

Entanglement Localization by a single defect in a spin chain

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- Aim:***
- 1) To describe the effect of a diagonal impurity in both the static and dynamical properties of entanglement;*
 - 2) To characterize the entanglement localization and the mirror effect induced by the defect.*

- Outline:***
- 1) The model and its solution;*
 - 2) Entanglement localization in the ground state;*
 - 3) Entanglement distribution in the presence of the defect:*
 - a) localization*
 - b) reflection and transmission*

Spin chain with one defect

We consider a 1-D XX spin $\frac{1}{2}$ closed chain placed in an external magnetic field, homogeneous everywhere but for a single site (diagonal impurity)

$$H = h \sum_{i=-\frac{N}{2}}^{\frac{N}{2}} \sigma_z^i - J \sum_{i=-\frac{N}{2}}^{\frac{N}{2}} (\sigma_x^i \sigma_x^{i+1} + \sigma_y^i \sigma_y^{i+1}) + \epsilon \sigma_z^l$$

defect

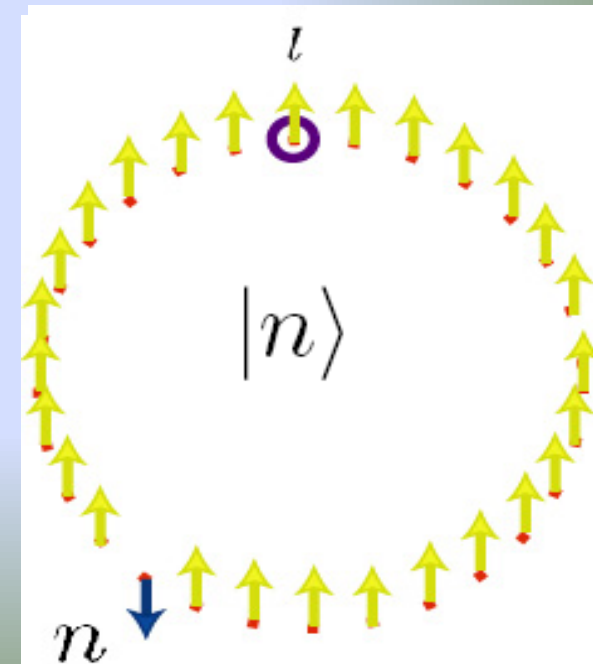
We assume $2h > J$ \longrightarrow unperturbed ground state:

$$|0\rangle^{\otimes N+1}$$

1-excitation sector

$$|k\rangle = \frac{1}{\sqrt{N+1}} \sum_n \exp\left[\frac{2\pi i k n}{N}\right] |n\rangle$$

$$E_k = 2h - J \cos\left(\frac{2\pi k}{N}\right)$$



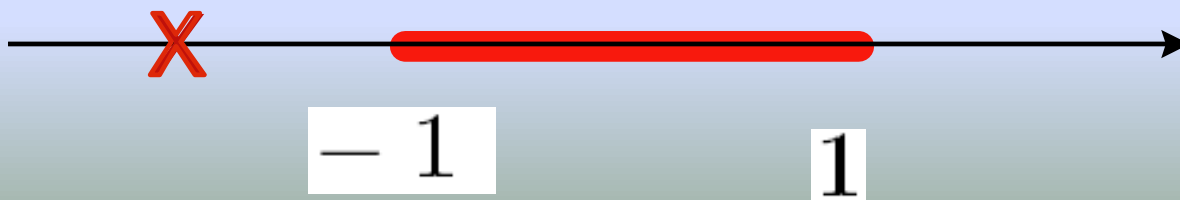
Green function approach (1-excitation sector)

$$G(z) = G_0(z) + G_0(z)|l\rangle \frac{2\epsilon}{1 - 2\epsilon G_0(l, l; z)} \langle l|G_0(z)$$

