Imperial College London

Cooling Using the Stark Shift Gate

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QUANTUM MECHANICS: FROM FUNDAMENTAL PROBLEMS TO APPLICATIONS
5/12/06

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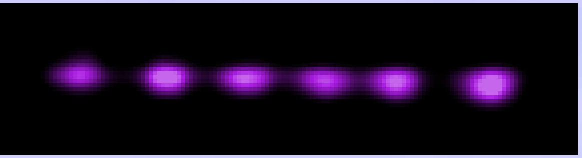
Cooling Using the Stark Shift Gate

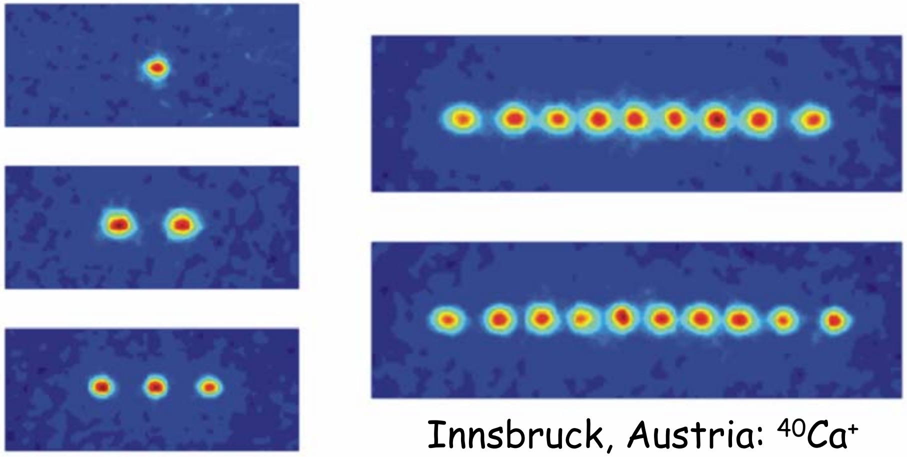
1 Introduction to Ion trap Quantum Computing

2 Startk Shift Gate Cooling

Cold ion crystals

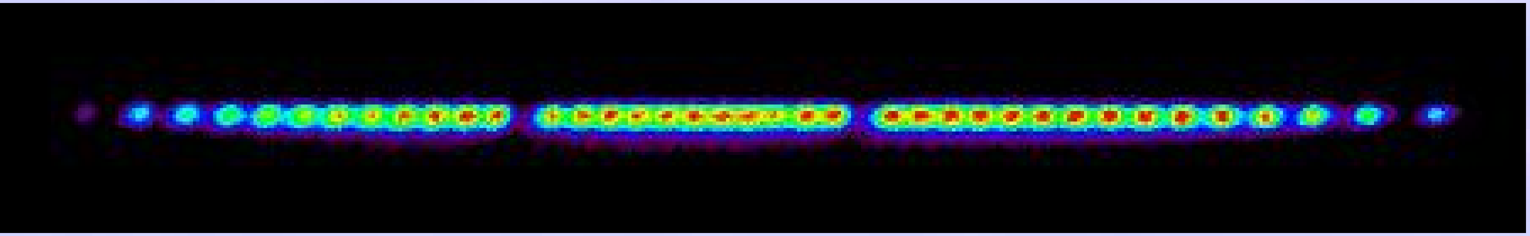


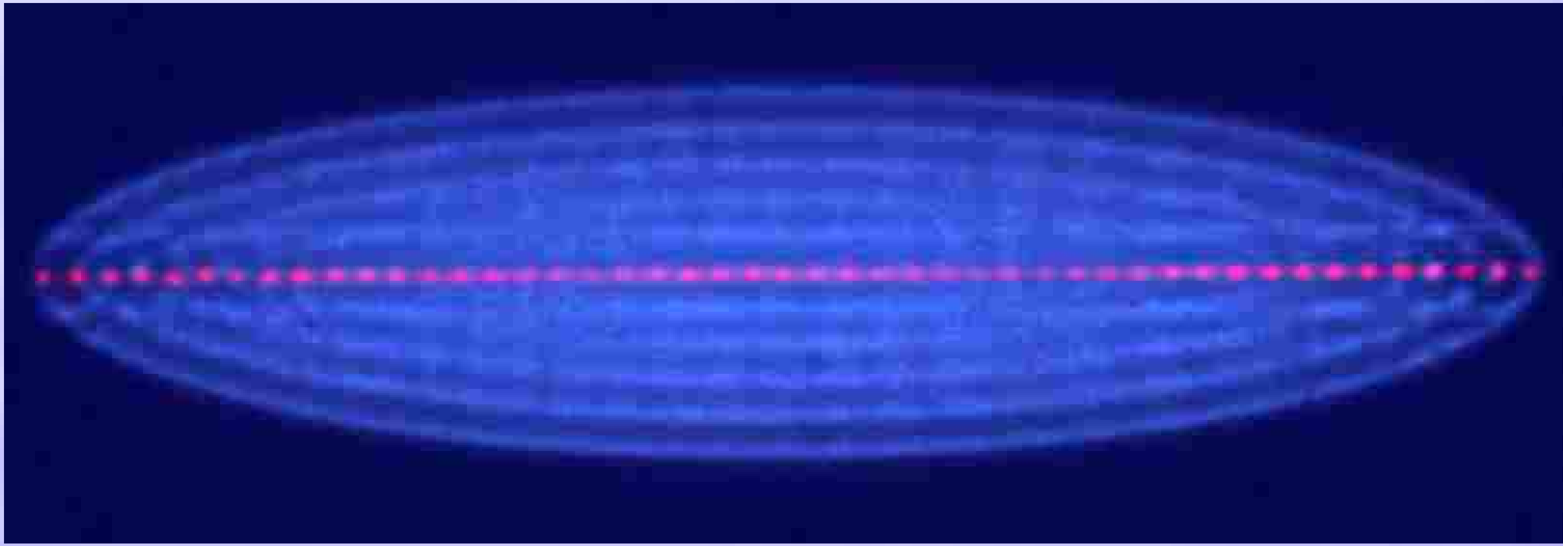
Oxford, England: $^{40}Ca^+$



Innsbruck, Austria: $^{40}Ca^+$

Boulder, USA: Hg^+ (mercury)



