Two-component Fermi systems with density imbalance: From ultracold atoms to semiconductors

P. Pieri, G.C. Strinati, and D. Neilson

Dipartimento di Fisica, U. of Camerino, Italy

UNICAM
Università di Camerino
1336

University of Camerino, Italy

Outline

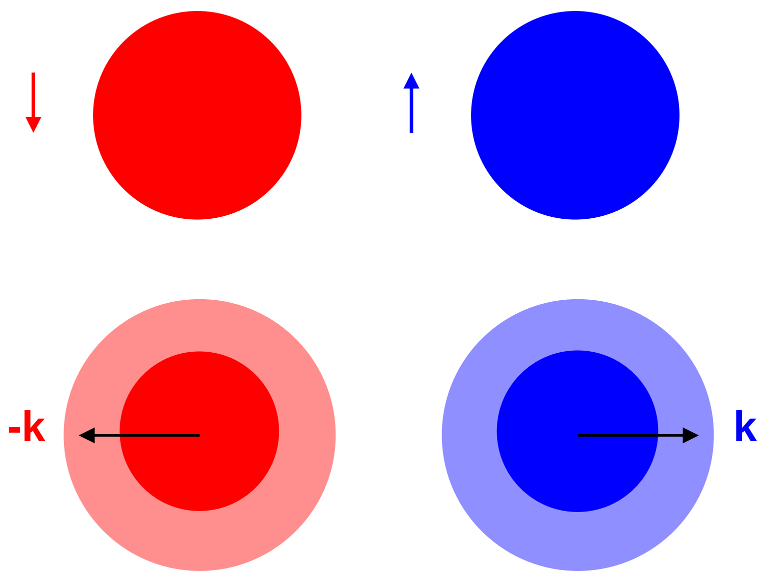
- Short introduction and some history of the problem
- Recent experiments with ultracold Fermi gases
- Our contribution for Fermi gases
- Electron-hole bilayer systems
- Conclusions

Introduction

What is the ground state of a two-component attractive Fermi system in the presence of a mismatch between the two populations?

Problem originally considered in the 60's for a weak-coupling superconductor in an external magnetic field (Sarma, Fulde & Ferrell, Larkin & Ovchinnikov, …).

When the two populations are perfectly matched: **BCS ground state**. Rearrangement of the two distributions in momentum space to allow pairing of one up fermion of momentum k with one down fermion of momentum -k in an energy window (of width Δ) around the Fermi surface.

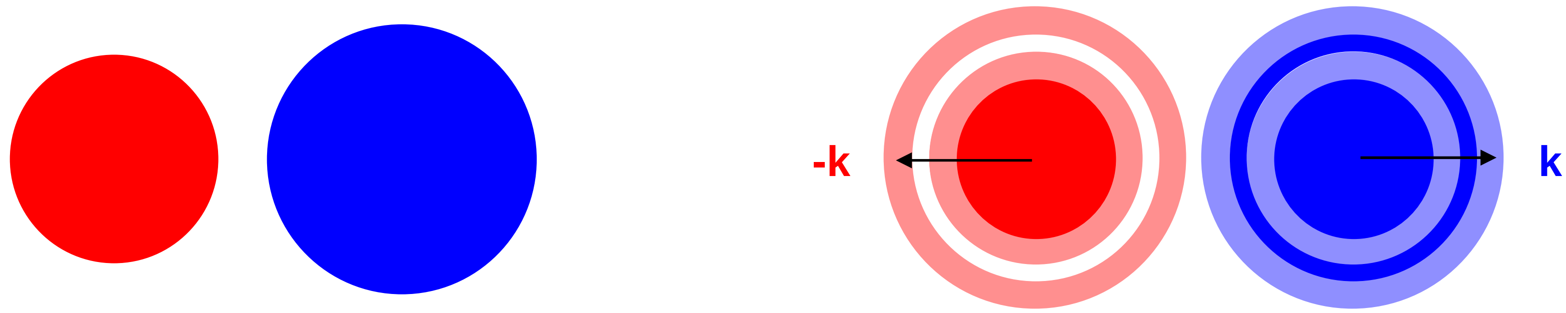


$$|\Phi\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{+}_{\mathbf{k}\uparrow} c^{+}_{-\mathbf{k}\downarrow}) |0\rangle$$

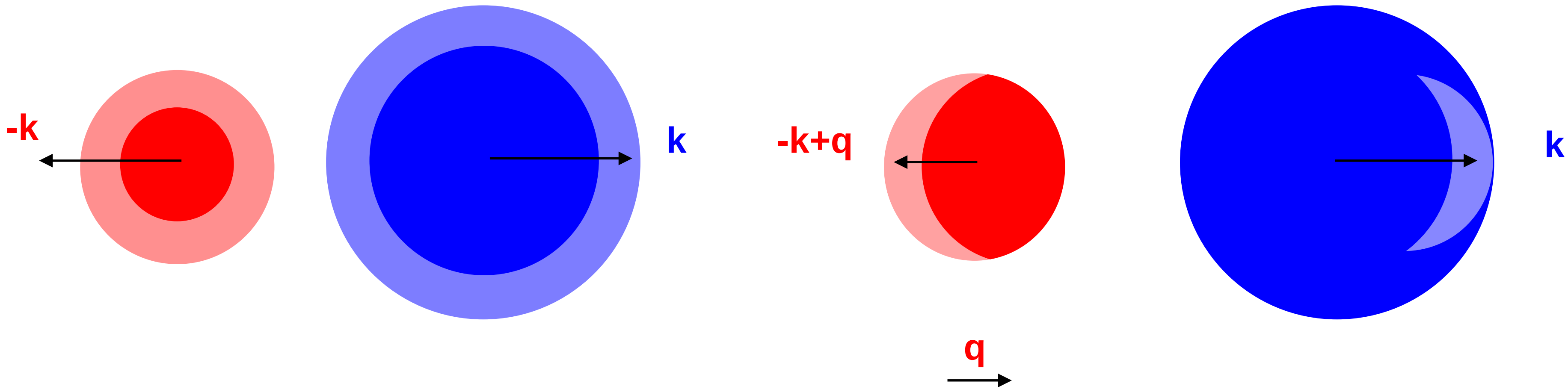
$$\langle\Phi| c^{+}_{\mathbf{k}\uparrow} c^{+}_{-\mathbf{k}\downarrow}) |\Phi\rangle = u_{\mathbf{k}} v_{\mathbf{k}} \neq 0$$

in a window about the Fermi energy where the Fermi distribution is smeared out.

For small imbalance between the two populations: BCS ground state can be modified to accomodate “excess” fermions where they cost less energy, still allowing paring between k and –k: “**Sarma phase**”.



