Dynamic Symmetry, Quantum Measurements and Ubiquitous Entanglement

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OUTLINE

1. What is erroneous with conventional picture of entanglement.

2. What is essential.

3. Observables, definition of quantum systems, and dynamic symmetry.

4. Definition of complete entanglement.

5. Corollaries: relativity of entanglement, single-particle entanglement, mini-max principle for robust entanglement.

Quantum entanglement is closely concerned with the emerging technologies of quantum cryptography and quantum computing. For example, the quantum key distribution has become recently an industrial product:

- A. Poppe, A. Fedrizzi, R. Ursin, H.R. Böhm, T. Lörunser, O. Maurhardt, M. Peev, M. Suda, C. Kurtsiefer, H. Weinfurter, T. Jennewein, and A. Zeilinger, Optics Express **12**, 3865 (2004)
- J. Ouelette, The Industrial Phys. **10**, 22 (2004)

At the same time, entanglement prompts a re-comprehension of quantum mechanics. It should begin with the very definition of quantum entanglement and its quantification.

The first principle that we use in our analysis is formulated as follows:

"We must not think of the things we could do with, but only of the things that we can't do without".

• Jerome K. Jerome, *Three Men in a Boat, to Say Nothing of the Dog* (1889)

Thus, as the first step, we should try to separate essential from accidental.

Conventional definitions of entanglement

Definition I. Entanglement is usually associated with quantum *nonlocality*. This simply means that measurements on *spatially separated* parts of a quantum system may instantaneously influence one another. Physically this is caused by quantum correlations between the parts of the system. Once created, those correlations keep on surviving even after spatial separation of parts.

Question:

Do we have entanglement if parts of a quantum system are correlated but not separated?

Answer:

Definitely yes!

In particular, this means that a single particle can be entangled with respect to its intrinsic degrees of freedom.

- H. Barnum, E. Knill, G. Ortiz, R. Somma, and L. Viola, Phys. Rev. Lett. **92**, 107902 (2004)
- M.A. Can, A.A. Klyachko, and A.S. Shumovsky, J. Opt. B: Quant. Semiclass. Opt. **7**, L1 (2005)
- S.J. van Enk, Phys. Rev. A **72**, 064306 (2005)
- A.A. Klyachko and A.S. Shumovsky, J. Phys: Conf. Series **36**, 87 (2006); *E-print quant-ph/0512213*

Definition II. Another common opinion is that the entanglement of multipartite systems defined in the Hilbert space

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \cdots$$

can be associated with *nonseparability* of states $\psi \in \mathcal{H}$ with respect to the parts of the system.

- E.g., see: D. Bruß, J. Math. Phys. **43**, 4237 (2002)

This statement, which is undoubtedly valid in the case of bipartite systems, does not have a lucid sense for multipartite entanglement. It also has no meaning in the case of single-particle entanglement.

Example

is provided by three-qubit states, whose classification has been constructed in

- A. Miyake, Phys. Rev. A **67**, 012108 (2003)

State	Type	3-part entanglement	2-part entanglement
GHZ	nonseparable	yes	no
W	nonseparable	no	yes
Bi-separable	separable	no	yes

Definition III. Entanglement is also associated with violation of different Bell-type conditions of classical realism. However, unentangled states can also manifest violation of those conditions.

• A.A. Klyachko, *E-print quant-ph/0206012*

• H. Barnum, E. Knill, G. Ortiz, and L. Viola, Phys. Rev. A **68** 032308 (2003)

• A.A. Klyachko, J. Phys.: Conf. Series **36**, 87 (2006)

Essential:

I. All entangled states of a given system are equivalent to within SLOCC. States from different classes are SLOCC nonequivalent.

- W. Dür, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314 (2000)
- F. Verstraete, J. Dehaene, and B. De Moor, Phys. Rev. A **68**, 012103 (2003)
- A. Miyake, Phys. Rev. A **67**, 012108 (2003)

$$\psi_E = (\widehat{SLOCC})\,\psi_{CE}.$$

II. CE is a manifestation of quantum fluctuations in a state ψ_{CE} where they come to their extreme.

- M.A. Can, A.A. Klyachko, and A.S. Shumovsky, Phys. Rev. A **66**, 02111 (2002)
- A.A. Klyachko and A.S. Shumovsky, J. Opt. B: Quant. and Semiclas. Optics **5**, S322 (2003)
- For a recent review, see: A.A. Klyachko and A.S. Shumovsky, J. Phys: Conf. Series **36**, 87 (2006); quant-ph/0512213

Basic observables:

Von Neumann theory of quantum measurements: all Hermitian operators represent measurable physical quantities.

