Impurities induced dephasing in solid state qubits

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"Quantum Mechanics: from fundamental problems to applications", Bertinoro 6/12/ 2006

Solid-state qubits…

☺ Scalability, integration, easy tunable

☹ BUT solid-state noise, crosstalk, …

Tasks of present research on SC nanostructures

- Identification of major noise sources and quantitative estimate of the ensuing decoherence
- Measurement of decoherence rate of single qubits

  Major activity of all experimental groups

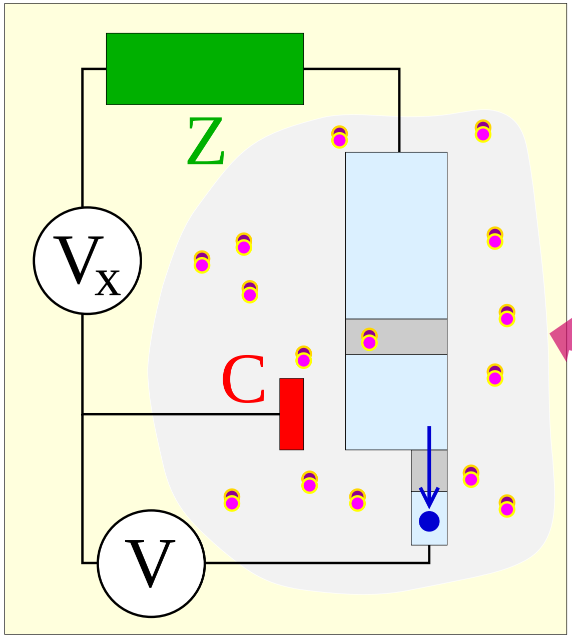
- Measurement/investigation of decoherence rates in two-qubit circuits

  Emerging activity

- Quantum control

  Major activities of all theory + experimental groups

Noise sources in SC qubits



Z(ω) electromagnetic fluctuations of the circuit (**gaussian**)

discrete noise due to fluctuating **background charges (BC)** trapped in the substrate or in the junction

- trapped flux in mesoscopic SQUIDs
- trapped flux at the junctions in smaller SQUIDs

quasiparticles/measurement

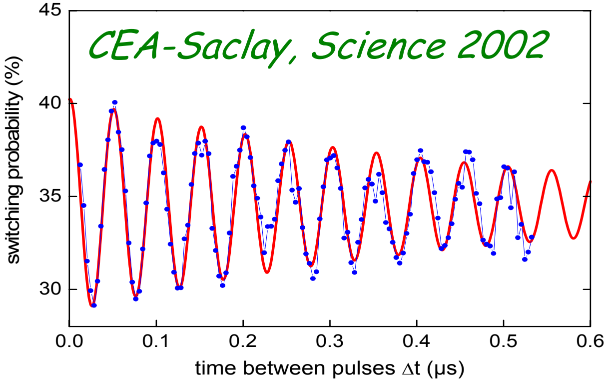
charge noise (1/f)

↔ switching impurities close to the device

Paladino, Faoro, Falci, Fazio, PRL 2002

➢ **THE** problem in high Q charge based qubits

➢ Affects two-qubit operations in spin-qubits



Noise characterization

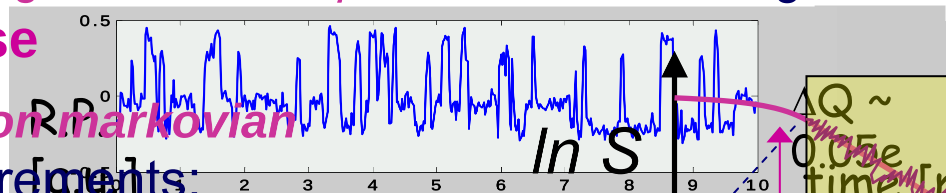
- Noise due to *charged bistable impurities → RTN*

- Distribution of *charged bistable impurities* with switching rates $P(\gamma) = C/\gamma$
  leads to **1/f noise**