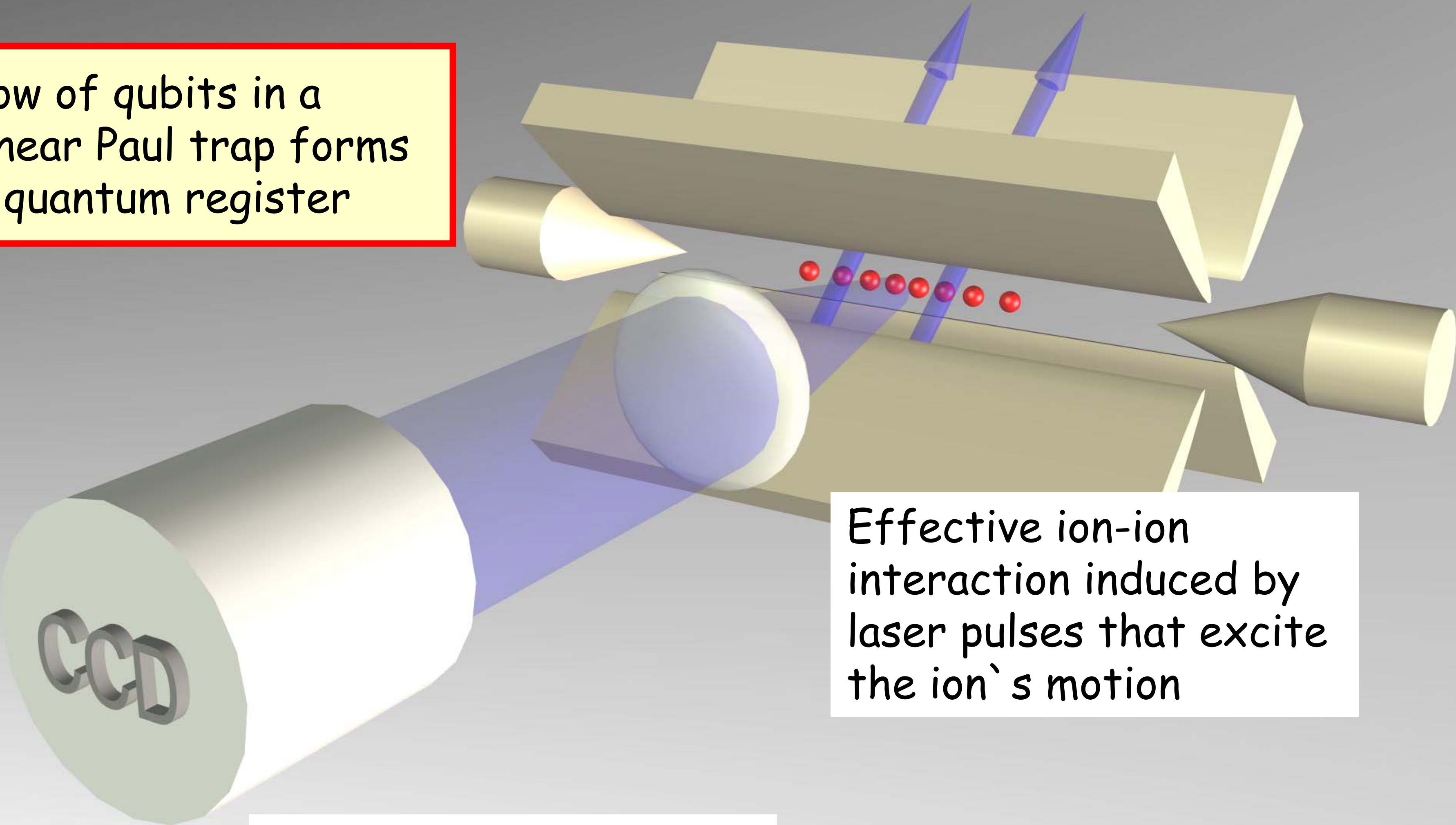


Aarhus, Denmark: $^{40}Ca^+$ (red) and $^{24}Mg^+$ (blue)

Ion Trap Quantum Processor

Laser pulses manipulate individual ions

row of qubits in a linear Paul trap forms a quantum register



Effective ion-ion interaction induced by laser pulses that excite the ion`s motion

A CCD camera reads out the ion`s quantum state

Courtesy of R. Blatt

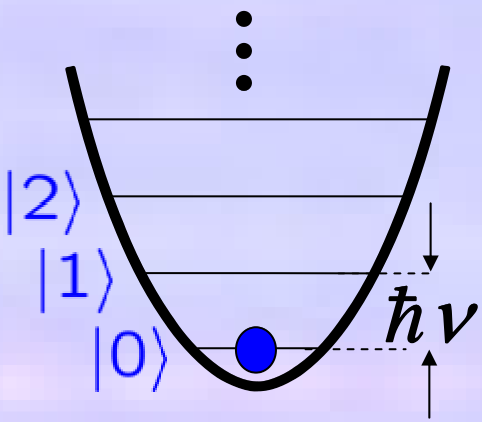
Orders of Magnitude

Physical size of the ground state:

$$\left.\begin{array}{l}\nu = (2\pi)\,1\text{MHz} \\ m = 40u\end{array}\right\} \quad \left\langle x^2 \right\rangle^{1/2} = \sqrt{\frac{\hbar}{2m\nu}} \approx 11nm$$

Size of the wave packet << wavelength of visible light

harmonic trap



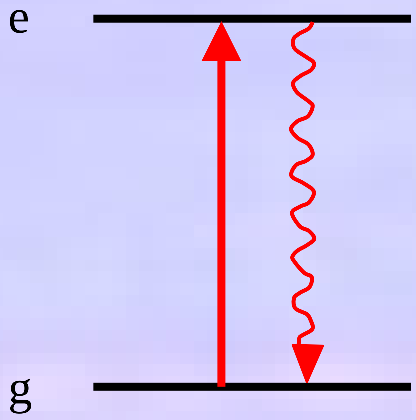
Energy scale of interest:

$$\hbar\nu = k_B T \longrightarrow T = \frac{\hbar\nu}{k_B} \approx 50\mu K$$

Seperation between ions:

$$d \approx 5\mu m$$

Detection of Ions



Lifetime of excited state: $\tau \approx 10ns$

Maximum photon scattering rate: $r = \frac{1}{2\tau} \approx 50MHz$

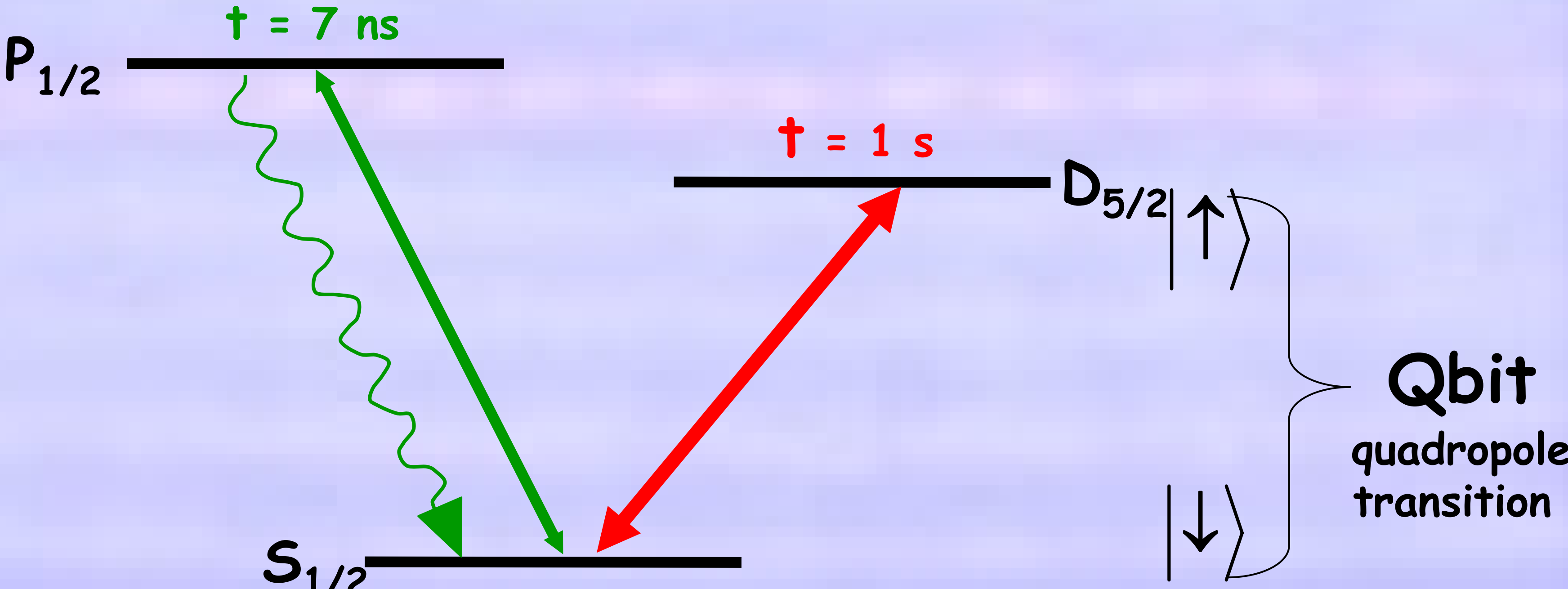
$\eta \approx 10^{-3}$

Rate of detected photons: $R = \eta r \approx 50kHz$

→ 50 photons per ms

Detection within 1 ms feasible provided that the background scattering rate is low.

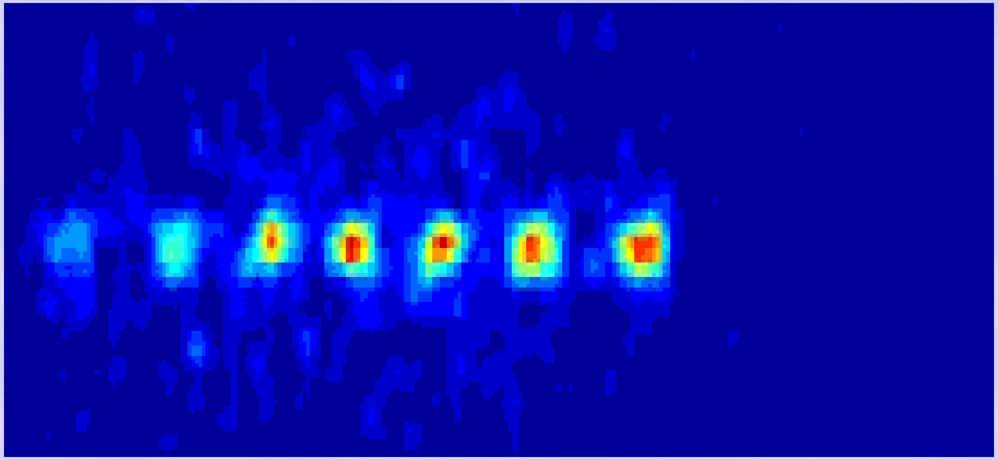
$^{40}Ca^{+}$: Important energy levels



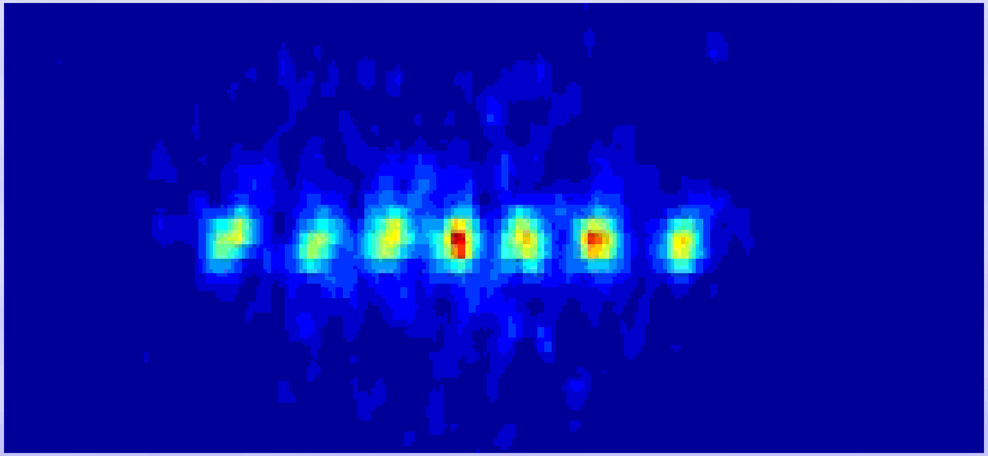
$$\phi = \alpha\left|\uparrow\right\rangle + \beta\left|\uparrow\right\rangle$$

Center-of-mass and breathing mode excitation

„center-of-mass mode“



„stretch mode“



Courtesy of R. Blatt

Laser – Ion Interactions

Hamiltonian: $$H = H^{(Internal)} + H^{(External)} + H^{(Interaction)}$$

$$H^{(Internal)} = \hbar \frac{\omega_0}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$$

$$H^{(External)} = \sum_i \hbar \nu_i a_i^\dagger a_i$$

mode frequencies

$$H^{(Interaction)} = \hbar\Omega\left(|g\rangle\langle e| + |e\rangle\langle g|\right)\cos(k\hat{x} - \omega t + \phi)$$

Rabi frequency

Laser frequency

Laser – Ion Interactions

$$H_{\text{int}} = \frac{\hbar\Omega}{2}\sigma_{+}\exp\left\{i\eta\left(e^{-i\nu t}a + e^{i\nu t}a^{\dagger}\right)\right\}e^{-i\delta t + i\phi} + h.c.$$

Lamb-Dicke parameter

$$\eta = kx_0 = k\sqrt{\frac{\hbar}{2m\nu}}$$

relates size of ground state
to wave length of light

In ion trap experiments,

usually $\eta \ll 1$

$$\delta = \omega - \omega_0$$

Detuning of laser with respect
to atomic transition

Lamb – Dicke regime

Taylor expansion of the exponentiel up to first order:

$$H_{\text{int}} = \frac{\hbar\Omega}{2}\sigma_+\left\{1 + i\eta\left(e^{-i\nu t}a + e^{i\nu t}a^+\right)\right\}e^{-i\delta t + i\phi} + h.c.$$

Carrier resonance:

$\boldsymbol{\delta = 0}$

$$H_{\text{int}} = \frac{\hbar\Omega}{2}\left\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\right\}$$

$$|g,n\rangle \leftrightarrow |e,n\rangle$$

Red sideband:

$\boldsymbol{\delta = -\nu}$

$$H_{\text{int}} = \frac{\hbar\Omega}{2}i\eta\left\{\sigma_+ a e^{+i\phi} - \sigma_- a^+ e^{-i\phi}\right\}$$

$$|g,n\rangle \leftrightarrow |e,n-1\rangle$$

Blue sideband:

$\boldsymbol{\delta = +\nu}$

$$H_{\text{int}} = \frac{\hbar\Omega}{2}i\eta\left\{\sigma_+ a^+ e^{+i\phi} - \sigma_- a e^{-i\phi}\right\}$$

$$|g,n\rangle \leftrightarrow |e,n+1\rangle$$

Vacuum Entanglement in an Ion Trap

1 Introduction to Ion trap Quantum Computing

2 Fast Cooling

The Stark Shift Gate

$$H = \Omega\sigma_x + \eta\Omega\sigma_y\left(ae^{-i\nu t} + a^+e^{+i\nu t}\right)$$