For larger imbalances pairing of k and –k no longer possible. However, one can consider pairing between k and –k+q. Pairs have a finite center of mass momentum **q:**
phase found theoretically by Fulde & Ferrel and independently by Larkin & Ovchinnikov (1964).



However…

It turns out that under the **original assumptions of weak attraction** and population imbalance induced by an external magnetic field (i.e., **fixed chemical potential difference**):

I) The **Sarma phase is a local maximum of the energy**

II) The **Fulde-Ferrell-Larkin-Ovchinnikov phase is stable only in a narrow range** of fields, close to the (usually huge) depairing field, where superconductivity is destroyed by the Zeeman splitting between up and down fermions

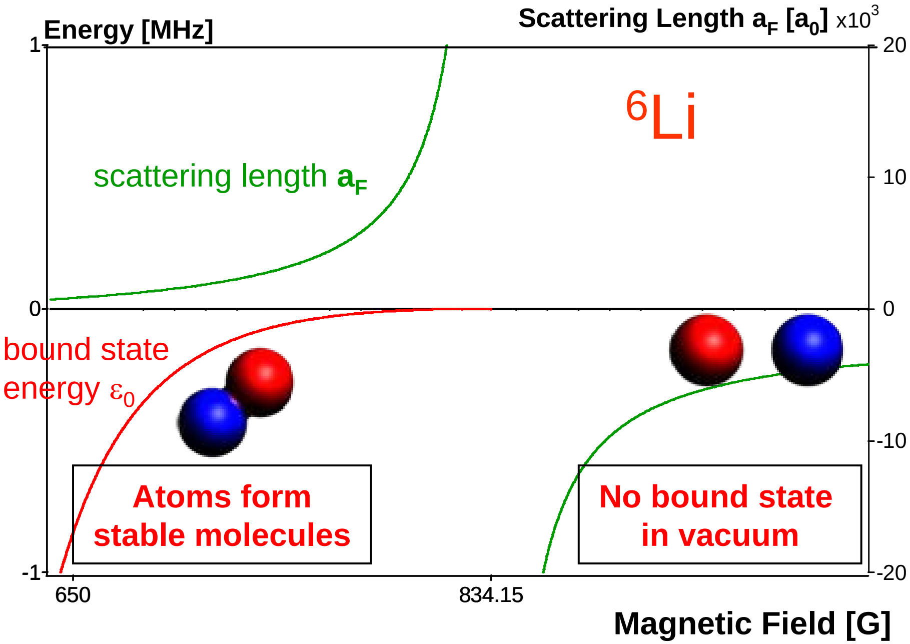
In addition, **orbital effects** by the magnetic field are normally **more important than Zeeman effect**: difficult to find superconductors which can reach fields close to the depairing field without losing the superconductivity.

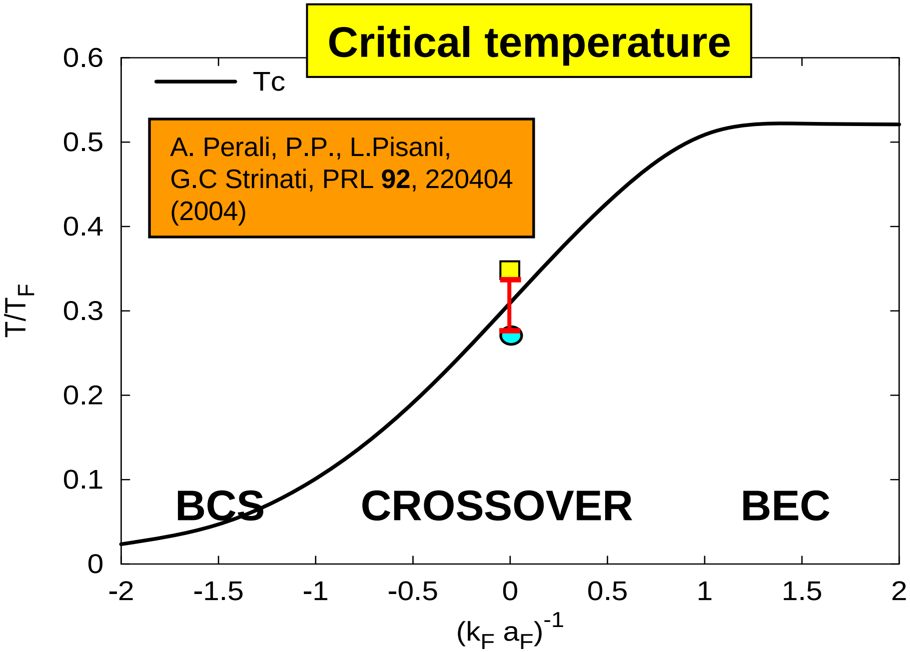
To summarize: **Sarma phase never observed. Some evidence of a FFLO phase in a heavy-fermion superconductor ($CeCoIn_5$) only recently (> 2003).**

Experiments with ultracold Fermi atoms

Fermionic atoms (40K or 6Li) are **cooled down** to ultralow temperatures (order of 100 nK) in a **harmonic** (magnetic and/or optical) **trapping potential**.

Ability to **vary the effective fermionic attraction from weak to strong** by using a Fano-Feshbach resonance.





J. Kinast *et al.*, Science **307**, 1296 (2005) (■); PRL **94**, 170404 (2005) (●)
M. W. Zwierlein *et al.*, Science **311, 492** (2006) (I)

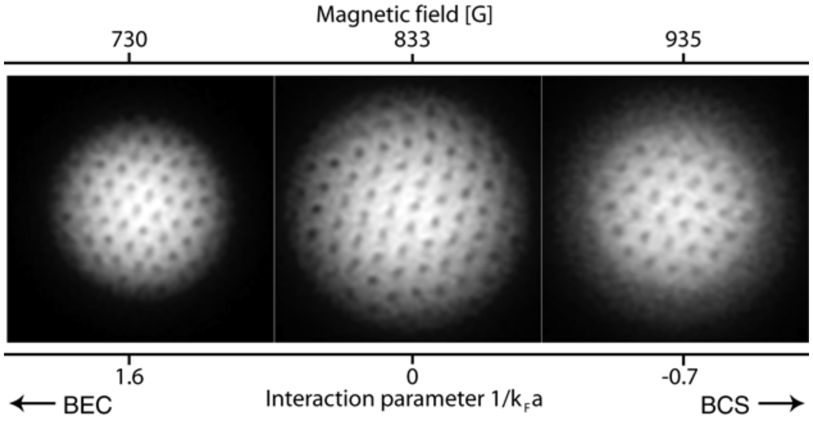
Superfluidity in trapped Fermi gases with equal populations

Indirect evidence of superfluid transition:

(2004)
- **Innsbruck:** pairing gap/coll. oscillations
- **JILA** and **MIT:** condensate ”projection” exps.
- **Duke U.**: specific heat/coll. oscillations