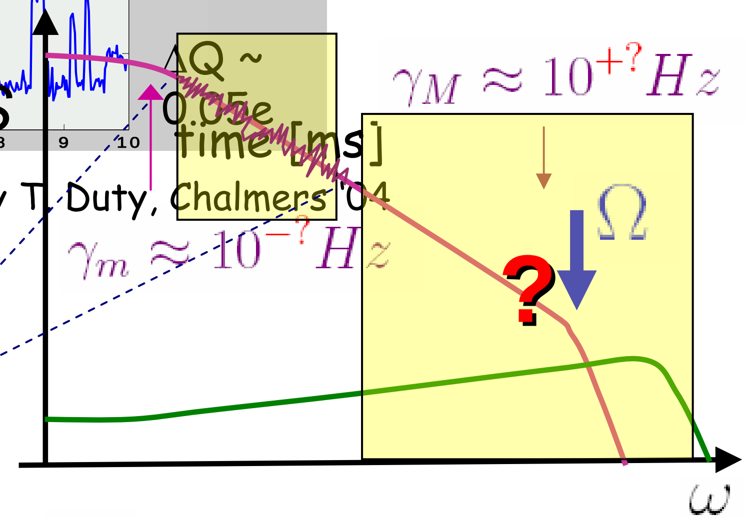
non gaussian, non markovian

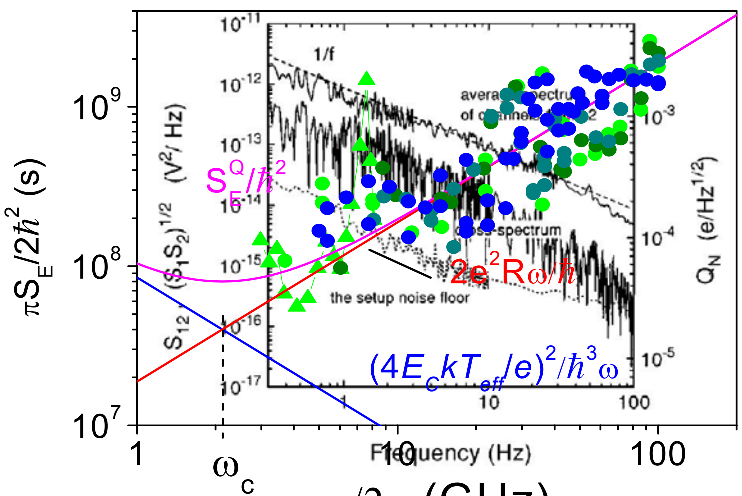
1/f noise measurements:

uncertainty on all high-frequency properties, eg. γ_M *vs* Ω



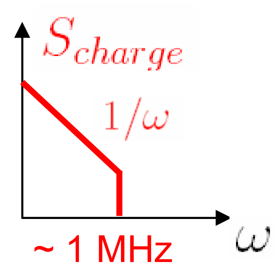
Courtesy T. Duty, Chalmers '04





Experiments in SET: *Zorin et al. PRB (1996)*

Experiments in charge qubit: *Astafiev et al., PRL 04*

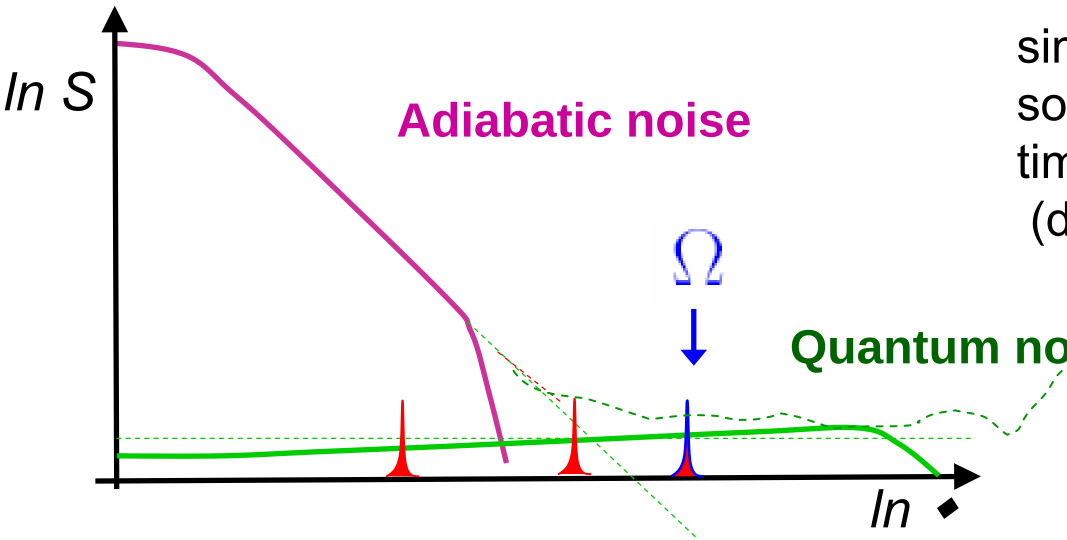


Saclay qubit

Ithier et al., PRB 05

Noise characterization

Variety of observations, material & device dependent



simultaneous presence of noise sources with different characteristic time scales
(different approximation schemes)

Strongly coupled noise

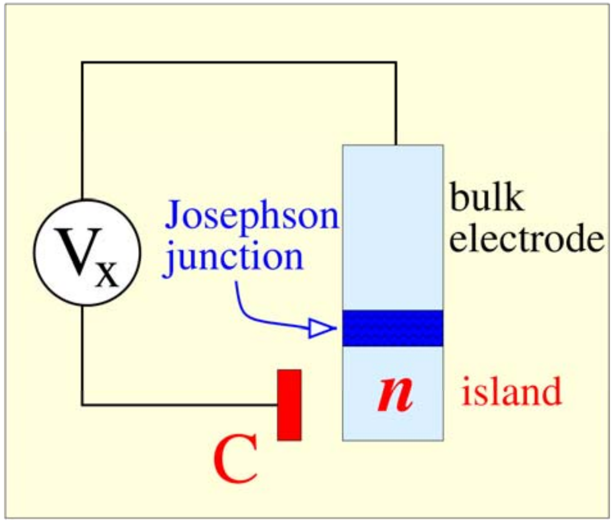
Falci, D'Arrigo, Mastellone, Paladino, PRL 2005