In the Frame Rotating with

$$H = i\eta\Omega\begin{bmatrix} e^{i(2\Omega-\nu)t}\sigma_+a - e^{-i(2\Omega-\nu)t}\sigma_-a^+ \\ e^{i(2\Omega+\nu)t}\sigma_+a^+ - e^{-i(2\Omega+\nu)t}\sigma_-a \end{bmatrix}$$

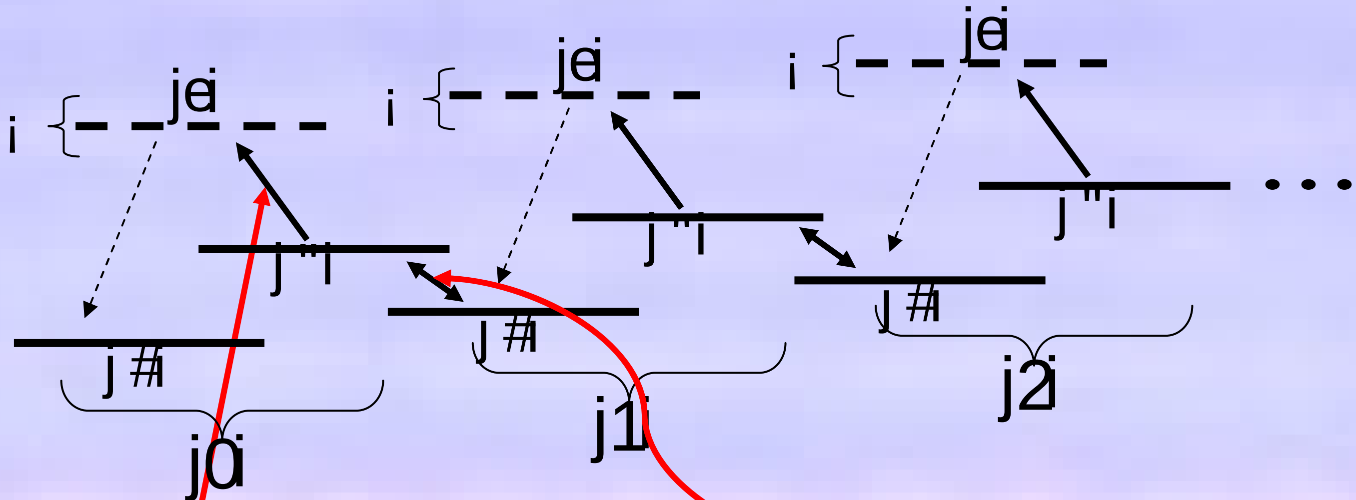
For: $\Omega = \frac{\upsilon}{2}$ in RWA

$$H_{ss} = \frac{i\eta\nu}{2}\left[\sigma_+a - \sigma_-a^+\right]$$

$$|-\rangle|n\rangle \leftrightarrow |+\rangle|n-1\rangle$$

D. Jonathan
M.B. Plenio
P.L. Knight
PRA(2001)

Regular Side Band Cooling



Coupling to a dissipative Level

Cooling Laser: $\Omega \ll \nu$
$|\downarrow\rangle|n\rangle \rightarrow |\uparrow\rangle|n-1\rangle$

Final Population and Final Rate:

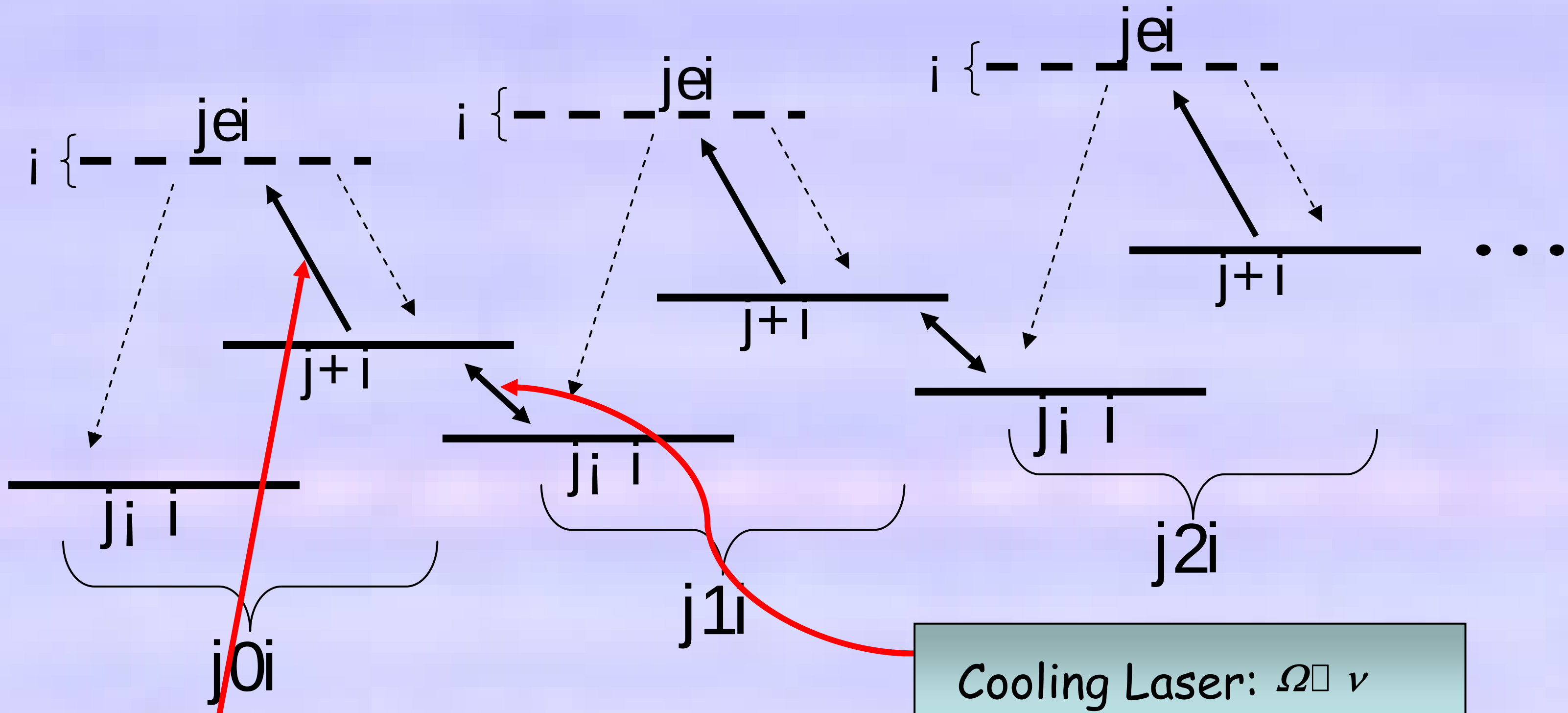
$$\langle n\rangle = \left(\frac{\Gamma}{\nu}\right)^2\left(\alpha + \frac{1}{4}\right), \quad W < \eta^2\Gamma$$

