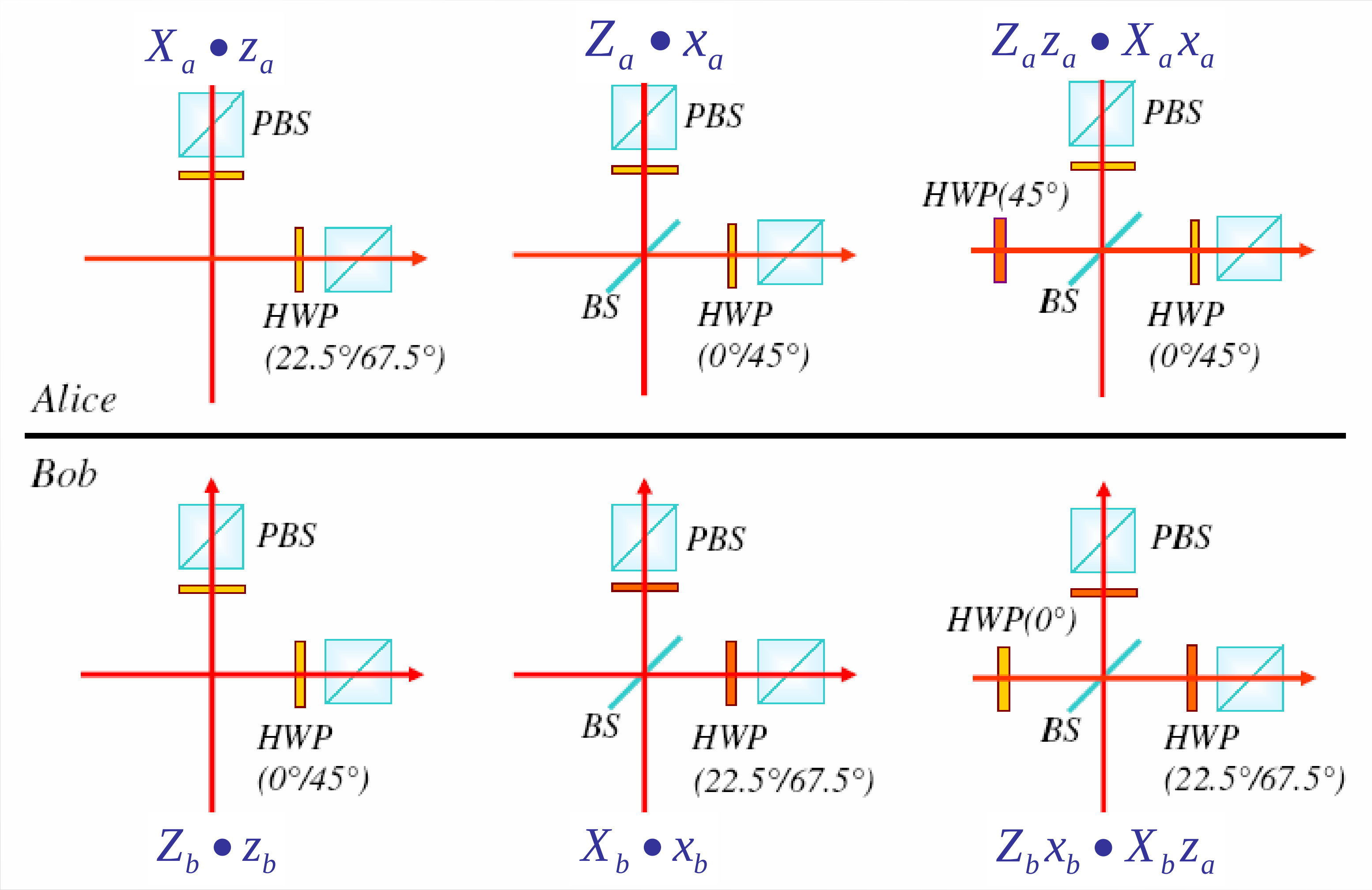
assigning to some observables a pre-definite value v:

$$v(Z_a)v(Z_b) = -1 \quad v(X_a)v(X_b) = -1$$
$$v(z_a)v(z_b) = -1 \quad v(x_a)v(x_b) = -1$$
$$v(Z_a z_a)v(Z_b)v(z_b) = 1$$
$$v(X_a x_a)v(X_b)v(x_b) = 1$$
$$v(X_a)v(z_a)v(X_b z_b) = 1$$
$$v(Z_a)v(x_a)v(Z_b x_b) = 1$$
$$v(Z_a z_a)v(X_a x_a)v(X_b z_b)v(Z_b x_b) = -1$$

By multiplying all the equalities but the last, one gets the inconsistancy:

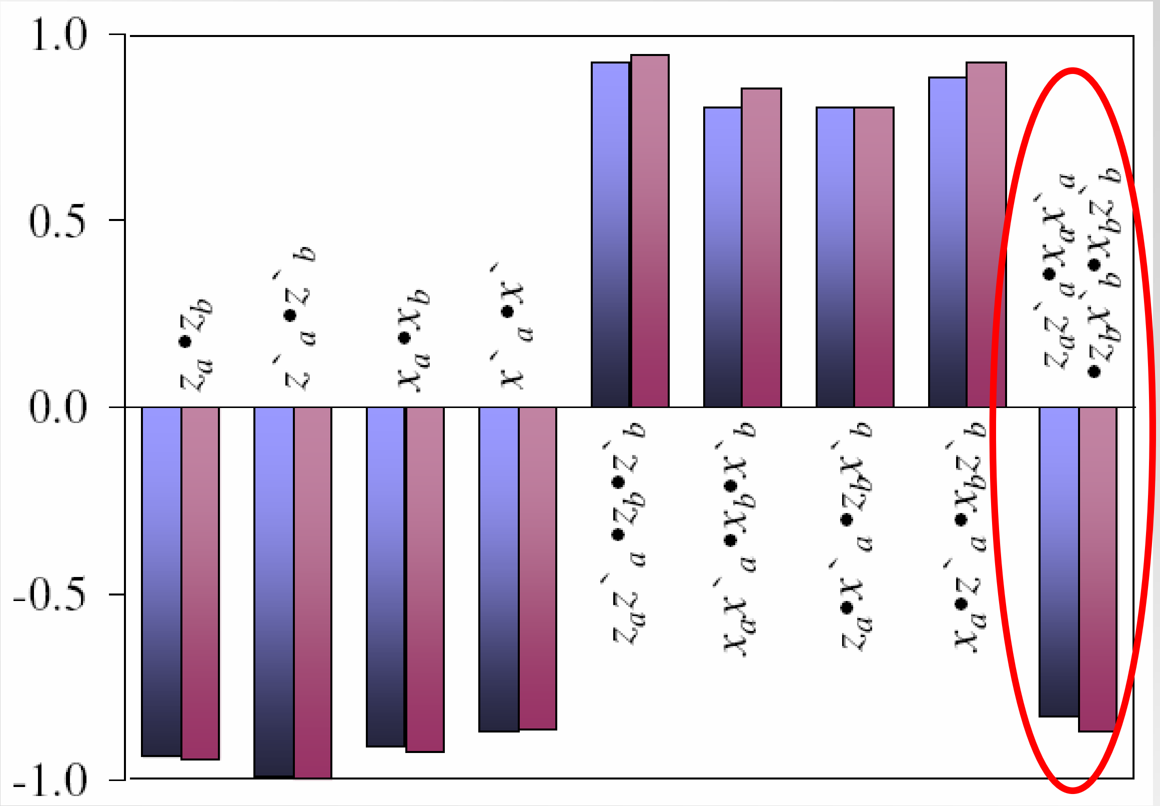
$$v(Z_a z_a)v(X_a x_a)v(X_b z_b)v(Z_b x_b) = 1$$

A.Cabello, PRL **87**, 010403 (2001)
Z. B. Chen, *et al*. PRL **90**, 160408 (2003)



Check all possible combinations between Alice and Bob

Experimental results:



define the operator:

$$\hat{O} = -Z_a \bullet Z_b - X_a \bullet X_b - z_a \bullet z_b - x_a \bullet x_b + Z_a z_a \bullet Z_b \bullet z_b + X_a x_a \bullet X_b \bullet x_b + X_a \bullet z_a \bullet X_b z_b + Z_a \bullet x_a \bullet Z_b x_b - Z_a z_a \bullet X_a x_a \bullet X_b z_b \bullet Z_b x_b$$

Perfect correlations are impossible → inequality is needed:

local realism bound: $\langle \hat{O} \rangle \leq 7$

QM bound: $\langle \hat{O} \rangle = 9$

experiment: $\langle \hat{O} \rangle = 8.114 \pm 0.01$

Inequality violated by 101-σ

Growing-with-size quantum nonlocality

- The magnitude of the violation of local realism can grow exponentially with the number *n* of particles (qubits) [Mermin: PRL. (1990)]

-Hyper-entanglement allows to replace *n* two-level systems by two *N*-level systems (# qubits = 2*N*) [Aravind: Found. Phys. Lett. (2002), Cabello, PRL, (2006)]

$$\left|\Pi^{-+}\right\rangle = \frac{1}{\sqrt{2}}\left[\left|H_a V_b\right\rangle - \left|V_a H_b\right\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[\left|l_a r_b\right\rangle + \left|r_a l_b\right\rangle\right]$$

1) Verify that this state violates two Bell's inequalities, for polarization (π) and momentum (**k**), separately

2) Demonstrate that Bell's inequalities violation grows exponentially with the number *N* of degrees of freedom

Define CHSH the Bell operators β_π and β_k for *polarization* and *momentum* observables:

$$A_\pi = |H\rangle\langle H| - |V\rangle\langle V|$$
$$a_\pi = |V\rangle\langle H| + |H\rangle\langle V|$$
$$B_\pi = \frac{1}{\sqrt{2}}\left[|H\rangle\langle H| - |V\rangle\langle V| + |V\rangle\langle H| + |H\rangle\langle V|\right]$$
$$b_\pi = \frac{1}{\sqrt{2}}\left[|V\rangle\langle V| - |H\rangle\langle H| + |V\rangle\langle H| + |H\rangle\langle V|\right]$$

$$\beta_\pi = -A_\pi \otimes B_\pi + A_\pi \otimes b_\pi + a_\pi \otimes B_\pi + a_\pi \otimes b_\pi$$

$$A_k = |l\rangle\langle r| + |r\rangle\langle l|$$
$$a_k = i\left[|r\rangle\langle l| - |l\rangle\langle r|\right]$$
$$B_k = \frac{1}{\sqrt{2}}\left[(i+1)|r\rangle\langle l| - (i-1)|l\rangle\langle r|\right]$$
$$b_k = \frac{1}{\sqrt{2}}\left[(i-1)|r\rangle\langle l| - (i+1)|l\rangle\langle r|\right]$$