Dyson

$$G_0^\pm(r, s; z) = \frac{(-x \pm i\sqrt{1-x^2})^{|r-s|}}{\pm iJ\sqrt{1-x^2}}, \quad z \in [2h-J, 2h+J]$$

$$G_0(r, s; z) = \frac{(-x + \sqrt{x^2-1})^{|r-s|}}{J\sqrt{x^2-1}}, \quad \text{otherwise}$$



$$x = \frac{z-2h}{J}$$

Energy band

distortion due to the defect

$$|\Psi(E)\rangle = \sum_n a_n(E) |n\rangle$$

$$a_n(E) = \frac{1}{\sqrt{N+1}} \left(e^{i\theta n} + \frac{\alpha e^{i|\theta||n-l|}}{i|\sin\theta| - \alpha} e^{i\theta l} \right)$$

$$\alpha = \frac{2\epsilon}{J}$$

where $\cos\theta = (2h - E)/J$

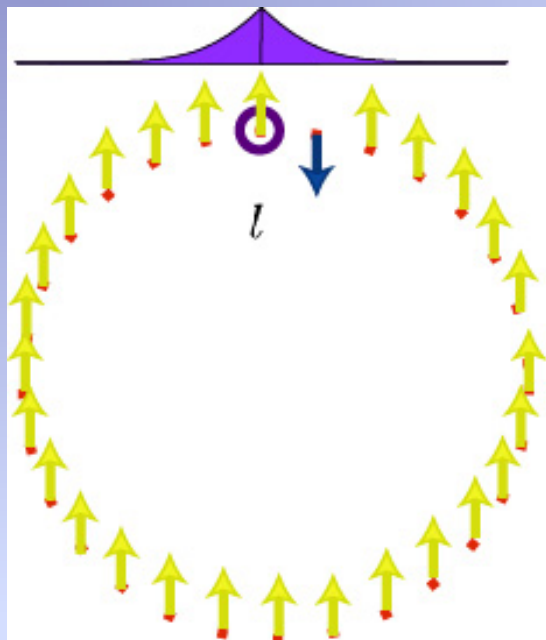
Localized state

$$|\Psi_{loc}\rangle = \sum_n b_n |n\rangle \quad E_{loc} = 2h \mp J\sqrt{1 + \alpha^2}$$

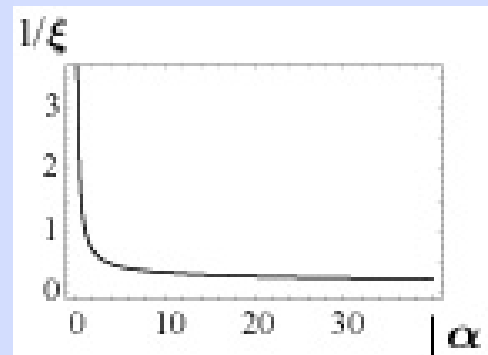
$$b_n = -\frac{\sqrt{|\alpha|}}{(1+\alpha^2)^{\frac{1}{4}}} \exp[-\xi(\alpha)|n-l|]$$

Inverse localization length

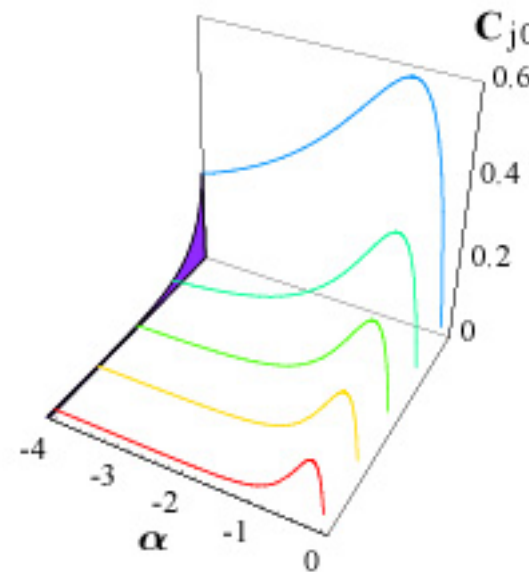
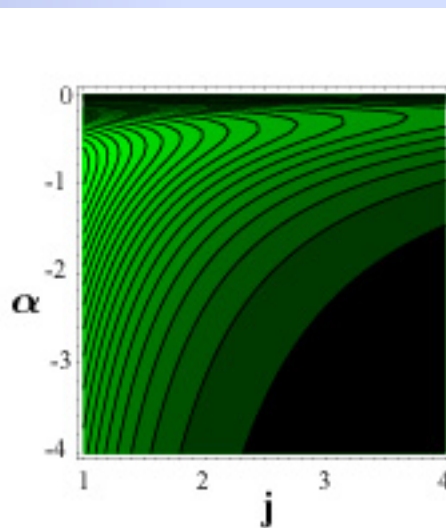
Localization