Winleand et. Al. PRL 40 - Two Level Side band cooling

Monroe et. Al. PRL 75 - Raman Side band cooling

Vuletic et. Al. PRL 81 - Side band cooling for Atoms

Stark Shift Cooling

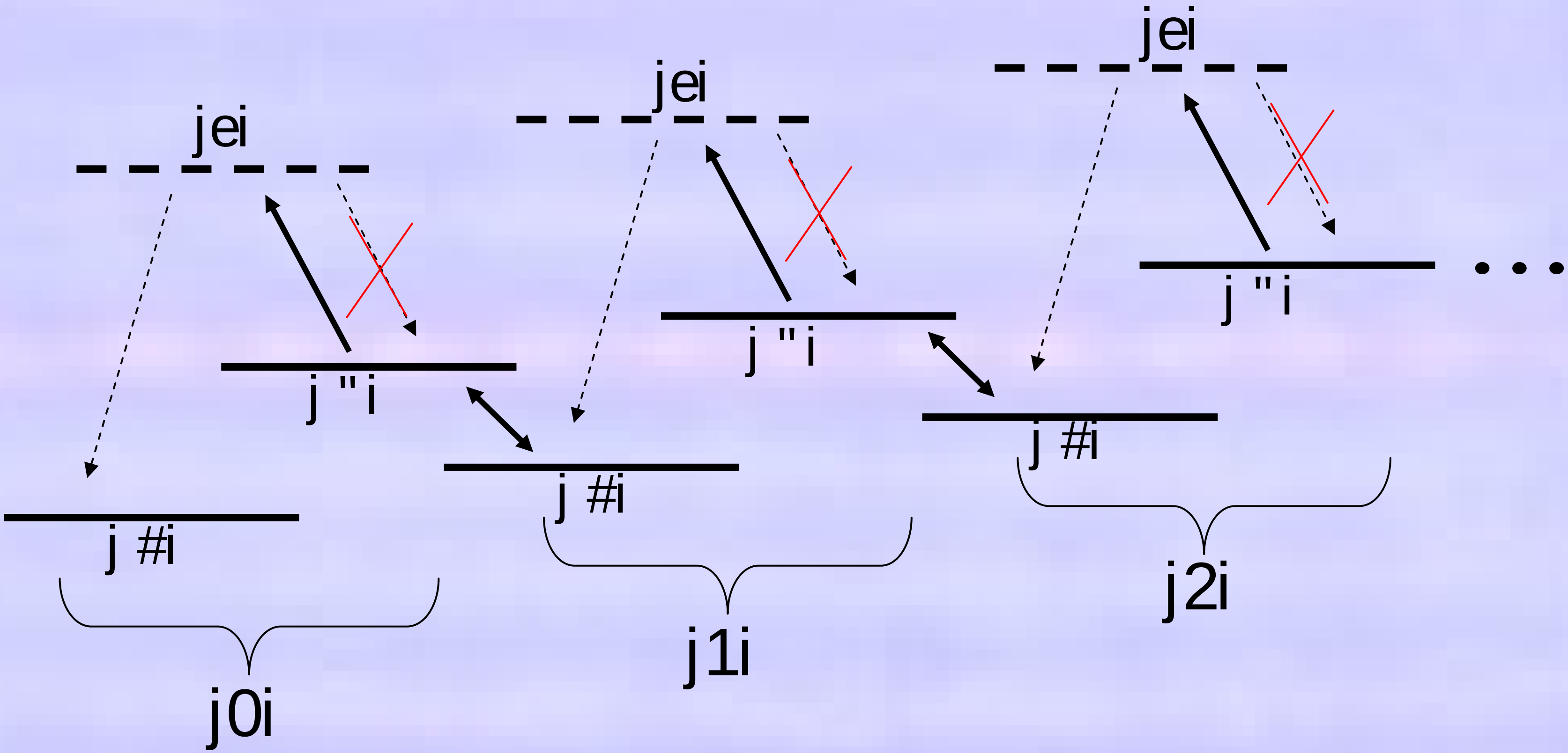


Coupling to a dissipative Level

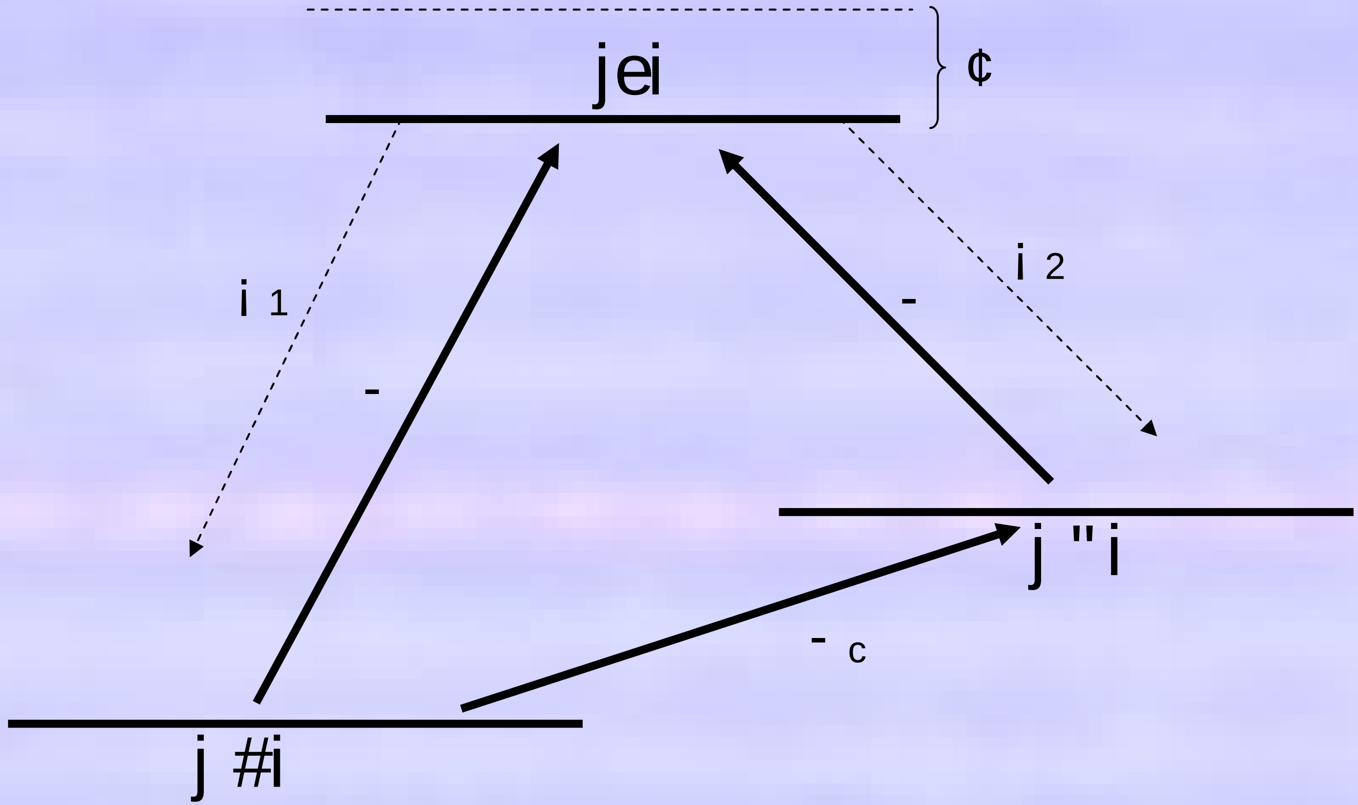
Cooling Laser: $\Omega \ll \nu$

$|-\rangle|n\rangle \rightarrow |+\rangle|n-1\rangle$

Stark Shift Cooling - Pulsed



The Hamiltonian



H = ! ₁j #i h# j + ! ₂j "i h" j + ! ₃jei hej + º aʸa

i - j #i heje^{i i(! 1 + ¢)t} + h:c i - j "i heje^{i i(! 2 + ¢)t} + h:c ¢ ¢ ¢

i - c j #i h" je^{i(kc x i ! c t)} + h:c :