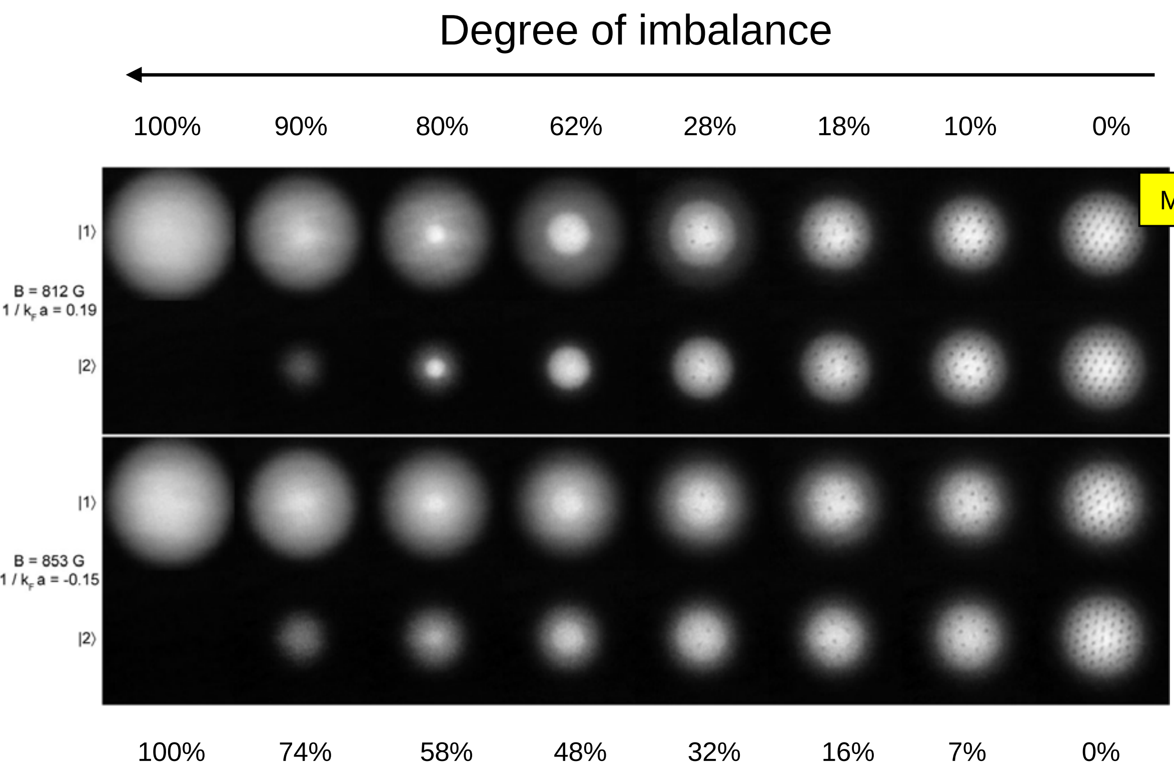
Definitive proof of superfluidity:

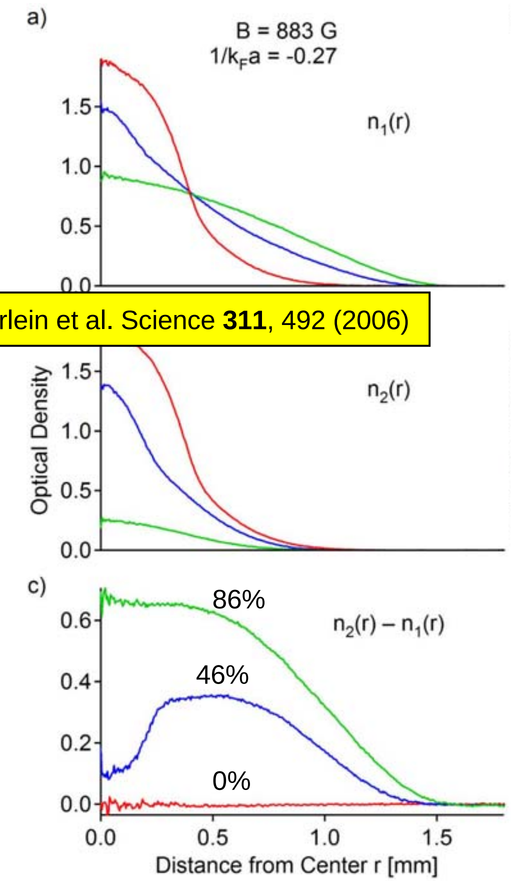
(2005) **MIT:** formation of Abrikosov vortex lattice under rotation.



M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle,
Nature **435**, 1047-1051 (2005)

Imbalanced systems

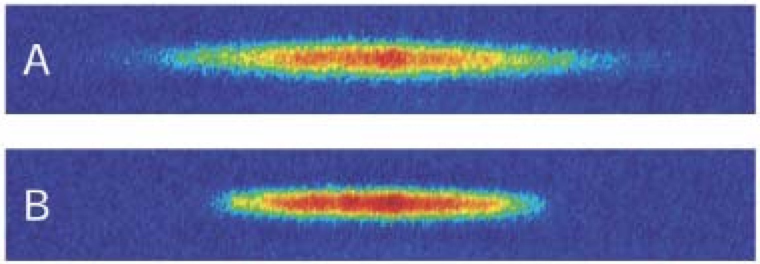




M.W. Zwierlein et al. Science **311**, 492 (2006)

First experiments focused around the crossover region $(k_F a_F)^{-1} = 0$ .The **superfluid** resides **at the trap centre** and is surrounded by a **shell of** unpaired **excess fermions**.

From the optical (column) **density impossible** to discerne **if the superfluid is partially polarized** (as it would be for the **Sarma** or **FFLO** phases) or not (ordinary BCS state).



G.B. Partridge et al. Science **311**, 503 (2006)

Imbalanced Fermi systems in the strong-coupling (BEC) limit

Approach the crossover region coming from the strong-coupling side. On physical grounds expect to find a **mixture of composite bosons and excess unpaired fermions**.

How is this limit recovered starting form a purely fermionic theory?