FOCUS: single impurity model

- Collection of impurities may originate 1/f + f noise
- Uncontrollable *dynamical impurities:*
  expected relevant limitation for *multi-qubit on-chip* devices

Qubit-impurity model

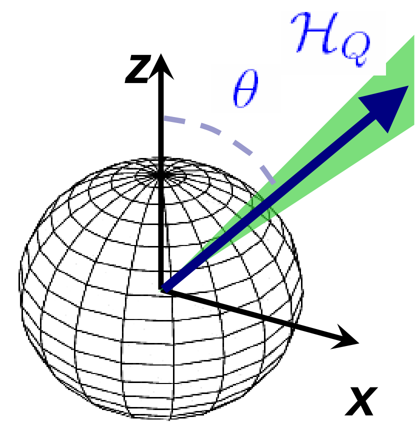
Experiments: V. Bouchiat et al., Journal of Superconductivity (1999)



$$\mathcal{H}_{qb} = -\frac{\epsilon_q}{2}\sigma_z - \frac{E_j}{2}\sigma_x$$

extra charge on the island

$$\{|0\rangle, |1\rangle\} \Longleftrightarrow \sigma_z$$



Charged impurity couples to σ_z

$$\mathcal{H}_{qb-imp} = -\frac{\epsilon_q}{2}\sigma_z - \frac{E_j}{2}\sigma_x - \frac{v}{2}\sigma_z\tau_z + \mathcal{H}_{imp}$$

Single impurity model

$$\mathcal{H}_{imp} = -\frac{\varepsilon}{2}\tau_z - \frac{\Delta}{2}\tau_x - \frac{1}{2}\hat{X}\,\tau_z + \mathcal{H}_E$$

spin-boson model

$$\mathcal{H}_E = \sum \omega_\alpha a^\dagger_\alpha a_\alpha \qquad \hat{X} = \sum \lambda_\alpha (a_\alpha + a^\dagger_\alpha)$$

$$\mathcal{H}_{qb-imp} = -\frac{\epsilon_q}{2}\sigma_z - \frac{E_j}{2}\sigma_x - \frac{v}{2}\sigma_z\,\tau_z + \mathcal{H}_{imp}$$

$$S(\omega) = \pi\, G(|\omega|)\, \coth\frac{\beta|\omega|}{2}$$

Qubit σ and bist τ le impurity

Idle qubit τ

target σ τ d control

$$\rho(t) = \mathrm{Tr}_{\tau,osc}\{W(t)\}$$

Qubit σ as measurement device

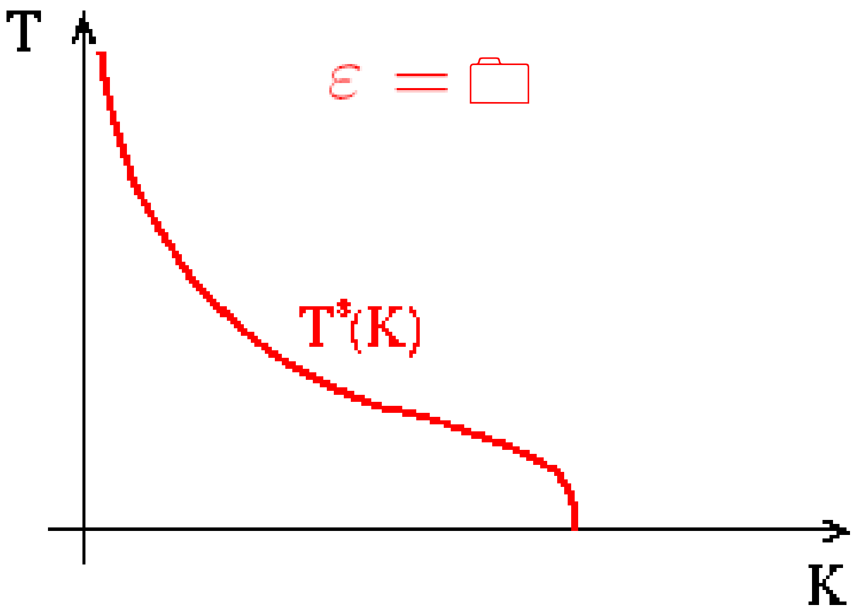
Two qubits, σ τ and coupled to a oscillator

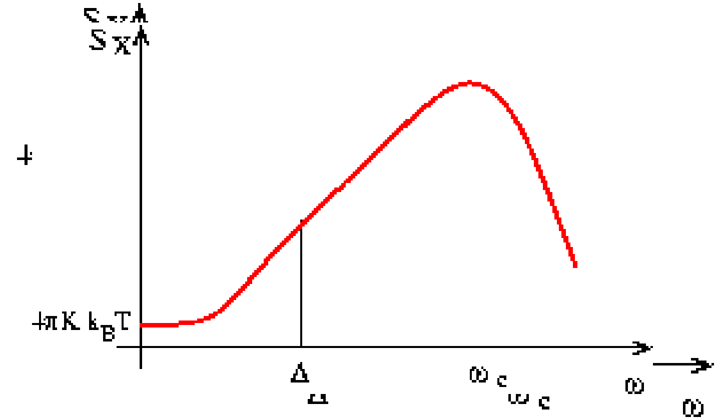
Paladino, Sassetti, Falci, Chem. Phys. 2005