The Master Equation

$$\frac{\partial}{\partial t}\rho = L\rho = \frac{1}{i}[H,\rho] + \frac{\Gamma}{2}\tilde{L}\rho$$

$$H_0 = \begin{pmatrix} \omega_1 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & \omega_3 \end{pmatrix} + \nu a^+ a$$

$$H_{int} = \underbrace{\Omega\left(P_{13}e^{-i\omega_{13}t}\right) + \Omega\left(P_{23}e^{-i\omega_{23}t}\right)}_{\text{EIT Lasers}} + \Omega_c \underbrace{\left(P_{12}e^{i(k_{12}x - \omega_{12}t)}\right)}_{\text{Cooling Laser}}$$

EIT Lasers

Cooling Laser

Relaxation part

$$\dot{\rho}\,|_{rel} = -\left(\Gamma_{13} + \Gamma_{23}\right)P_{33}\rho P_{33}$$
$$+\Gamma_{13}\int \frac{d\Omega(q_{13})}{4\pi}\phi(q_{13})e^{iq_{13}x}P_{13}\rho P_{31}e^{-iq_{13}x}$$
$$+\Gamma_{23}\int \frac{d\Omega(q_{23})}{4\pi}\phi(q_{23})e^{iq_{23}x}P_{23}\rho P_{32}e^{-iq_{23}x}$$
$$-\frac{\Gamma_{13} + \Gamma_{23}}{2}\left(P_{33}\rho P_{11} + P_{11}\rho P_{22} + P_{33}\rho P_{22} + P_{22}\rho P_{33}\right)$$

$$P_{ij} = |i\rangle\langle j|$$

The Master Equation

$$\frac{\partial}{\partial t}\rho = L\rho = \frac{1}{i}[H,\rho] + \frac{\Gamma}{2}\tilde{L}\rho$$

Steady State Solution: $\frac{1}{i}[H,\rho] + \frac{\Gamma}{2}\tilde{L}\rho = 0$

Expansion in the Lamb Dicke Parameter:

$$L = L_0 + \eta L_1 + \eta^2 L_2 + \ldots$$

$$\rho = \rho_0 + \eta\rho_1 + \eta^2\rho_2 + \ldots$$

L_0: No coupling

L_1: Rabi flipping

L_2: Second order rotation + dissipation

ρ_0: Dark State × Mixture of number states

$$\eta^0 : L_0(\rho_0) = 0$$

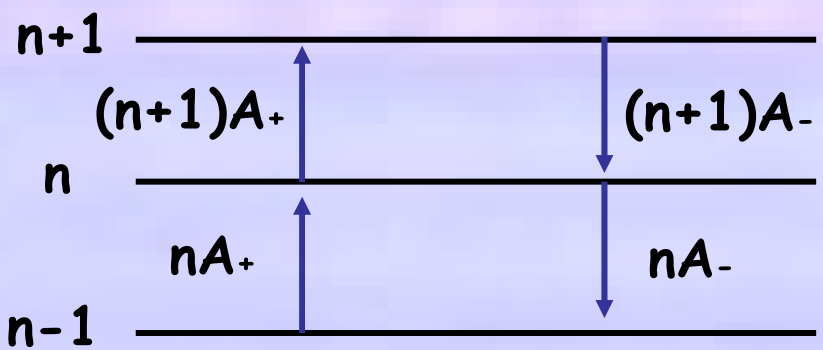
$$\eta^1 : L_1(\rho_0) + L_0(\rho_1) = 0$$

$$\eta^2 : L_2(\rho_0) + L_1(\rho_1) + L_0(\rho_2) = 0$$

The Solution

The expansion is valid under the conditions: $\frac{\Gamma\nu}{\Omega^2}\eta, \frac{\nu^2}{\Omega^2}\eta, \frac{\Delta\nu}{\Omega^2}\eta \ll 1$

$$\frac{d}{dt}P(n)=\eta^2\left\{A_-\Big((n+1)P(n+1)-nP(n)\Big)+A_+\big(nP(n-1)-(n+1)P(n)\big)\right\}$$



$$\Rightarrow \quad p(n)=(1-q)q^n$$

$$q=\frac{A_+}{A_-}$$

$$A_-(\nu)=\frac{2\Gamma\Omega^2\Omega_c^2}{\Gamma^2\left(\nu-2\Omega_c\right)^2+\left(2\Omega^2+\left(\nu-2\Omega_c\right)\left(\Delta-\nu\Omega_c\right)\right)^2}$$

$$A_+=A_-(-\upsilon)$$

Final Temperature and Rate

$$\langle n \rangle = \frac{A_+}{A_- - A_+}$$

$$W = \eta^2 \left(A_- - A_+ \right)$$

The Optimal Point:

$$W \approx \eta \Omega$$

This point is achieved for the validity conditions:

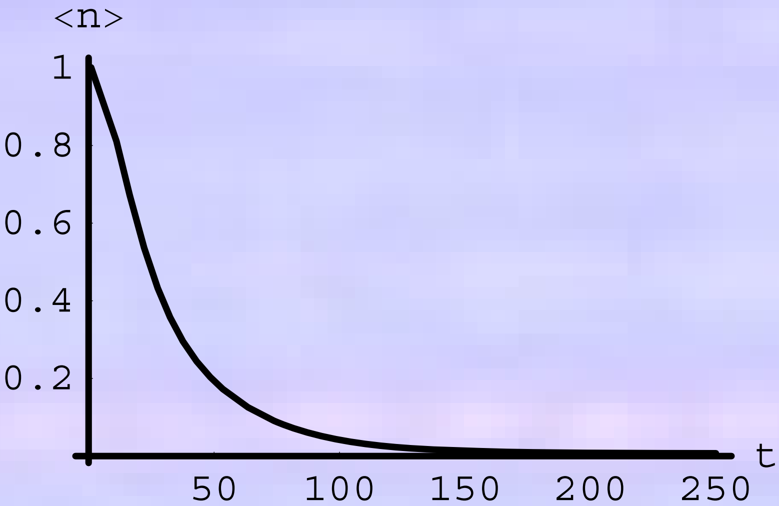
$$\frac{\Gamma \nu}{\Omega^2}\eta, \frac{\nu^2}{\Omega^2}\eta, \frac{\Delta \nu}{\Omega^2}\eta \approx 1$$