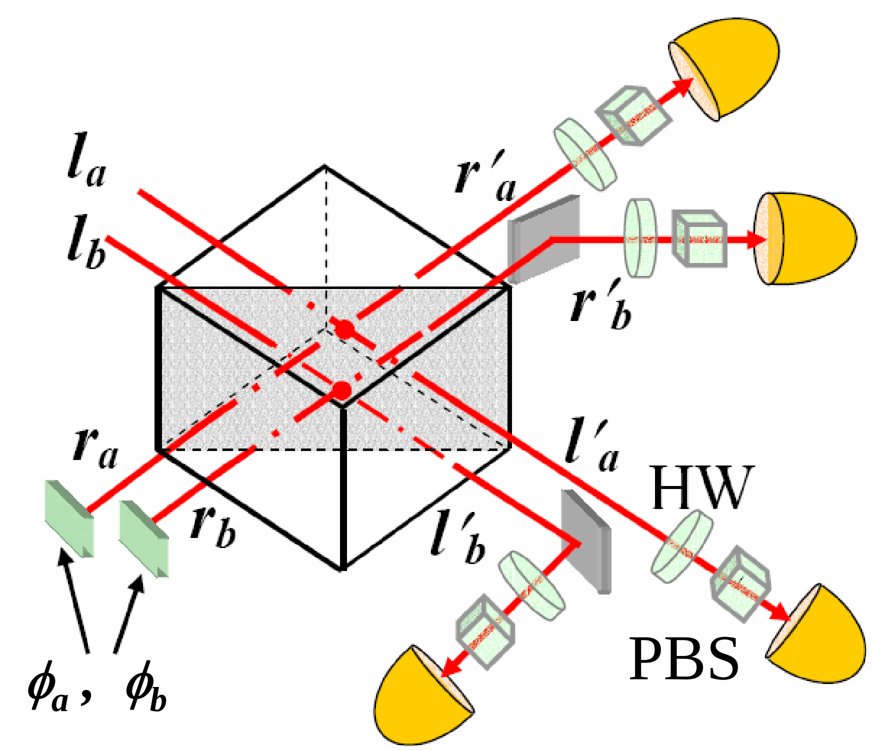
$$\beta_k = A_k \otimes B_k - A_k \otimes b_k + a_k \otimes B_k + a_k \otimes b_k$$

→ LR theory:
$$|\langle \beta_\pi \rangle| \le 2$$
$$|\langle \beta_k \rangle| \le 2$$

$$\beta = \beta_\pi \otimes \beta_k$$ →

LR theory: $\beta \le 4$

QM: $\beta = 8$



Photon a: $A_\pi A_k, A_\pi a_k, a_\pi A_k, a_\pi a_k$

Photon b: $B_\pi B_k, B_\pi b_k, b_\pi B_k, b_\pi b_k$

(16 experimental configurations)

$|\langle \beta_\pi \rangle| = 2.5762 \pm 0.0068$ (85-σ violation)

$|\langle \beta_k \rangle| = 2.5658 \pm 0.0067$ (84-σ violation)