Impurity dynamical regimes

$$\mathcal{H}_{imp} = -\frac{\varepsilon}{2}\tau_z - \frac{\Delta}{2}\tau_x - \frac{1}{2}\hat{X}\,\tau_z + \mathcal{H}_E$$

$\varepsilon = $





T<T*: time scale

time scale

$$\Delta_T = \Delta_r (2\pi k_B T/\Delta_r)^K$$

T>T* time scale

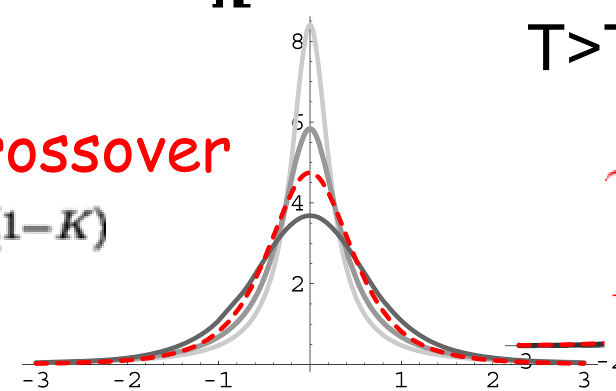
Large T

coherent-incoherent crossover

Low T

$$T_c < \Delta_r = \Delta(\Delta/\omega_c)^{K/(1-K)}$$

$$k_B T^*(K) \approx \Delta_r/\pi K$$



$$\gamma(T) = 2\pi K k_B T$$

$$\rightarrow \frac{\Delta_r}{K}\left(\frac{\Delta_r}{2\pi k_B T}\right)^{1-2K}$$

Kondo/Zeno

Impurity time scales

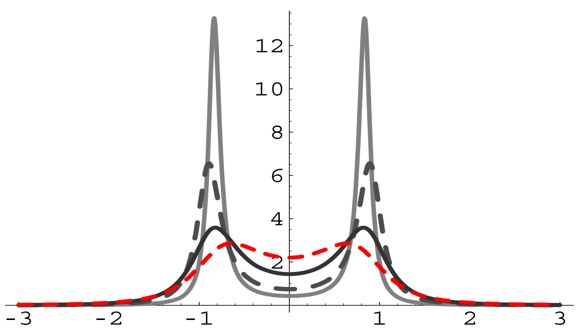
$$\mathcal{H}_{qb-imp} = -\frac{\epsilon_q}{2}\sigma_z - \frac{E_j}{2}\sigma_x - \frac{v}{2}\sigma_z\tau_z + \mathcal{H}_{imp}$$

$$S_\tau(\omega) = v^2 \int_{-\infty}^{\infty} dt \, \frac{1}{2} \left(\langle \tau_z(t)\tau_z(0) + \tau_z(0)\tau_z(t) \rangle - \langle \tau_z \rangle_\infty^2 \right) e^{i\omega t}$$

Impurity "correlation time"

$$\tau_c^{-1} \approx \Delta_T$$

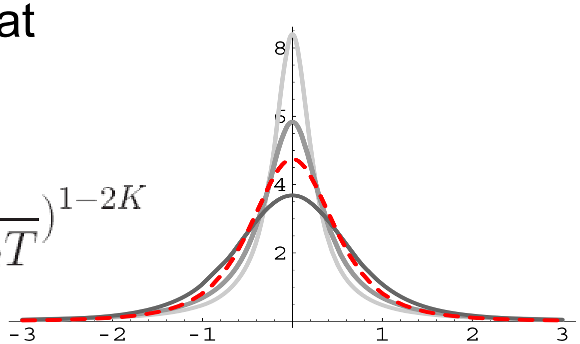
$$\tau_c^{-1} \approx \gamma(T)$$



Kondo behavior at

$$T \gg T^*(K)$$

$$\gamma(T) \approx \frac{\Delta_r}{K}\left(\frac{\Delta_r}{2\pi k_B T}\right)^{1-2K}$$



T

Expected dynamical regimes for the qubit

$$\mathcal{H}_{qb-imp} = -\frac{\epsilon_q}{2}\sigma_z - \frac{E_j}{2}\sigma_x - \frac{v}{2}\sigma_z \tau_z + \mathcal{H}_{imp}$$

Weak-coupling $v\,\tau_c^{-1} \ll 1$

$$\Gamma_r = \frac{1}{2}\sin\theta^2 S_\tau(\Omega_1)$$

$$\Gamma_\phi = \frac{1}{4}\sin\theta^2 S_\tau(\Omega_1) + \frac{1}{2}\cos\theta^2 S_\tau(0)$$

$$\Omega_1 = \sqrt{\epsilon_q^2 + E_J^2} \rightarrow \Omega_1 + shift$$

$$\delta\Omega_1 = -\frac{1}{4}\sin^2\theta\,\mathcal{P}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{S_\tau(\Omega_1)}{\omega-\Omega}$$