Start form the Bogoliubov de Gennes equations for superfluid fermions in a trap. By using the assumptions valid in the BEC limit:

i) the chem. potential of the minority fermions approaches the binding energy: $\mu_\downarrow \simeq -\varepsilon_0$

ii) the chem. potential of the majority fermions approaches the Fermi energy associated with the number of excess fermions: $\mu_\uparrow \simeq E_F[N_\uparrow - N_\downarrow]$

iii) the order parameter is much smaller than the binding energy: $\Delta << \varepsilon_0$

obtain two coupled eqs. for the condensate amplitude $\Phi(\mathbf{r}) \equiv \sqrt{\frac{m^2 a_F}{8\pi}}\Delta(\mathbf{r}) = \sqrt{n_\downarrow(\mathbf{r})}$ and the excess fermion density $\delta n(\mathbf{r}) = n_\uparrow(\mathbf{r}) - n_\downarrow(\mathbf{r})$

$$-\frac{\nabla^2}{2m_B}\Phi(\mathbf{r}) + \left[2V(\mathbf{r}) + \frac{3\pi a_{BF}}{m}\delta n(\mathbf{r})\right]\Phi(\mathbf{r}) + \frac{4\pi a_B}{m_B}|\Phi(\mathbf{r})|^2\Phi(\mathbf{r}) = \mu_B\Phi(\mathbf{r})$$

$$\delta n(\mathbf{r}) = \frac{(2m)^{3/2}}{6\pi^2}\left[\mu_\uparrow - V(\mathbf{r}) - \frac{3\pi a_{BF}}{m}|\Phi(\mathbf{r})|^2\right]^{3/2}$$

$$\mu_B = \mu_\uparrow + \mu_\downarrow + \varepsilon_0$$

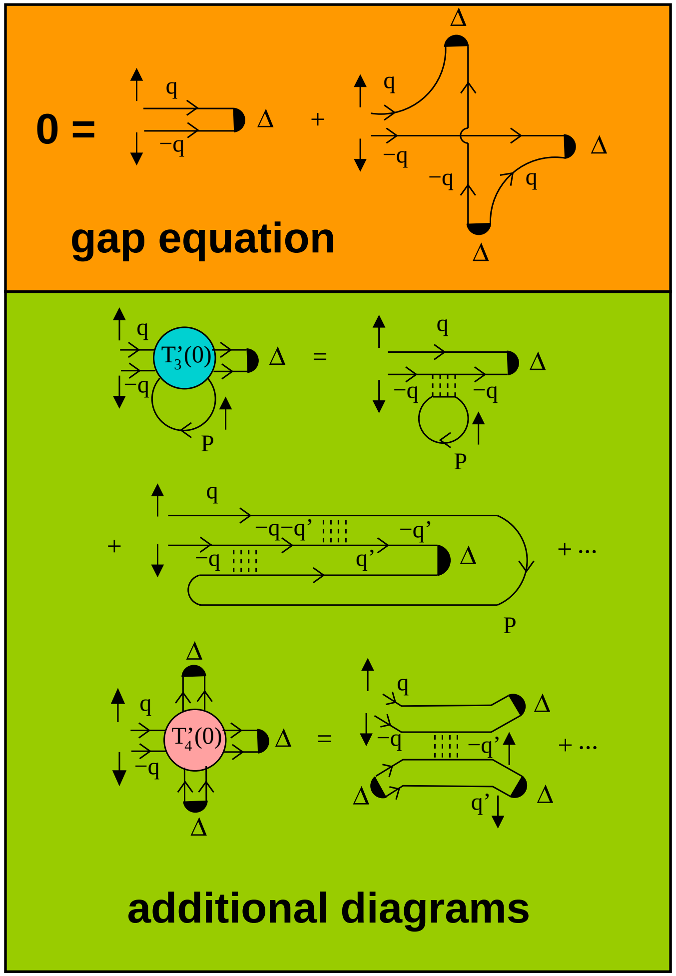
$$m_B = 2m$$

However, the values obtained for the scattering lengths: $a_{BF} = \frac{8}{3} a_F ; \quad a_B = 2a_F$ are only approximate.

Exact values obtained from the solution of the 3- and 4-body Schroedinger eqs.:

$a_{BF} = 1.18 a_F$ (1957: Skorniakov & Ter-Martirosian)

$a_B = 0.6 a_F$ (2004: Petrov, Salomon, Shlyapnikov)



These values can be recovered in the many-body fermionic theory by **modifying the gap equation** to include the diagrams which lead to the correct scattering lenghts in the zero-density limit.

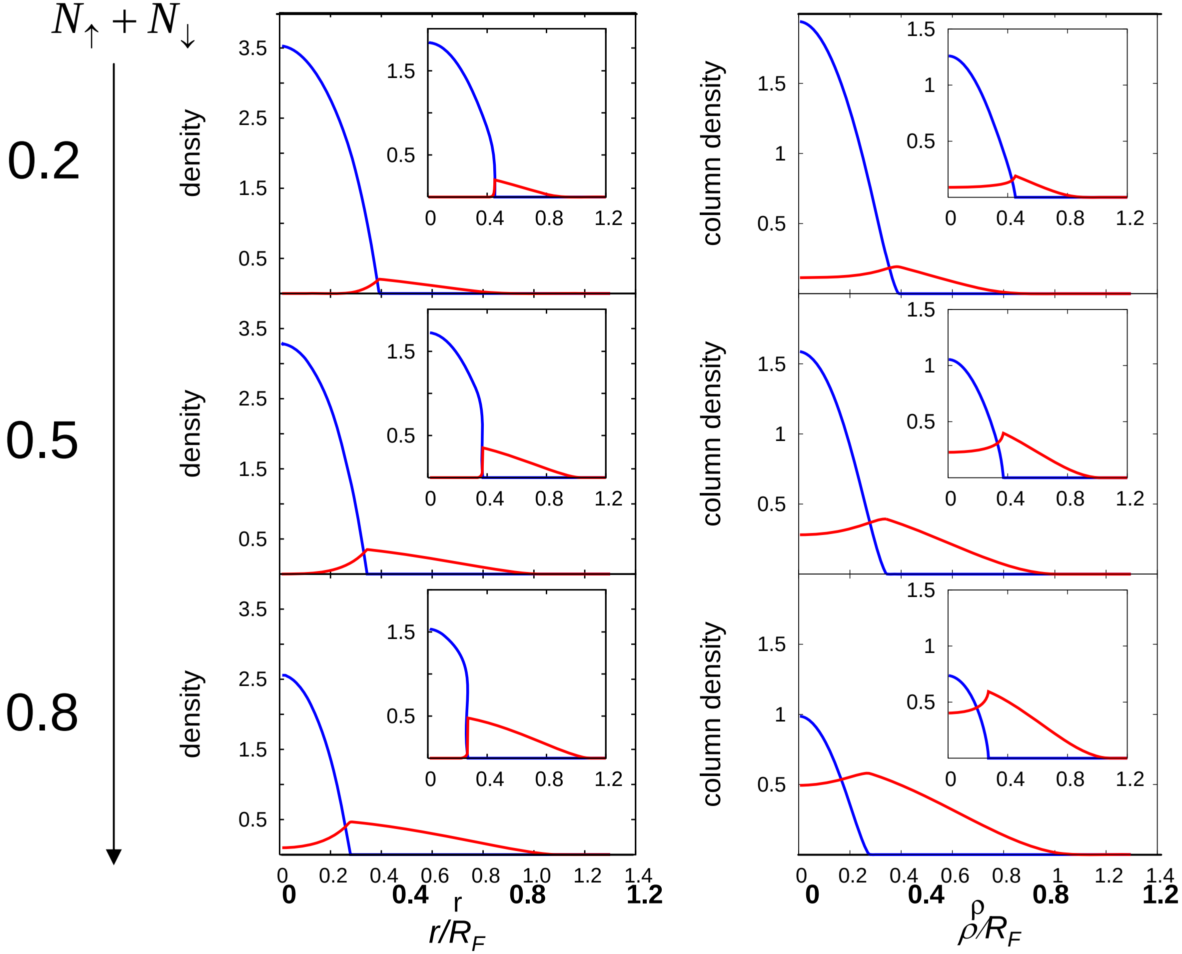
The series of diagrams $T_3(0)$ and $T_4(0)$ leading to the correct values of a_{BF} and a_B were recently identified.