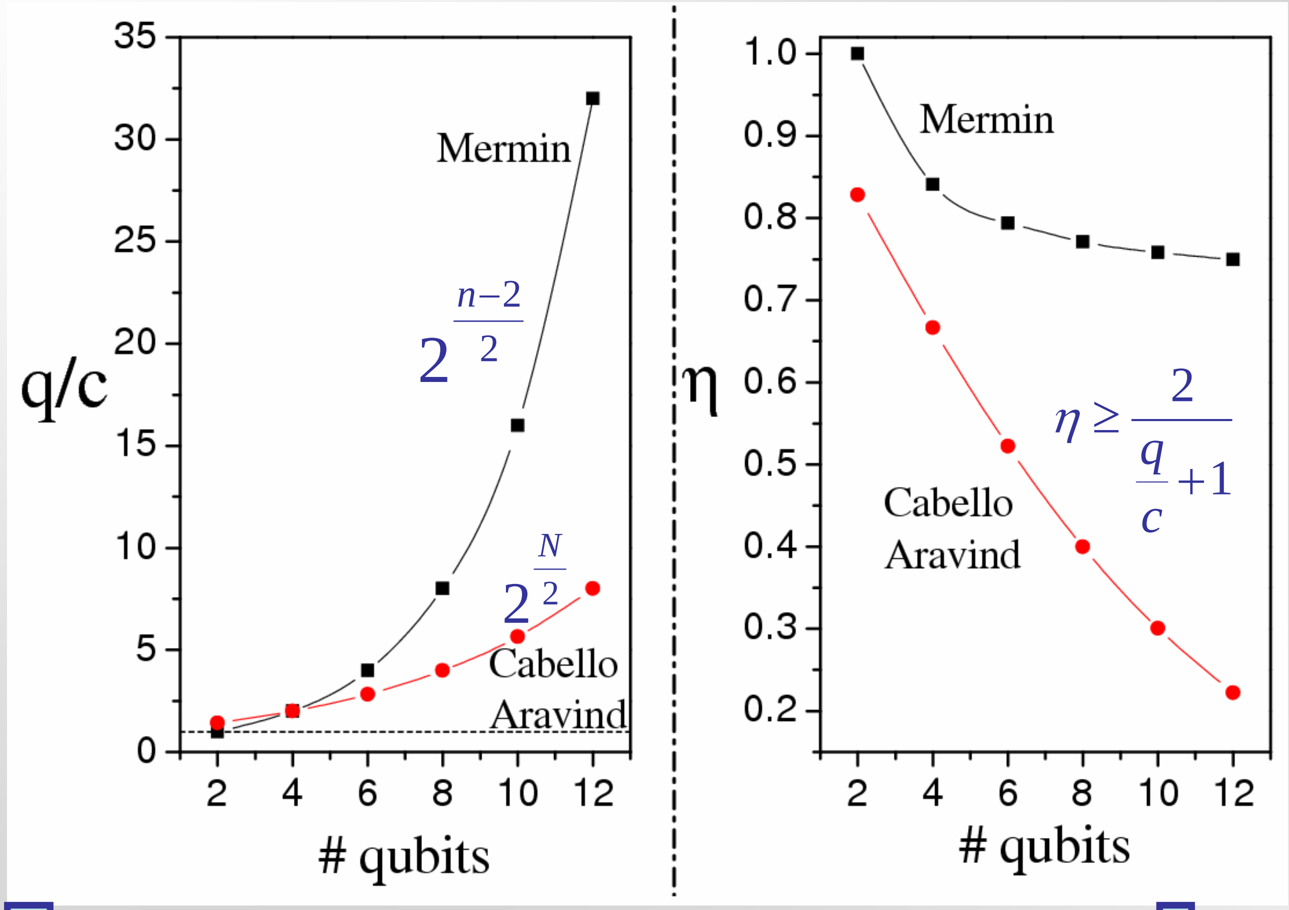
$|\langle \beta \rangle| = 7.019 \pm 0.015$ (196-σ violation)

Note:

$$\frac{|\langle \beta \rangle|}{4} > \frac{|\langle \beta_\pi \rangle|}{2}$$

$$\frac{|\langle \beta \rangle|}{4} > \frac{|\langle \beta_k \rangle|}{2}$$

$$|\langle \beta \rangle| > |\langle \beta_\pi \rangle| |\langle \beta_k \rangle| = 6.610$$



$$N = 1 \Rightarrow \frac{q}{c} = \sqrt{2}$$

$$N = 2 \Rightarrow \frac{q}{c} = 2$$

$$N = 3 \Rightarrow \frac{q}{c} = 2\sqrt{2}$$

No *fair sampling* assumption

Bell states analysis for quantum communication protocols

More secure protocols (QKD, dense coding) and larger robustness to noise achievable by using *d*-level quantum systems (qu-dits, $d = 3,4$)
[Bechmann-Pasquinucci, A. Peres, PRL (2000); Bruss, Macchiavello, PRL (2002)]

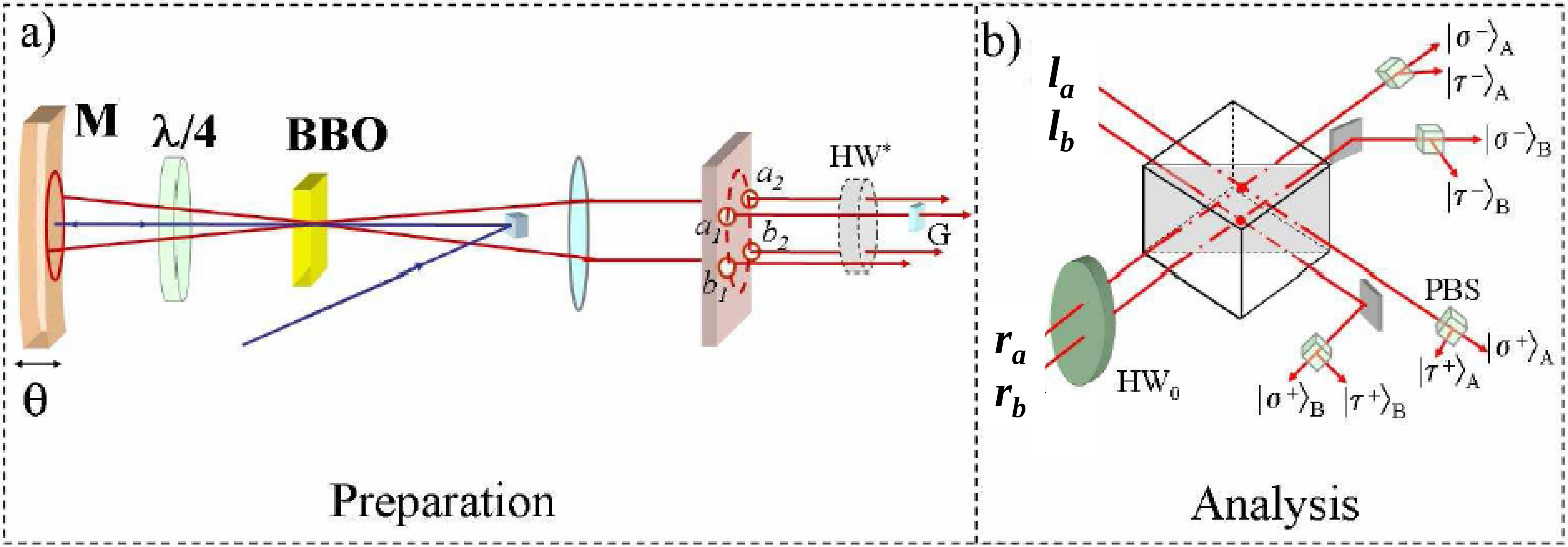
Quantum cryptographic schemes realized with ququarts ($d = 4$) based on mutually unbiased bases made by two-photon Bell states:

$$|\xi\rangle = \alpha|00\rangle + \beta|11\rangle + \gamma|01\rangle + \delta|10\rangle$$

$$|0\rangle \equiv |H\rangle$$
$$|1\rangle \equiv |V\rangle$$

Complete Bell state analysis not achievable by a single joint measurement on the two particles (max. 50%).

⇒ Further deg. of freedom besides polarization are needed (linear momentum assisted measurement)



Analysis based on single photon Bell states:

$$\left|\sigma^{\pm}\right\rangle = \left(\left|Hl\right\rangle \pm \left|Vr\right\rangle\right)$$

$$\left|\tau^{\pm}\right\rangle = \left(\left|Vl\right\rangle \pm \left|Hr\right\rangle\right)$$

$$\left|\Pi^{++}\right\rangle = \left|\Phi^{+}\right\rangle \otimes \left|\psi^{+}\right\rangle = \frac{1}{2}\left[\left|\sigma^{+}\right\rangle_a \left|\tau^{+}\right\rangle_b - \left|\sigma^{-}\right\rangle_a \left|\tau^{-}\right\rangle_b + \left|\tau^{+}\right\rangle_a \left|\sigma^{+}\right\rangle_b - \left|\tau^{-}\right\rangle_a \left|\sigma^{-}\right\rangle_b\right]$$

$$\left|\Pi^{-+}\right\rangle = \left|\Phi^{-}\right\rangle \otimes \left|\psi^{+}\right\rangle = \frac{1}{2}\left[-\left|\sigma^{+}\right\rangle_a \left|\tau^{-}\right\rangle_b + \left|\sigma^{-}\right\rangle_a \left|\tau^{+}\right\rangle_b + \left|\tau^{+}\right\rangle_a \left|\sigma^{-}\right\rangle_b - \left|\tau^{-}\right\rangle_a \left|\sigma^{+}\right\rangle_b\right]$$

$$\left|\Xi^{++}\right\rangle = \left|\Psi^{+}\right\rangle \otimes \left|\psi^{+}\right\rangle = \frac{1}{2}\left[\left|\sigma^{+}\right\rangle_a \left|\sigma^{+}\right\rangle_b - \left|\sigma^{-}\right\rangle_a \left|\sigma^{-}\right\rangle_b + \left|\tau^{+}\right\rangle_a \left|\tau^{+}\right\rangle_b - \left|\tau^{-}\right\rangle_a \left|\tau^{-}\right\rangle_b\right]$$

$$\left|\Xi^{-+}\right\rangle = \left|\Psi^{-}\right\rangle \otimes \left|\psi^{+}\right\rangle = \frac{1}{2}\left[-\left|\sigma^{+}\right\rangle_a \left|\sigma^{-}\right\rangle_b + \left|\sigma^{-}\right\rangle_a \left|\sigma^{+}\right\rangle_b - \left|\tau^{+}\right\rangle_a \left|\tau^{-}\right\rangle_b + \left|\tau^{-}\right\rangle_a \left|\tau^{+}\right\rangle_b\right]$$

Analysis:

1) HW_0 operation:

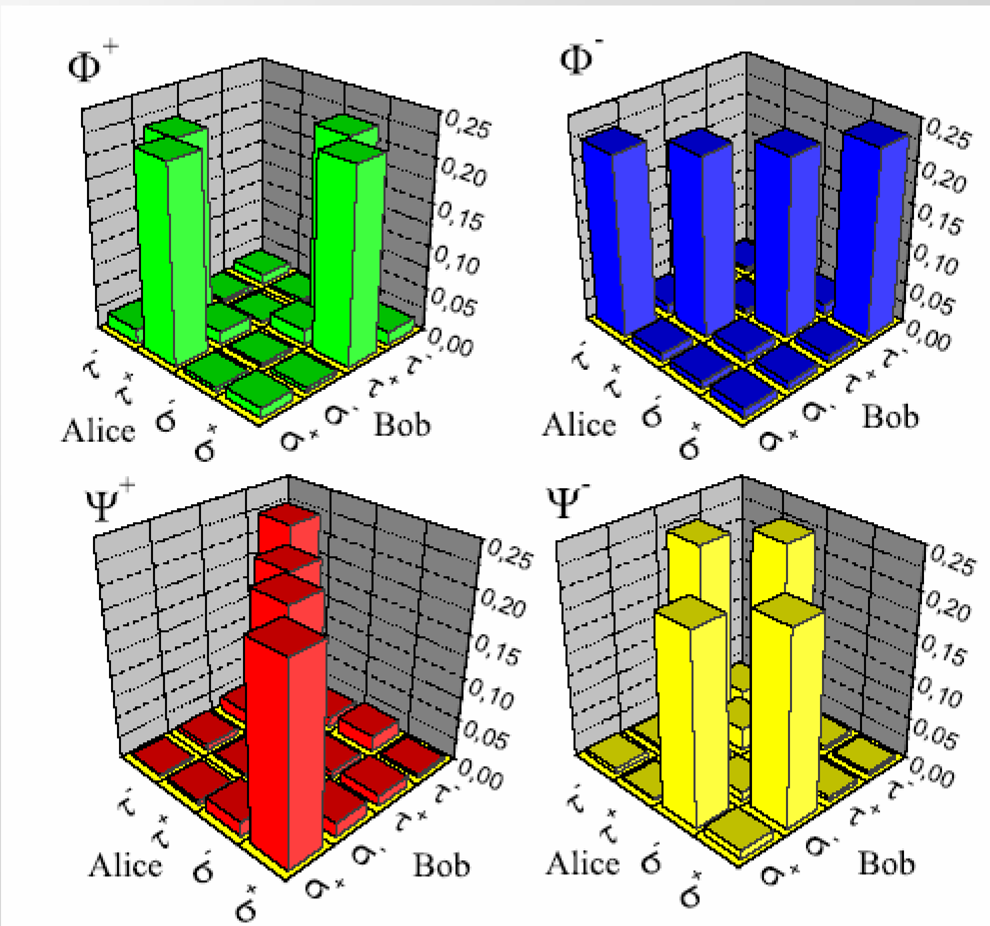
$$|\sigma^{\pm}\rangle \Rightarrow |H\rangle \otimes \frac{1}{\sqrt{2}}\left(|l\rangle \pm |r\rangle\right)$$

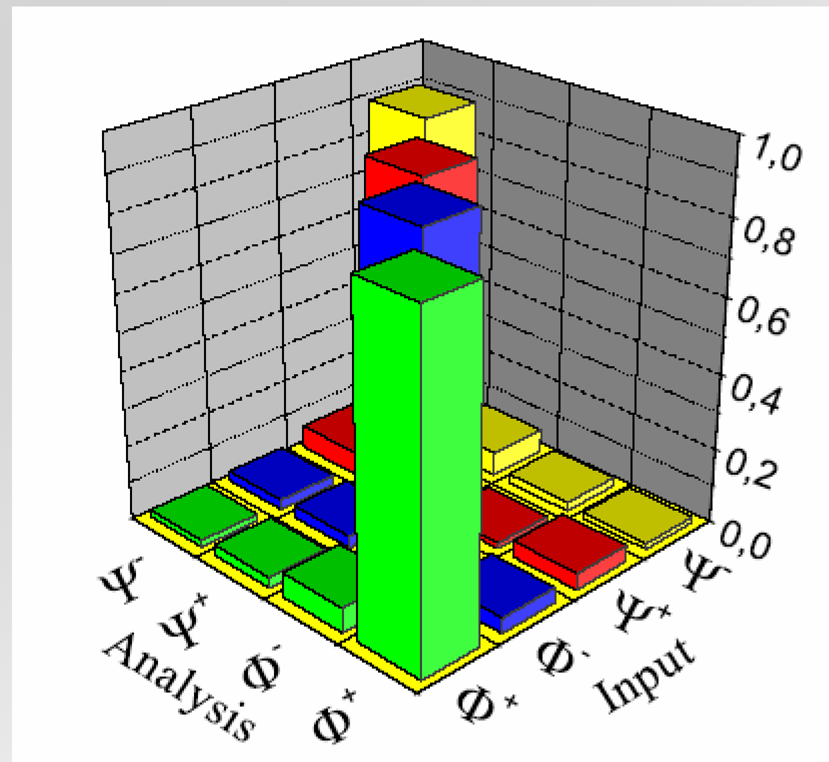
$$|\tau^{\pm}\rangle \Rightarrow |V\rangle \otimes \frac{1}{\sqrt{2}}\left(|l\rangle \pm |r\rangle\right)$$

2) BS discrimination:

$|l\rangle + |r\rangle$ from $|l\rangle - |r\rangle$

3) Pol. analysis performed by PBS's

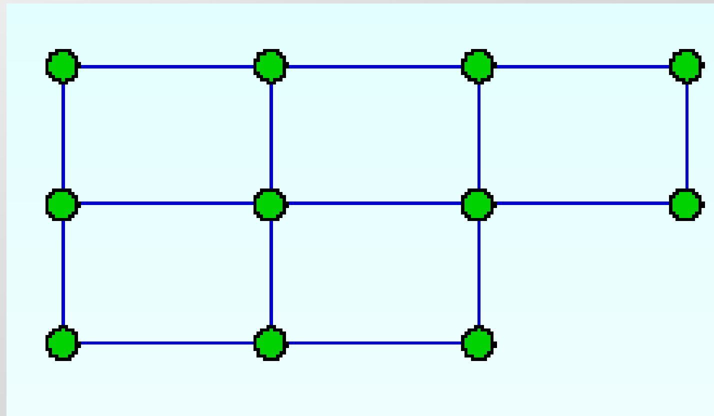




average fidelity F = 0.889 ± 0.010

Two photon linear cluster states

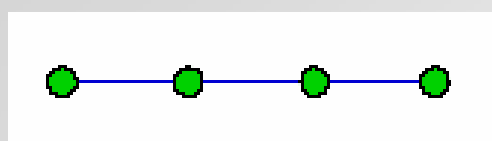
Cluster state: quantum state obtained by starting from a finite n-dimensional lattice



Each dot corresponds to the qubit: $\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

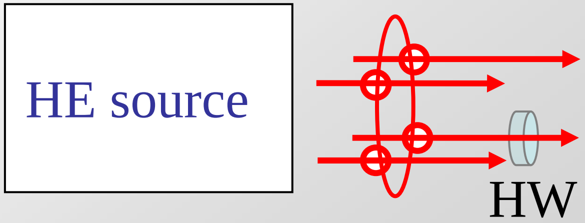
Each link corresponds to a Control Phase (CP) gate: $CP = |0\rangle_1\langle 0| \otimes \mathbb{1}_2 + |1\rangle_1\langle 1| \otimes \sigma_z^{(2)}$

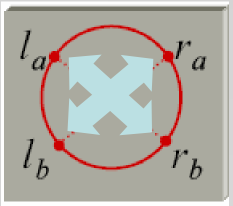
4-qubit linear cluster state:



4-photons: $\frac{1}{4}\left[|H_aH_bH_cH_d\rangle + |H_aH_bV_cV_d\rangle + |V_aV_bH_cH_d\rangle - |V_aV_bV_cV_d\rangle\right]$

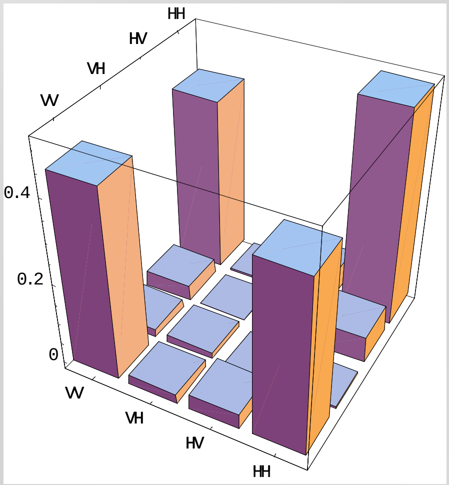
Create 2-photons linear cluster states (starting by hyperentanglement):



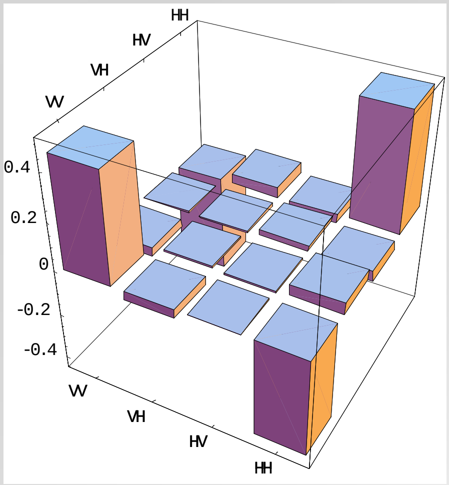


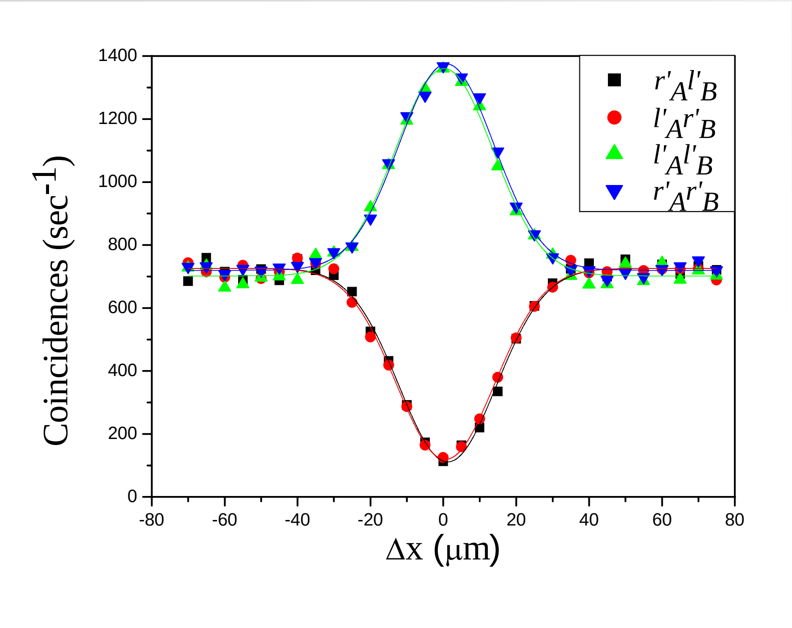
$$\frac{1}{4}\left[\left|H_a l_a\right\rangle\left|H_b r_b\right\rangle+\left|H_a r_a\right\rangle\left|H_b l_b\right\rangle+\left|V_a l_a\right\rangle\left|V_b r_b\right\rangle-\left|V_a r_a\right\rangle\left|V_b l_b\right\rangle\right]$$