Valid only if the Environment is fast

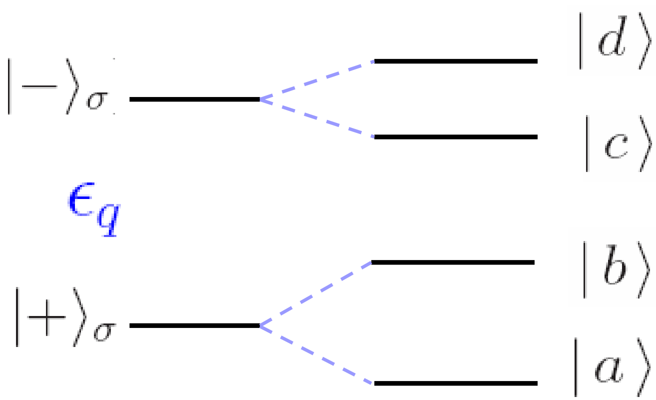
$$\tau_c \ll \Delta t \ll \Gamma_\phi^{-1}$$

Modulable correlation time $\tau_c(T) \longrightarrow v\,\tau_c^{-1} \geq 1$

qubit dynamics ??

Extended Hilbert space qubit+impurity

Pure dephasing $\mathcal{H}_0 = -\frac{\epsilon_q}{2}\sigma_z - \frac{\varepsilon}{2}\tau_z - \frac{\Delta}{2}\tau_x - \frac{v}{2}\sigma_z\tau_z$



$\sigma_z|\pm\rangle_\sigma = \pm|\pm\rangle_\sigma$

$$E_{-\pm} = \frac{1}{2}(\epsilon_q \mp \Omega_-)$$

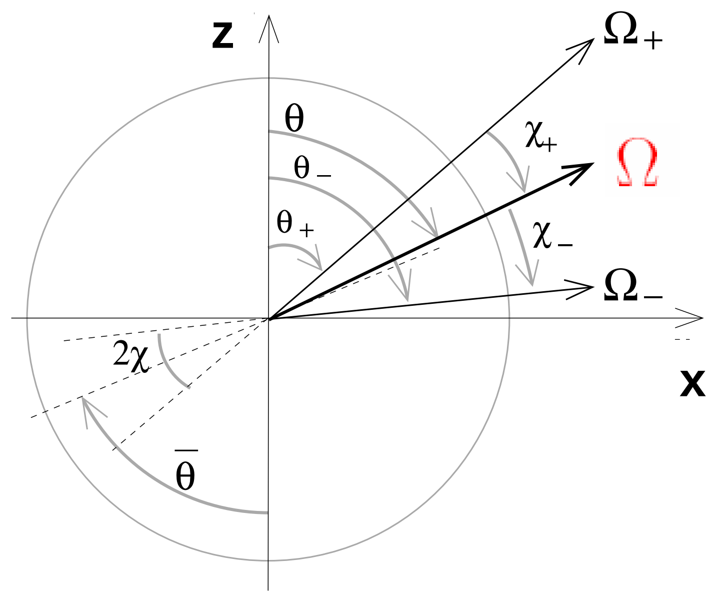
$$E_{+\pm} = -\frac{1}{2}(\epsilon_q \pm \Omega_+)$$

$-\frac{1}{2}\hat{X}\,\tau_z$

intra-doublet transitions

conditional level splittings

$$\Omega_\pm = \sqrt{(\varepsilon \pm v)^2 + \Delta^2}$$



Theory for general T and coupling v

$\rho(t)$ qubit+impurity density matrix

At pure dephasing $[\sigma_z, \mathcal{H}] = 0$

relevant dynamical quantities

$$\langle \sigma_- \rangle = \mathrm{Tr}\,(\rho\,\sigma_-) = \cos\chi\,(\rho_{ac} + \rho_{bd}) + \sin\chi\,(\rho_{ad} - \rho_{bc})$$

coherences

Master equation in the enlarged Hilbert space

$$\partial_t \rho(t) = -i[\mathcal{H}_0, \rho(t)] - \int_0^\infty dt' \{ \tfrac{1}{4} S(t')\,[\tau_z, [\tau_z(t'), \rho(t)]] + \tfrac{i}{2}\chi(t')\,[\tau_z, [\tau_z(t'), \rho(t)]_+]\}$$

with factorised initial condition

$$\rho(0) = \rho_\sigma(0) \otimes \rho_\tau(0) \qquad \rho_\tau(0) = \frac{1}{2}\hat{I} + \frac{1}{2}\delta p(0)\,\tau_z$$

Coherences (secular approximation)