The rate at this point:

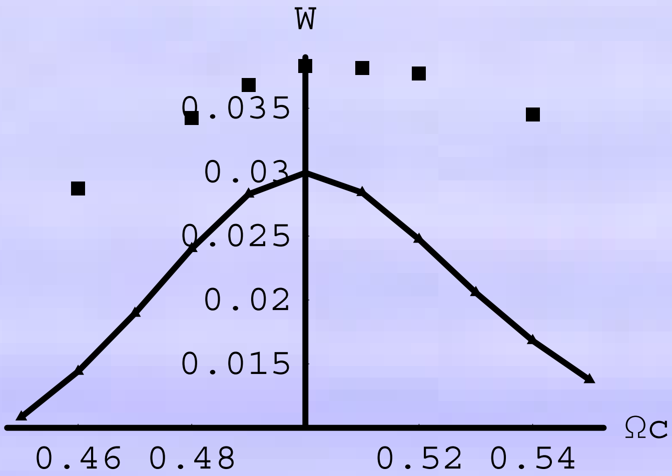
$$\frac{1}{8}\eta \Omega_c$$

Gate period: $\frac{2\pi}{\eta \Omega_c}$

Numerical Results - Rate



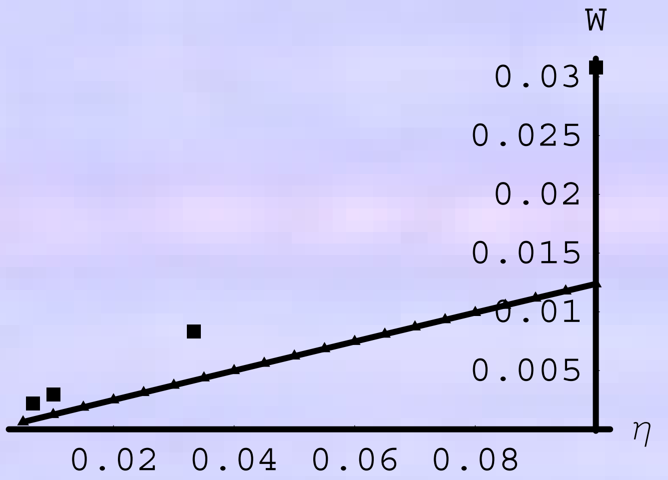
Solution of the Master Equation – Cooling of One Phonon

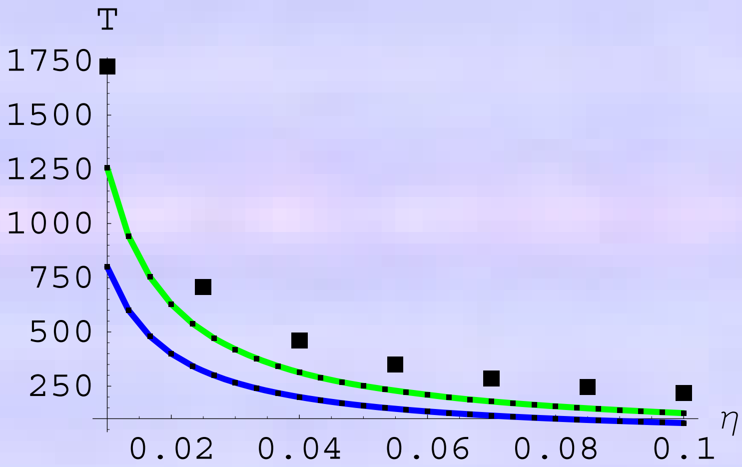


The rate as a function of the Rabi Frequency, Comparison to Numerical Results

The Rate at the Optimal Point

$$W \approx \eta\Omega$$

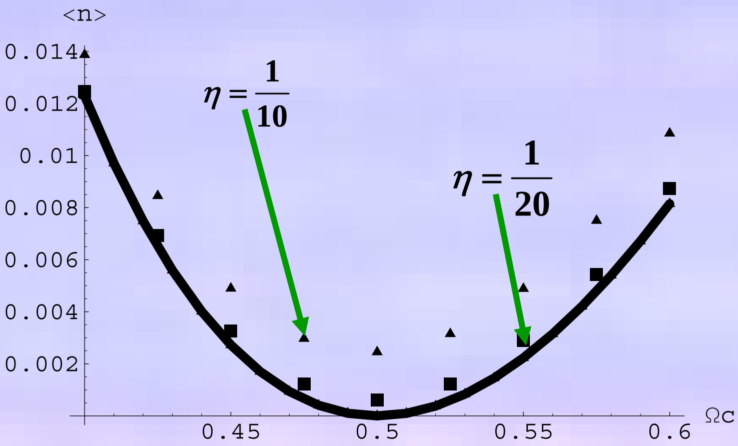




The analytical result versus the numerics. As can be seen from this figure the rate is proportional to the Lamb Dicke parameter and the fit between the rate equation results and the numerical results improves with the decrease of the Lamb Dicke parameter.

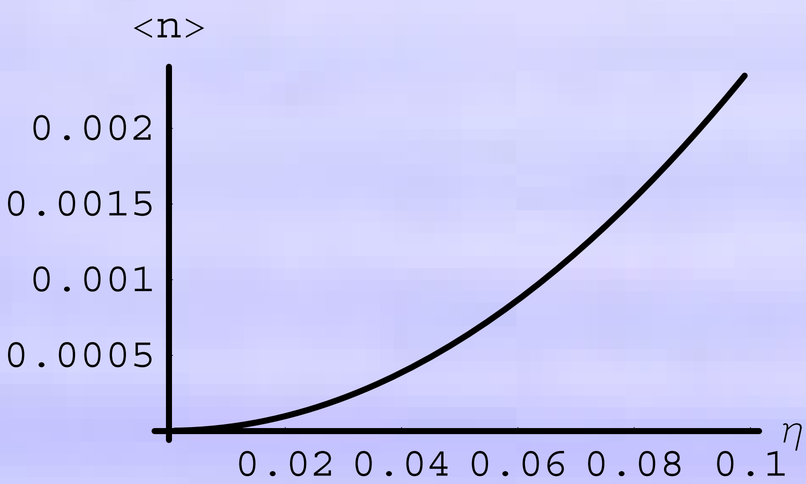
The squares are Tc, Tc is the time that takes to reduce the population from 1 to 0.01.
The green line corresponds to two periods of a Rabi frequency and the blue line to the analytical cooling rate

The Final Population



The final population as a function of the Rabi frequency. The result of the rate equation in comparison with the solution of the Master equation

- = 1=10; º = 1; ¡ = 10; ¢ = 0



The final population as a function of the Lamb Dicke Parameter. Solution of the Master equation