$T_3(0)$: Bedaque & van Kolck, Phys. Lett. B (1998)

$T_4(0)$: Brodsky et al., JETP Letters (2005)

Density profiles

$$\alpha = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



Main panels: $(k_F a_F)^{-1} = +3$

Insets: $(k_F a_F)^{-1} = +1$

Condensate

δn

P.P, G.C. Strinati, PRL **96**, 150404 (2006)

Results obtained from the numerical solutions of our coupled eqs. (with the exact values of scattering lengths).

In the BEC limit partial penetration of the excess fermions in the superfluid core (BEC limit of the **Sarma phase**).
When approaching the crossover region $(k_F a_F)^{-1} = 0$ the excess fermions are completely expelled from the superfluid core: **phase separation** between a perfectly matched BCS superfluid core and a fully polarized normal shell.

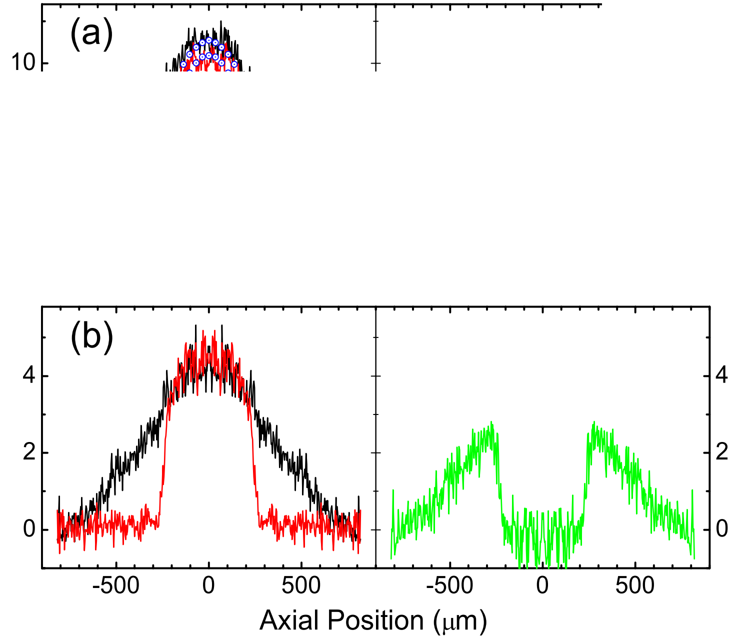
Last few months experiments

$(k_F a_F)^{-1} = 0$
$\alpha = 0.58$

MIT group

Y. Shin et al., PRL **97**, 030401 (2006).

Our **theoretical prediction** fully **confirmed** by two subsequent experiments that were able to measure the (nonintegrated) density difference: **complete phase separation between a superfluid core with equal populations and a fully polarized normal shell**.



$(k_F a_F)^{-1} = 0$
$\alpha = 0.35$

Rice U. group

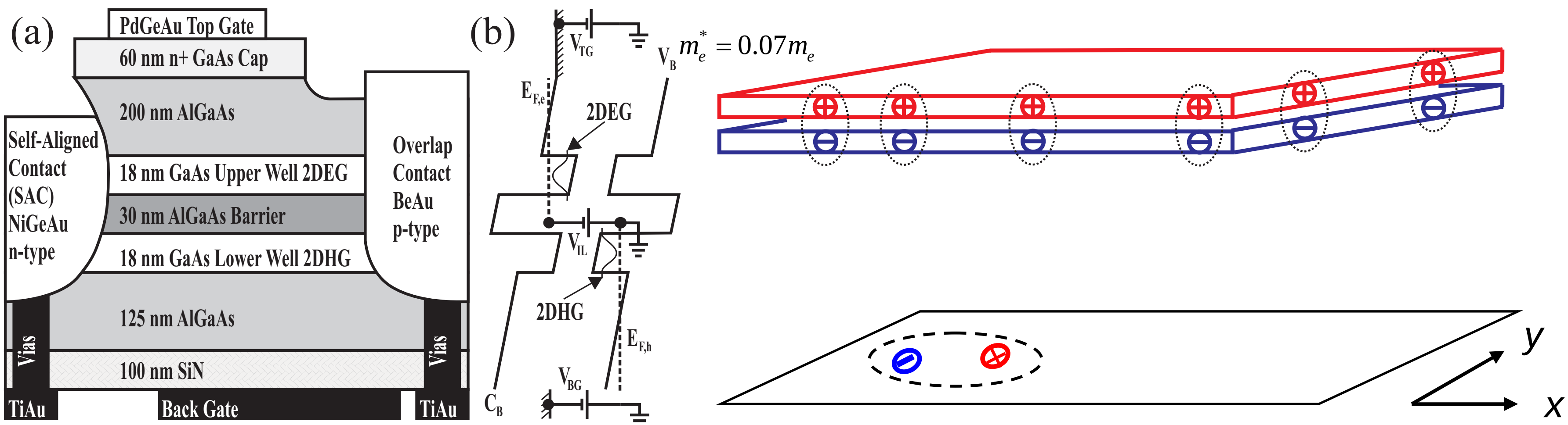
G.B. Partridge et al., PRL **97**, 190407 (2006).

Electron-hole bilayer systems

Semiconductor heterostructures. Two layers separated by a distance *d.*
Hole carriers in one layer and **electron** carriers in the other layer. Electron and hole **densities can be varied independently** in the two layers by electric contacts.
Effectively 2-dimensional problem. Electron and holes attract each other via the Coulomb potential:

$$V(r) = -\frac{1}{4\pi\varepsilon_0\varepsilon}\frac{e^2}{\sqrt{d^2+r^2}}$$

where *r* is the distance between the e and the h in the *xy* plane.