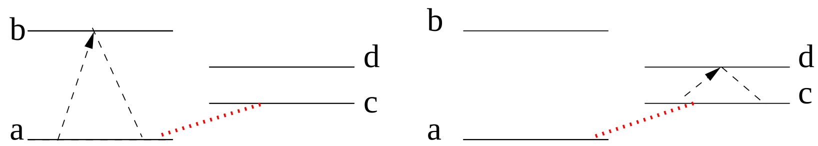
$$\begin{pmatrix} \dot\rho_{ac} \\ \dot\rho_{bd} \end{pmatrix} = \begin{pmatrix} -i\delta - \Gamma_1 & \Gamma_{12} \\ \Gamma_{21} & i\delta - \Gamma_2 \end{pmatrix} \begin{pmatrix} \rho_{ac} \\ \rho_{bd} \end{pmatrix} \qquad \delta = (\Omega_+ - \Omega_-)/2$$

$$\begin{pmatrix} \dot\rho_{ad} \\ \dot\rho_{bc} \end{pmatrix} = \begin{pmatrix} -i\Omega - \Gamma_3 & \Gamma_{34} \\ \Gamma_{43} & i\Omega - \Gamma_4 \end{pmatrix} \begin{pmatrix} \rho_{ad} \\ \rho_{bc} \end{pmatrix} \qquad \Omega = (\Omega_+ + \Omega_-)/2$$

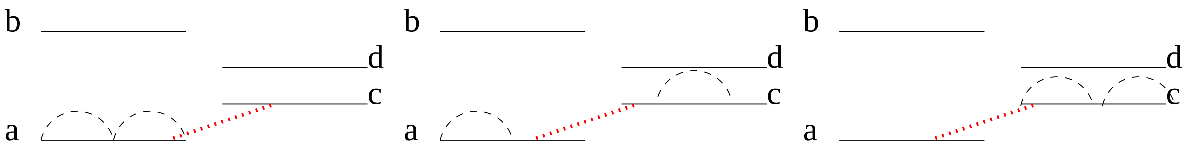
$$\Gamma_{21} = \tfrac{1}{2}\tau_{ab}\tau_{cd}\left[\Gamma_-(\Omega_+) + \Gamma_-(\Omega_-)\right]$$

absorption/emission rates

$$\Gamma_\pm(\omega) = \pi G(|\omega|)sgn(\omega)[\coth(\beta\omega/2)\pm 1]$$



Γ_1



Coherences

$$\langle \sigma_- \rangle_t = \frac{e^{i\epsilon_q t}}{2}\left[\cos\chi^2\left(e^{\lambda_1 t} + e^{\lambda_2 t} - \frac{\Gamma_{12}+\Gamma_{21}}{\Lambda}\left(e^{\lambda_1 t} - e^{\lambda_2 t}\right)\right)\right.$$
$$\left. + \sin\chi^2\left(e^{\lambda_3 t} + e^{\lambda_4 t} + \frac{\Gamma_{34}+\Gamma_{43}}{\Sigma}\left(e^{\lambda_3 t} - e^{\lambda_4 t}\right)\right)\right]$$

$$\lambda_{1/2} = -\frac{\Gamma_1+\Gamma_2}{2} \pm \frac{\sqrt{(2i\delta+\Gamma_2-\Gamma_1)^2+4\Gamma_{12}\Gamma_{21}}}{2}$$

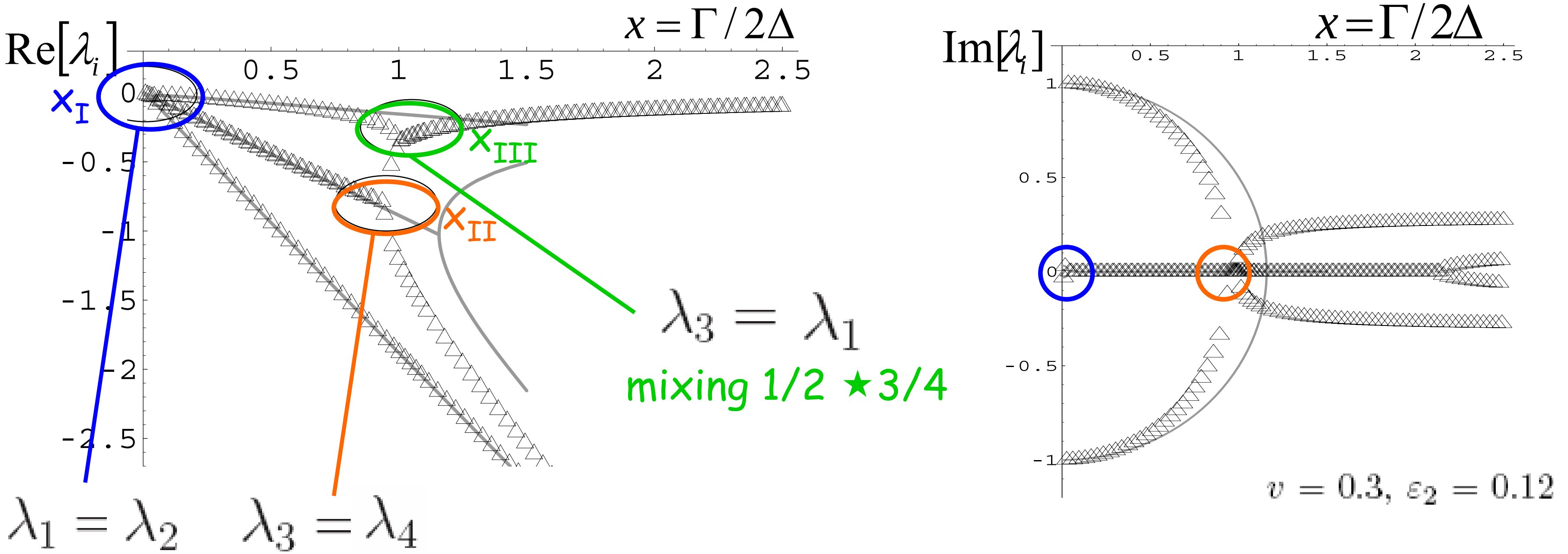
$$\lambda_{3/4} = -\frac{\Gamma_3+\Gamma_4}{2} \pm \frac{\sqrt{(2i\Omega+\Gamma_4-\Gamma_3)^2+4\Gamma_{34}\Gamma_{43}}}{2}$$