Modes r_a- l_b



Modes l_a- r_b



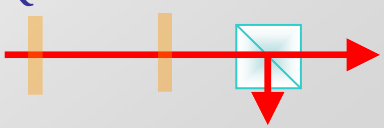


Characterization and nonlocality test with linear cluster states:

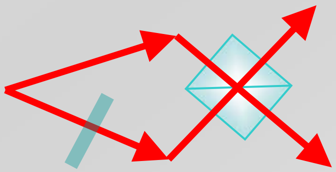
- Entanglement Witness [Kiesel *et* al. PRL (2005)]
- Stronger two observer AVN violation of Local Realism (provides larger amount of LR violation) [Cabello, PRL (2005)]

Both require to measure further operators: σ_Y (Y) and σ_y (y)

σ_Y:



σ_y:



$$\hat{S} = X_a x_a Y_b y_b - Y_a x_a X_b y_b + X_a X_b z_b + Y_a Y_b z_b$$

$$2 \leq \langle S \rangle \leq 4 \quad \Rightarrow \quad \langle S \rangle = 3.4145 \pm 0.0053$$

$$\hat{W} = \frac{1}{2}\left[4 - Z_a Z_b + z_a z_b - Z_a x_a x_b - Z_b x_a x_b - X_a X_b z_b + X_a X_b z_a\right]$$

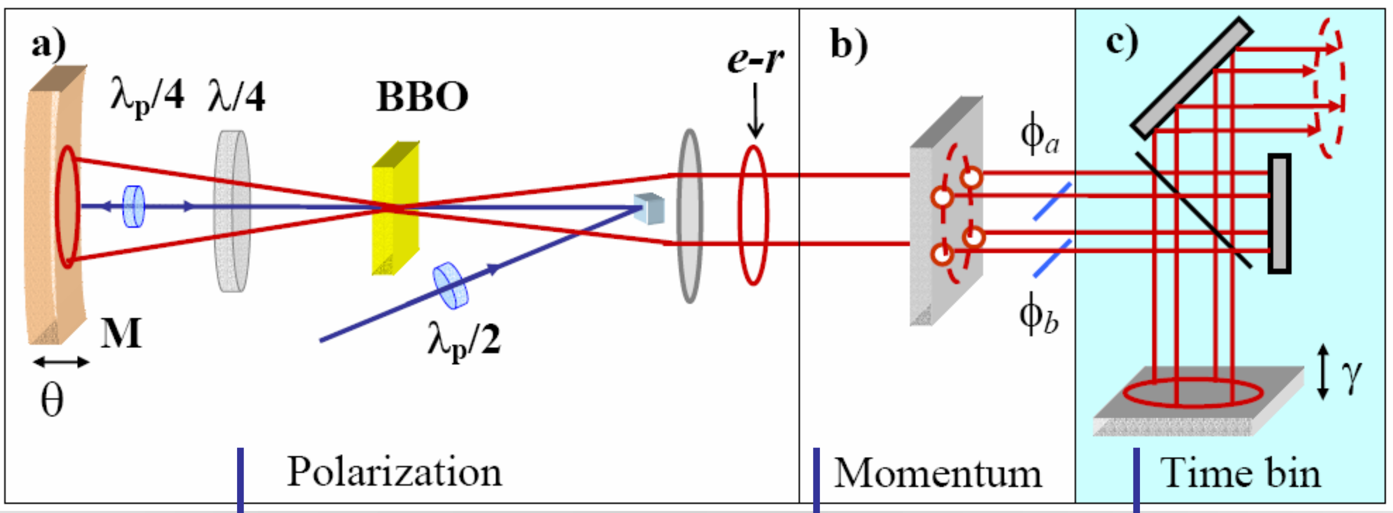
$$\langle W \rangle_{Cluster} = -1$$

$$\langle W \rangle_{HE} = +1$$

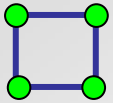
$$\Rightarrow \quad \langle W \rangle = -0.6814 \pm 0.0071$$

Perspectives

1) Enhancing quantum nonlocality by further degrees of freedom:



$$\left[|H_aH_b\rangle + e^{i\theta}|V_aV_b\rangle\right]\otimes\left[|l_ar_b\rangle + e^{i\phi}|r_al_b\rangle\right]\otimes\left[|L_aL_b\rangle + e^{i\gamma}|S_aS_b\rangle\right]$$

2) Realization of the box linear cluster states: 

3) Use two photon cluster states for one-way quantum computation?

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- M. Barbieri, *et al.*, "*Enhancing the Violation of the Einstein-Podolsky-Rosen Local Realism by Quantum Hyper-entanglement*", Phys. Rev. Lett. **97**, 140707 (2006).
- M. Barbieri, *et al.*, "*Complete and Deterministic Bell state identification by two photon hyper-entanglement"*, quant-ph/0609080.