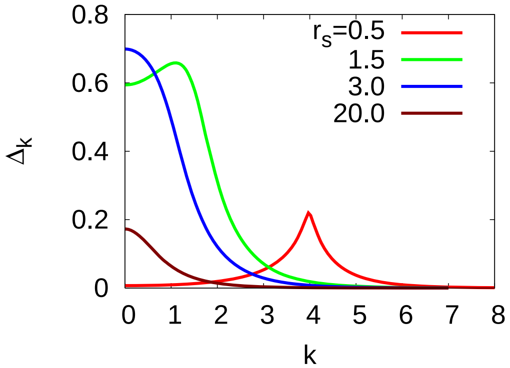
For GaAs-AlGaAs: $\varepsilon = 13;\ m_e^* = 0.07 m_e;\quad m_h^* = 0.30 m_e;$ effective Bohr radius $a_0^* = 12\,\mathrm{nm}.$

Mean-field equations for the **BCS** or **Sarma** phases:

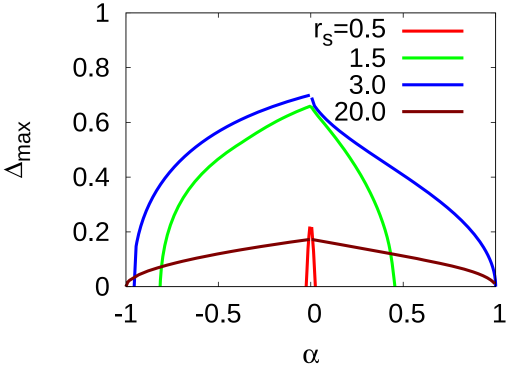
$$\Delta_{\mathbf{k}} = -\frac{1}{\Omega}\sum_{\mathbf{k}'} V^{\rm eh}_{\mathbf{k}-\mathbf{k}'}\frac{\Delta_{\mathbf{k}'}}{2E_{\mathbf{k}'}}[1 - f(E^+_{\mathbf{k}'}) - f(E^-_{\mathbf{k}'})]$$

$$n_{\rm e} = \frac{1}{\Omega}\sum_{\mathbf{k}}\left[u^2_{\mathbf{k}} f(E^+_{\mathbf{k}}) + v^2_{\mathbf{k}}(1 - f(E^-_{\mathbf{k}}))\right]$$

$$n_{\rm h} = \frac{1}{\Omega}\sum_{\mathbf{k}}\left[u^2_{\mathbf{k}} f(E^-_{\mathbf{k}}) + v^2_{\mathbf{k}}(1 - f(E^+_{\mathbf{k}}))\right]$$



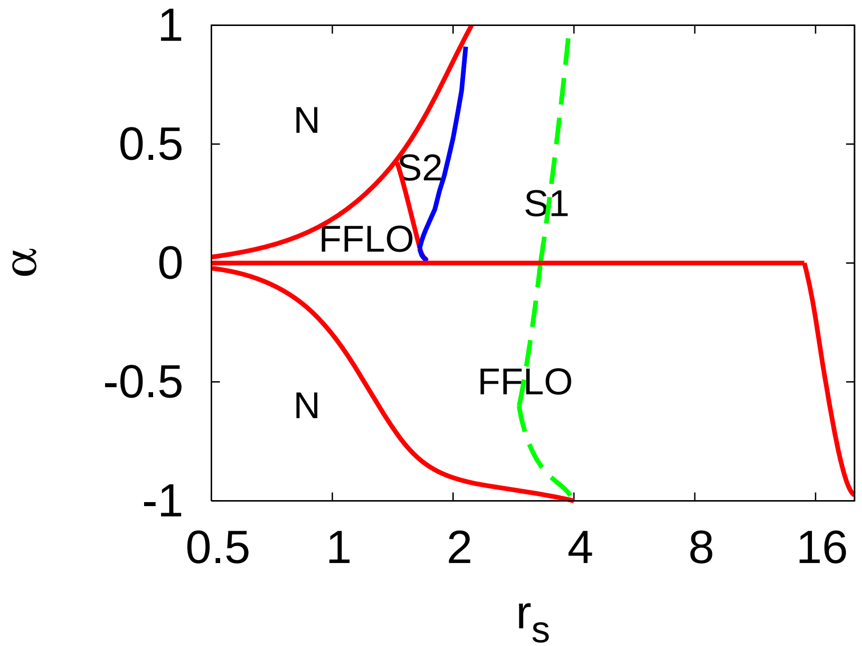
The **crossover from** the **BCS to** the **BEC** limit is **controlled** here **by the parameter r_s** (average interparticle distance in units of the effective Bohr radius)



Instability towards the **FFLO phase** is signalled by a **negative superfluid density**:

$$\rho_s = m_{\rm e}n_{\rm e} + m_{\rm h}n_{\rm h} - \frac{1}{4\pi}\sum_{j,\lambda}\frac{(k^\lambda_j)^3}{\left|\frac{dE^\lambda_k}{dk}\right|_{k=k^\lambda_j}} \quad < 0$$

Phase diagram at T=0



Rich phase diagram where **both Sarma** (S2 or S1) and **FFLO phases** find place.

Phase separation suppressed by intra-layer Coulomb repulsion.

The different effective electron and hole masses act to favor the Sarma phases for positive imbalance and the FFLO phase for negative imbalance.

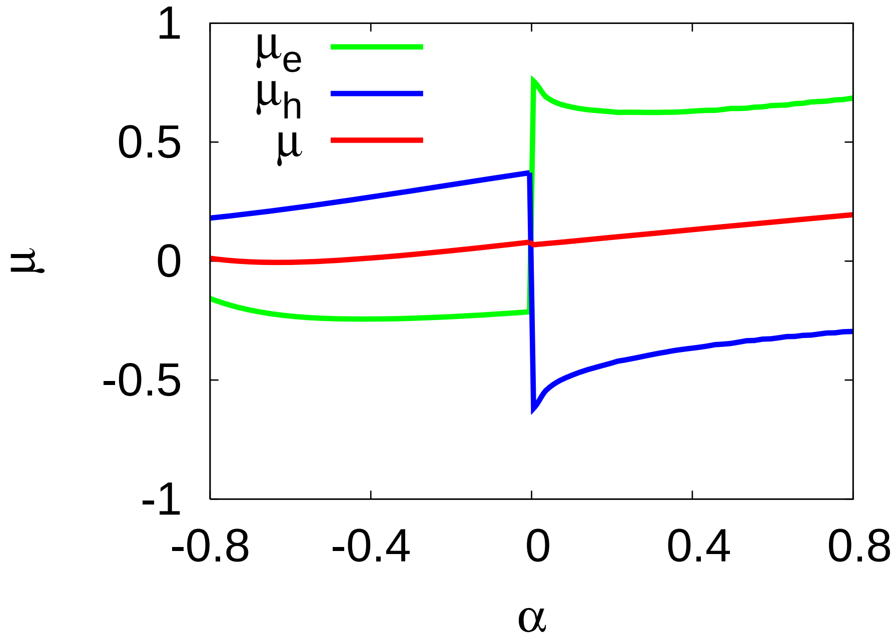
Conclusions

- Recent experiments with trapped Fermi atoms prompted interest in two-component Fermi systems in the presence of density imbalance.