The qubit dynamics is conditioned by crossings of λ_i changing T and v

✓ NOTE: ME interpolates between systematic expansions valid for T→0 (systematic weak damping approximation) and in the high-T limit (NIBA)

High-temperatures

White noise $S(\omega) = 2\,\Gamma$ (with $\Gamma = 2\pi K k_B T$)



$x_I < x < x_{III}$
$x > x_{III}$

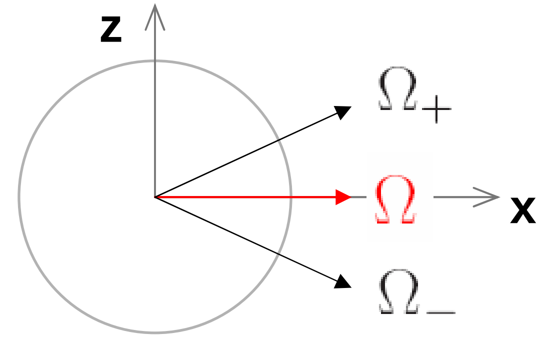
$$\langle \sigma_- \rangle \approx e^{i\Omega_1 t - \Gamma_Z t}\left\{ A e^{i\Omega_Z t} + B\, e^{-i\Omega_Z t} \right\}\ {}_{1/2^t}$$

$$- \sin\chi \left|\rho_{ad}(0) e^{-i\Omega_{2\text{eff}} t} - \rho_{bc}(0) e^{i\Omega_{2\text{eff}} t}\right| e^{-\mathcal{R}e\,\lambda_{3/4} t}\}$$

slow modulation & beatings at δ

Counterintuitive: τ slowly relaxing with $\Gamma_Z \approx \Delta^2/(2\Gamma)$ modulates the phase of σ at $\Omega_Z \approx \bar{\Omega} \sin\bar{\theta} \sin\chi + \delta \cos\bar{\theta} \cos\chi$ independent on damping!!!!

Coherences at $\epsilon = 0$



Additional symmetries

$$\delta = 0 \qquad \Omega_+ = \Omega_- = \Omega = \sqrt{v^2 + \Delta_2^2}$$

Three complex λ_i

$$\lambda_2 = -\gamma_\phi \qquad \lambda_{3/4} = -\frac{\gamma_r}{2} \pm \sqrt{\left(\frac{\gamma_r}{2}\right)^2 - \Omega^2}$$

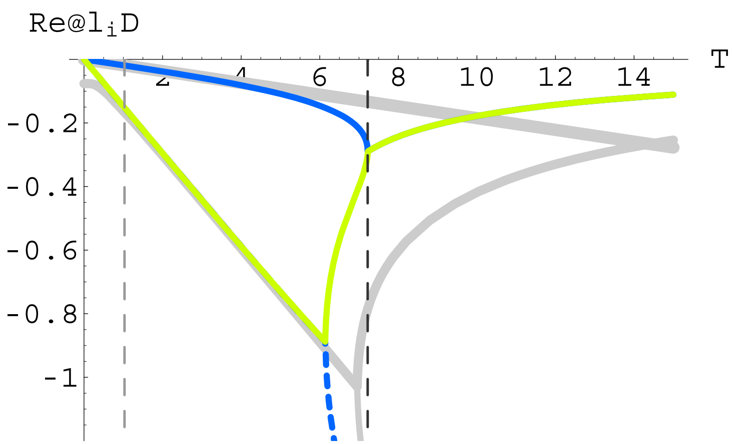
Simply related to GR relaxation and dephasing rate for τ

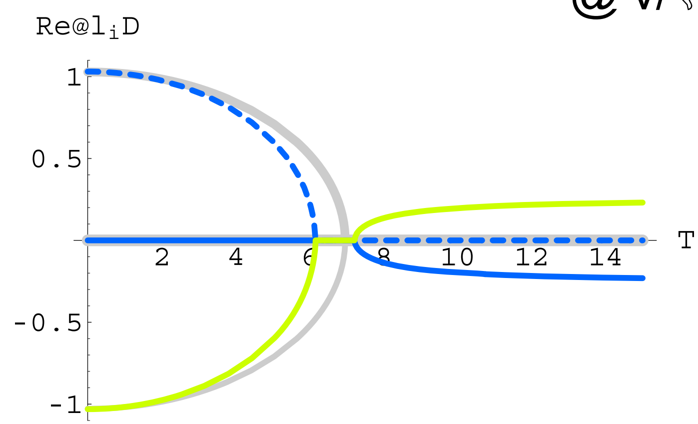
$$\gamma_r = \frac{1}{2}\sin\theta^2 S(\Omega) \qquad \gamma_\phi = \frac{1}{2}\cos\theta^2 S(0)$$