- = 1=10; º = 1; ¡ = 6; ¢ = 0; - c = 1=2

The Final Population

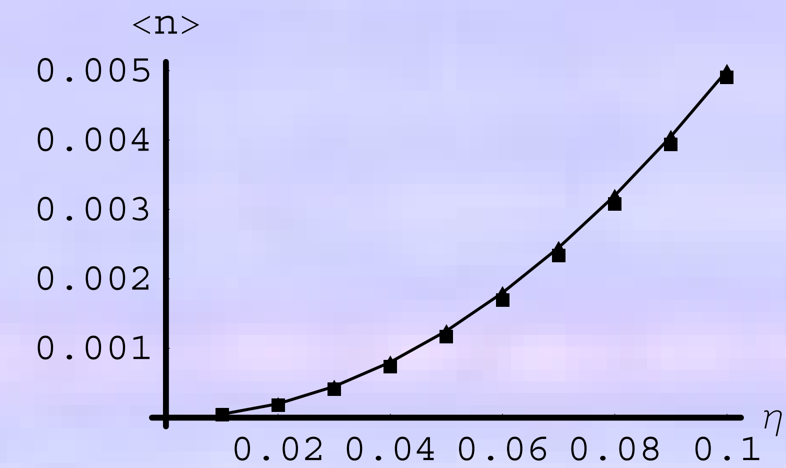
The final population at the optimal point: $\frac{\Gamma\nu}{\Omega^2}\eta, \frac{\nu^2}{\Omega^2}\eta, \frac{\Delta\nu}{\Omega^2}\eta \approx 1$

$$\langle n \rangle = \frac{\Omega^4}{\nu\left(\nu\Gamma^2 + \left(\Delta + \frac{3}{2}\nu\right)\left(\nu^2 + 2\left(\frac{1}{2}\left(\Delta + \frac{\nu}{2}\right)\nu - \Omega^2\right)\right)\right)}$$

$$\langle n \rangle \approx \eta^2$$

Recoil Energy

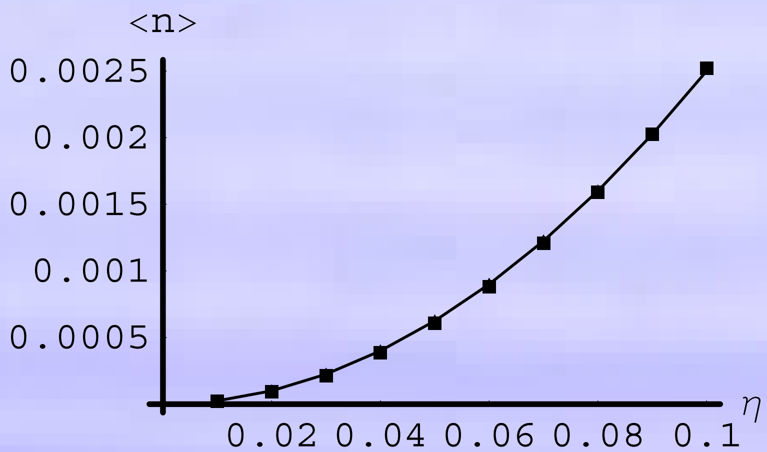
The final population and the optimal point - Numerics



Numerics at the point:

$$\Omega = \sqrt{\Gamma \nu \eta}, \quad \Delta = 0$$

Versus: $\frac{1}{2}\eta^2$



Numerics at the point:

$$\Omega = \sqrt{\Gamma \nu \eta}, \quad \Delta = \Gamma$$

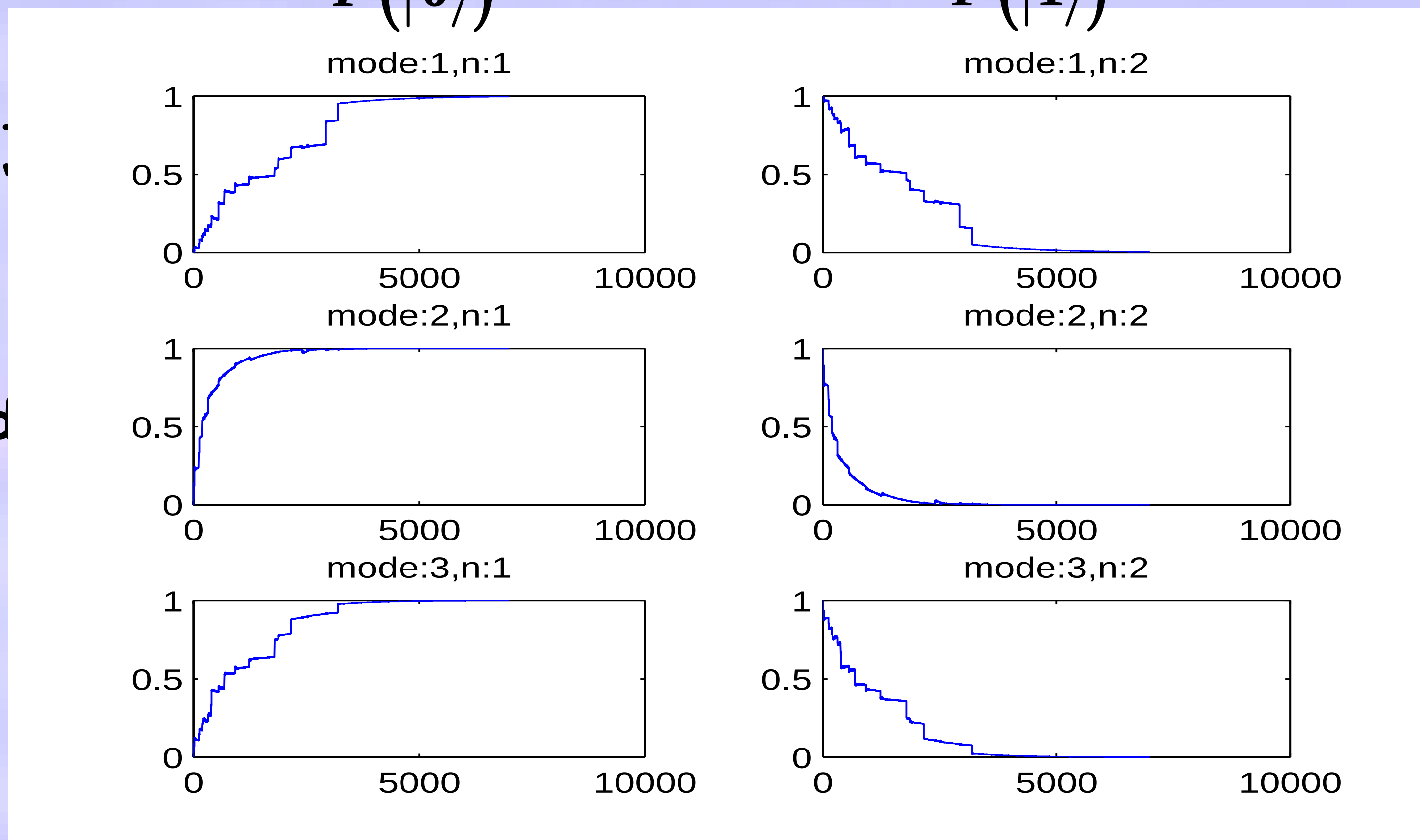
Versus: $\frac{1}{4}\eta^2$

Cooling of a Chain

$P(|0\rangle)$ $P(|1\rangle)$

„center-of-
mass mode"

„stretch mod



Monte Carlo Simulation of cooling three modes simultaneously. The Rabi Frequency is set to the third mode

Summary

1 The Cooling Time ≈ Gate Time

2 The Final Temperature is the Recoil below Energy

3 Cooling of few modes or even the whole chain is possible