- Phase separation dominates the phase diagram in trapped Fermi gases: No room for “exotic” phases (Sarma, FFLO).

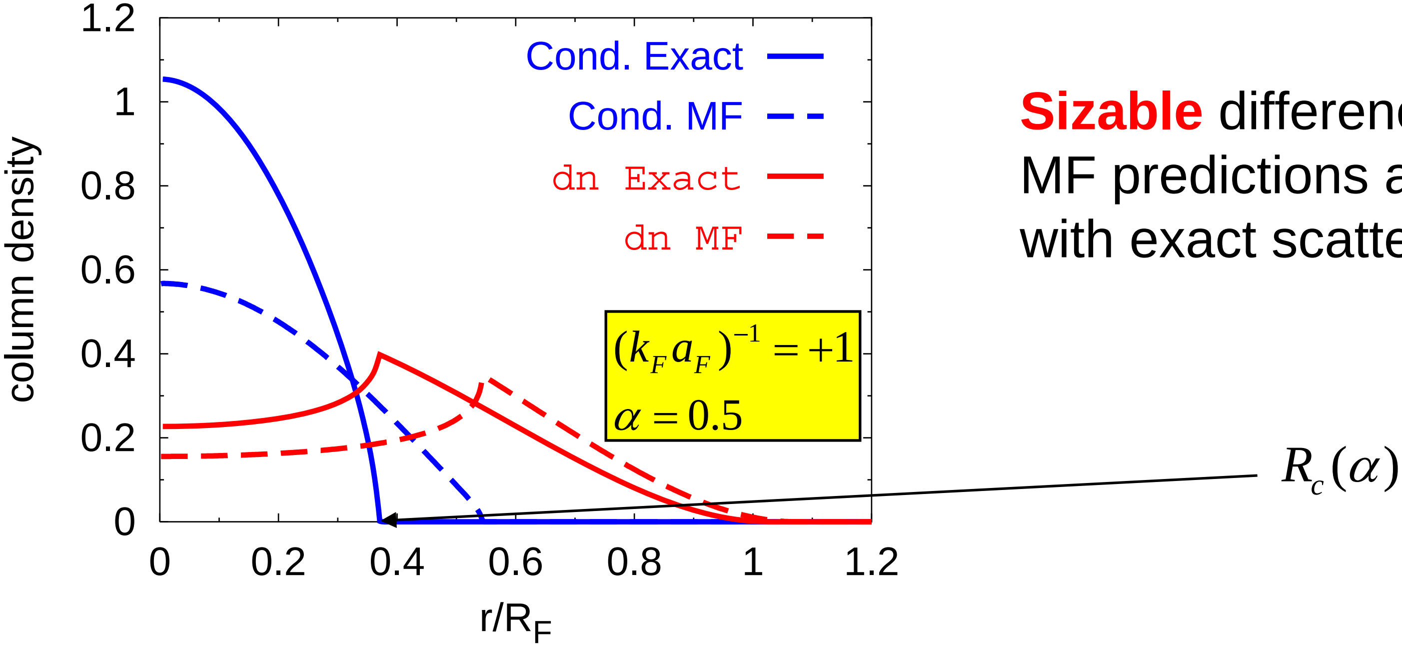
- Electron-hole bilayers are promising candidates for revealing both FFLO and Sarma phases due to the concurrence of several favorable factors (different effective electron and hole masses, Coulomb repulsion, momentum dependent gap function).

Supplementary material

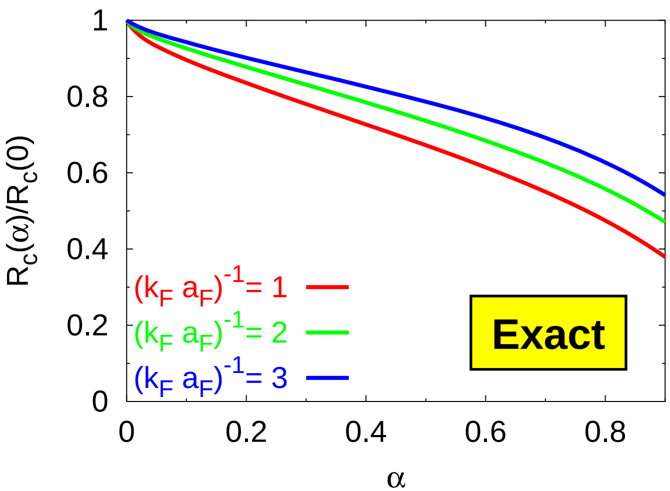


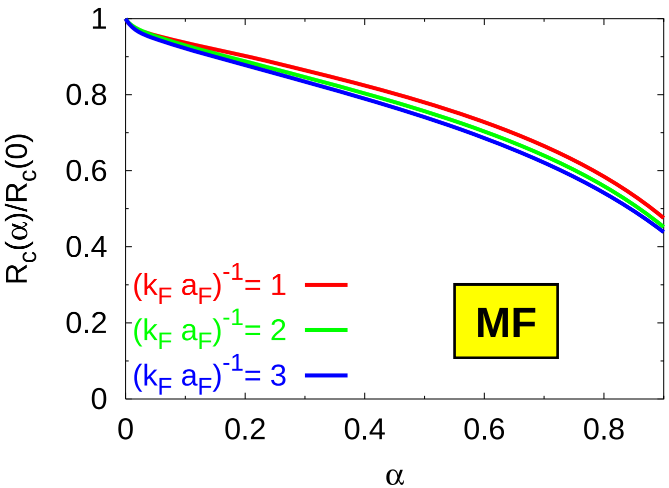
1
0.5
0
-0.5
-1
-0.8
-0.4
0
0.4
0.8
μ
α
μe
μh
μ

Comparison with MF predictions



Sizable difference between MF predictions and solutions with exact scattering lengths.