Crossings: one real $\rightarrow$ two complex conjugate dominant eigenvalues

Coherences

@ v/☟=0.25

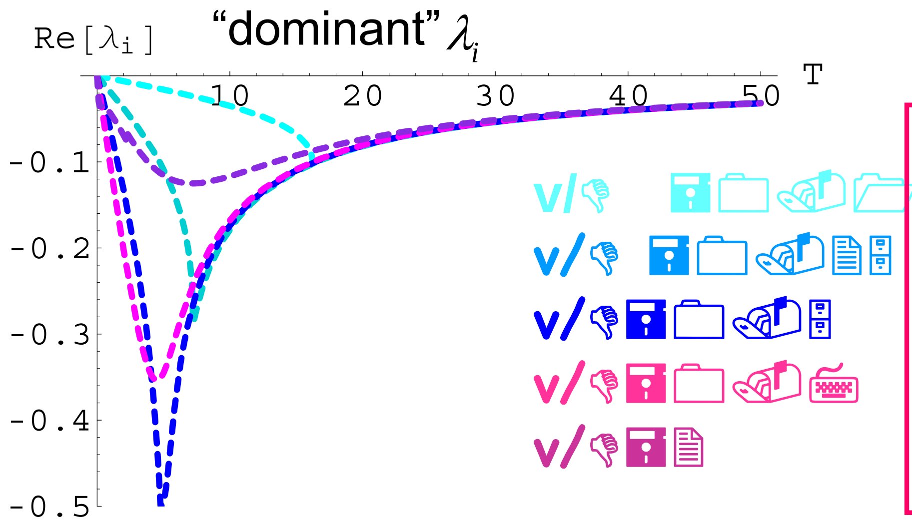




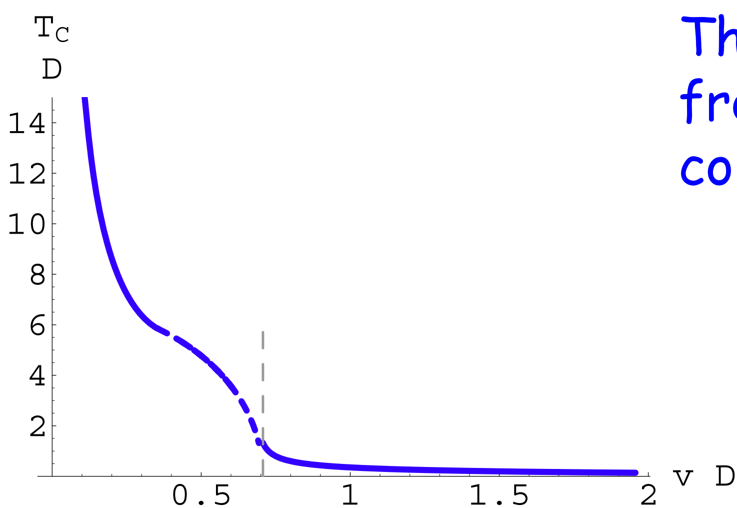
Small temperatures : single real dominat eigenvalues

High temperature: two complex conjugate eigenvalues

Crossover modulated by T and v



- Possible strong dephasing also at small T (from SMALL couplings v)
- Saturation of dephasing at high T (Kondo/Zeno behavior) independent on v

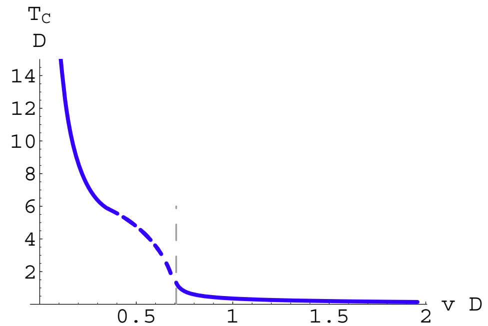


The crossover temperature follows from the condition on the impurity correlation time (if v/👎<<1)

$$v\,\tau_c^{-1} = 1$$

Conclusions & Perspectives

- ✓ Impurity model showing a complex dynamics
- ✓ Crossover effects in the qubit dynamics



Hints:

- High temperatures saturation of dephasing → dynamical decoupling techniques for "dirty" qubits
- Very low temperatures → small dephasing for "clean" qubits

Paladino, Sassetti, Falci, Weiss, submitted 2006

- ➢ Effect of a distribution of impurities?
- ➢ Quantitative indication of the quantum/classical dynamics of the impurity? (purity, concurrence....) c.f. Experiment: Simmonds et.al. PRL 2004