$$\xi(\alpha) = -\ln(\sqrt{1 + \alpha^2} - |\alpha|)$$



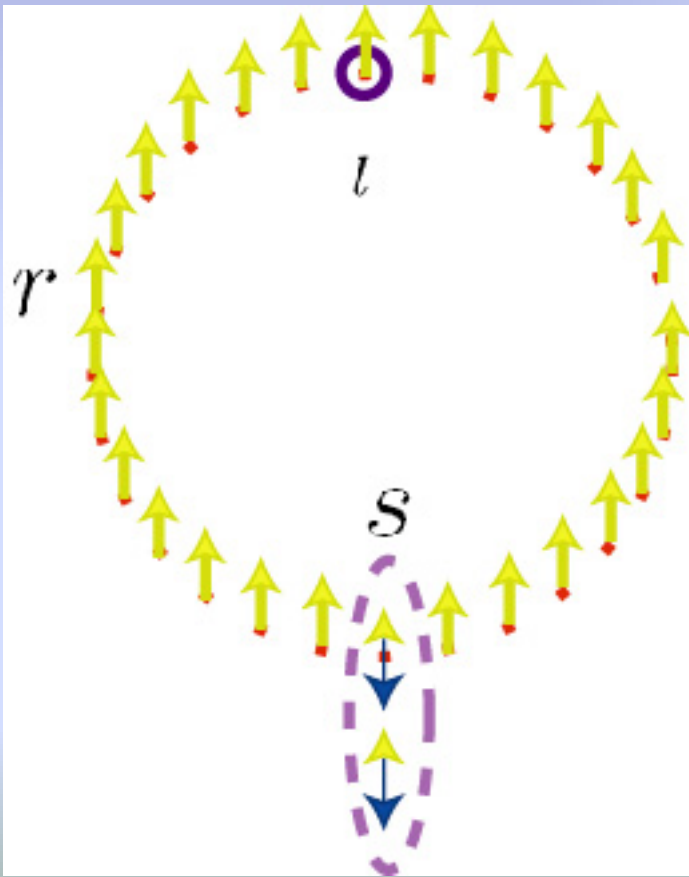
Entanglement



$$C_{ij} = 2\text{Res}[G(i, j; E_{loc})] = \frac{2|\alpha|}{\sqrt{1 + \alpha^2}} e^{-\xi(|i| + |j|)}$$