Anisotropic trap

$$\mu(\mathbf{r}) = \mu - \frac{1}{2} m\omega_r^2 (x^2 + y^2 + \lambda^2 z^2) \qquad \lambda = \omega_z / \omega_r$$

$$Z = \lambda z$$

$$N = \int dxdydz\, n(x,y,z) = \frac{1}{\lambda} \int dxdydZ\, n(\sqrt{x^2 + y^2 + Z^2})$$

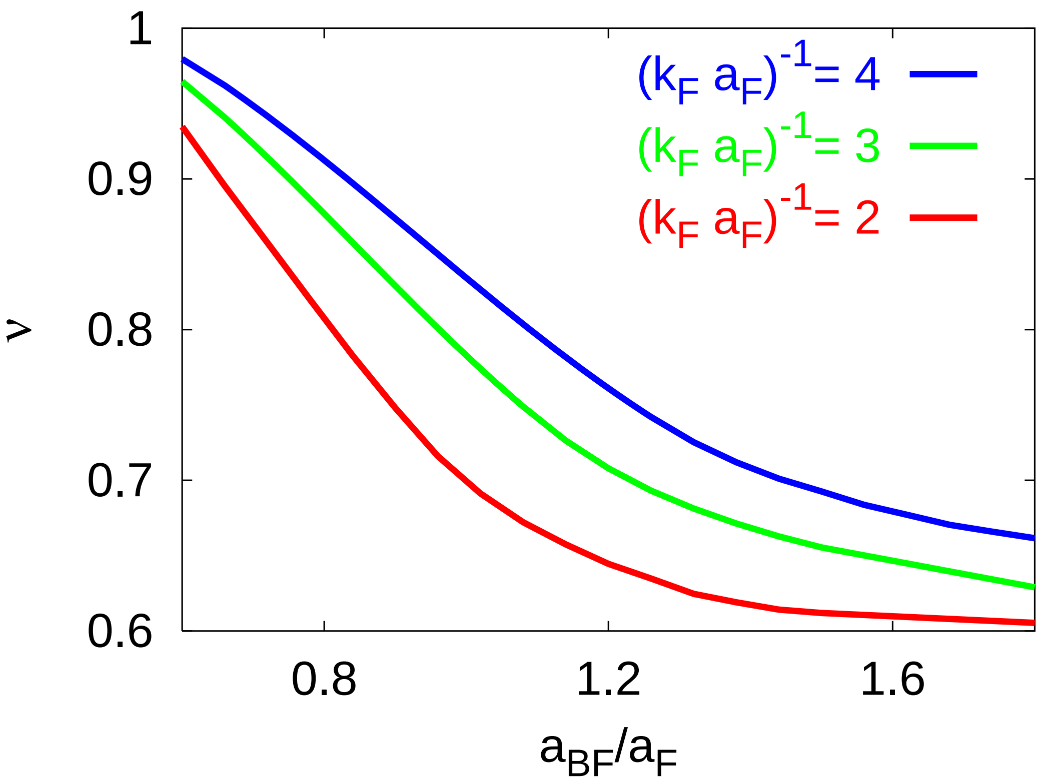
➡ equivalent isotropic problem with number of particles $N^{eff} = \lambda N$
and frequency $\omega_0 = \omega_r$

➡ $$E_F^{eff} = (3N^{eff})^{1/3} \hbar\omega_r = (3\lambda N)^{1/3} \hbar\omega_r = E_F$$

Results for the isotropic case can be easily rescaled to the anisotropic case.

Parameter $k_F a_F$ not affected by rescaling: phase diagram universal.

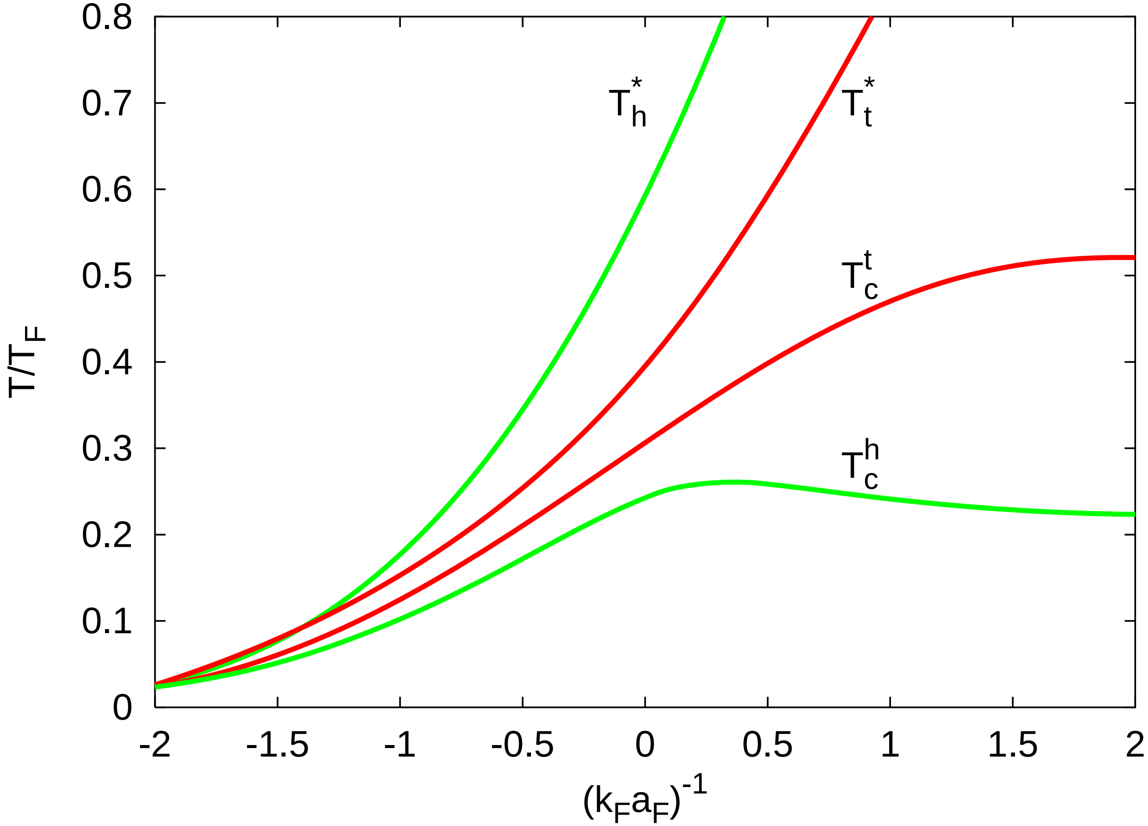
Calibration curves to extract a_{BF} from experiments



Measure ratio ν between column density δn at $r = 0$ and at its peak value for e.g. $\alpha = 0.5$ and extract a_{BF} from calibration curves.

Verify in this way a **long-standing** (half a century old!) prediction for a_{BF} .

Phase diagram



T^*: pairing temperature
T_c: superfluid critical temperature
—— : homogeneous gas (h)
—— : trapped gas (t)

A. Perali et al., PRL **92**, 220404 (2004)

"Precursor-pairing" region reduced with respect to the homogeneous case.

T_c increases monotonically in the trapped case.