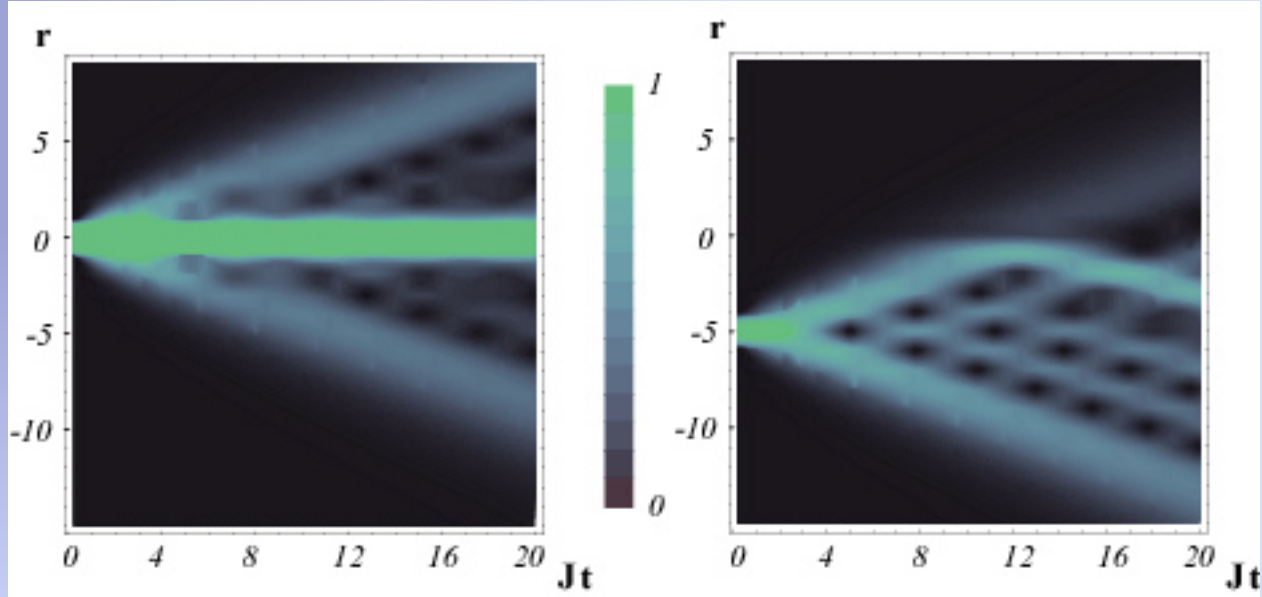
Entanglement distribution

$$C_r(t) = \left| \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} \left\{ e^{i\theta(r-s)} + e^{i\theta(l-s)} g_{r,l}^{(+)} + e^{i\theta(r-l)} g_{l,s}^{(-)} + g_{r,l}^{(+)} g_{l,s}^{(-)} \right\} e^{-iEt} + \text{Res}[G(r, s; E_{loc})] e^{-iE_{loc}t} \right|$$



$$g_{i,j}^{(\pm)}(E) = \frac{\alpha G_0^{(\pm)}(i, j; E)}{1 - \alpha G_0^{(\pm)}(l, l; E)} = \frac{\alpha e^{\pm i|\theta||i-j|}}{\pm i \sin |\theta| - \alpha}$$

$$|\psi, t=0\rangle = \frac{1}{\sqrt{2}} \left(|0_{ext}, 1_s\rangle - |1_{ext}, 0_s\rangle \right)$$

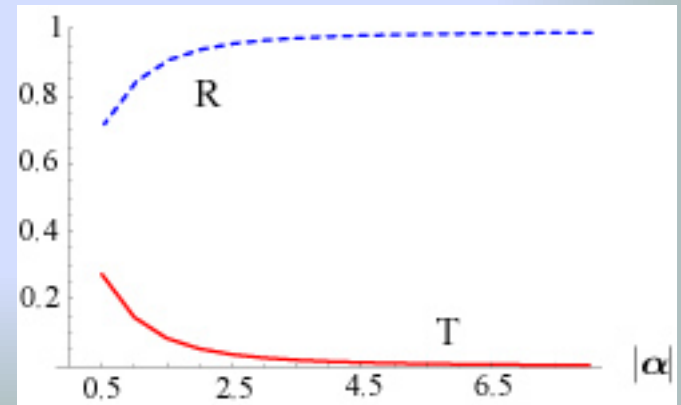


$$C_r \simeq \begin{cases} 1 - \frac{1}{2\alpha^2} & r = 0 \\ \frac{r}{|\alpha|\tau} J_r(\tau) & r \neq 0 \end{cases}$$

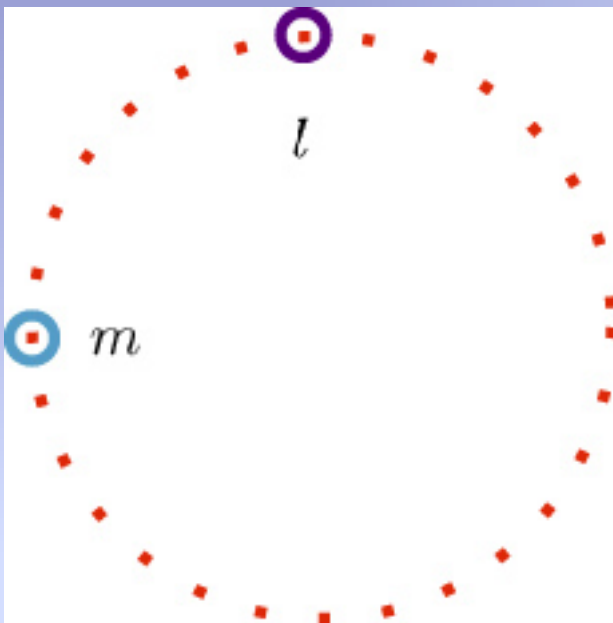
$$C_r = \left| (-1)^s J_{r-s}(\tau) - J_{r+s} - i \frac{J_{r+s+1} + J_{r+s-1}}{2\alpha} \right|$$

Reflection and transmission

$$T = \lim_{t \rightarrow \infty} \sum_{r > 0} C_r^2(t), \quad R = \lim_{t \rightarrow \infty} \sum_{r < 0} C_r^2(t)$$



Two defects (in progress...)



$$|\psi_{loc}\rangle = \frac{1}{N} \sum_n \left(e^{-\xi|l-n|} + K e^{-\xi|m-n|} \right) |n\rangle$$