This assumption has been put into question by Wick, Wightman and Wigner

- G.C. Wick, A.S. Wightman, and E.P. Wigner, Phys. Rev. **88**, 101 (1952).

In the case of conventional bipartite entanglement, only local observables give information about quantum correlations between parts of the system.

Hermann's conjecture: for a given quantum system, measurable observables form a Lie algebra $\mathcal{L}$ of Hermitian operators acting in Hilbert space $\mathcal{H}$ of system under consideration.

- R. Hermann , *Lie Groups for Physicists* (Benjamin, New York, 1966)

We choose orthogonal basis X_i of $\mathcal{L}$ as *basic observables*, whose measurement give us the whole allowed information about a given state of the system. The corresponding Lie group

$$G = \exp(i\mathcal{L})$$

determines the dynamic symmetry of the system.

• A.A. Klyachko, *E-print quant-ph/0206012*

• A.A. Klyachko and A.S. Shumovsky, J. Opt. B: Quant. and Semiclas. Optics **5**, S322 (2003)

• A.A. Klyachko and A.S. Shumovsky, J. Phys: Conf. Series **36**, 87 (2006); *E-print quant-ph/0512213*

Example:

A qubit (state in two-dimensional Hilbert space $\mathcal{H}_2$).
$\star$ Dynamic symmetry $G = \mathrm{SU}(2)$
$\star$ Basic observables = three Pauli operators $X_i = \sigma_i$, $(i = x, y, z)$.

N qubits (state in $\mathcal{H} = \bigotimes_{j=1}^{N} \mathcal{H}_2$)
$\star$ Dynamic symmetry $G = \prod_{j=1}^{N} \mathrm{SU}(2)$
$\star$ Basic observables = $3N$ pauli operators (three Pauli operators for each part).

Note:

$$(S\widehat{LOCC}) = g^c \in G^c = \exp(\mathcal{L} \otimes \mathbb{C}).$$

- F. Verstraete, J. Dehaene, and B. De Moor, Phys. Rev. A **68**, 012103 (2003)

Definition of CE states:

$$\psi = \psi_{CE} \in \mathcal{H}$$

iff

$$\langle\psi|X_i|\psi\rangle = 0 \qquad \forall X_i.$$

- A.A. Klyachko, *E-print quant-ph/0206012*
- M.A. Can, A.A. Klyachko, and A.S. Shumovsky, Phys. Rev. A **66**, 02111 (2002)
- A.A. Klyachko and A.S. Shumovsky, J. Opt. B: Quant. and Semiclas. Optics **5**, S322 (2003)

Total variance

$$\mathbb{V}(\psi) = \sum_i (\langle\psi|X_i^2|\psi\rangle - \langle\psi|X_i|\psi\rangle^2).$$

Hence

$$\mathbb{V}(\psi_{CE}) = \max_{\psi\in\mathcal{H}} \mathbb{V}(\psi).$$

Casimir operator $C_{\mathcal{H}} = \sum_i X_i^2$ acts as a scalar in any irreducible representation $G : \mathcal{H}$. Thus,

$$\mathbb{V}(\psi_{CE}) = C_{\mathcal{H}}.$$

Corrolaries:

$\ast$ Measure of entanglement of pure states:

$$\mu(\psi) = \sqrt{\frac{\mathbb{V}(\psi) - \mathbb{V}_{\mathrm{min}}}{\mathbb{V}_{\mathrm{max}} - \mathbb{V}_{\mathrm{min}}}}$$

• A.A. Klyachko, B. Öztop, and A.S. Shumovsky, Appl. Phys. Lett. **88**, 124102 (2006)

In particular case of bipartite entanglement, this measure coincides with general concurrence of Ref.

• P. Rungta, V. Bužek, C.M. Caves, M. Hillery, and G.J. Milburn, Phys. Rev. A, **64**, 042315 (2001)

$*$ Relativity of entanglement with respect to G:

A qutrit

$$|\psi\rangle = \sum_{s=-1}^{1} \psi_s |s\rangle \in \mathcal{H}_3.$$

The two symmetries are allowed: $G = \mathrm{SU}(3)$ (the true qutrit). Basic observables are given by eight Gell-Mann matrices.

C.M. Caves and G.J. Milburn, Opt. Commun. **179**, 439 (2000)

$G' = \mathrm{SU}(2)$ (the spin-qutrit). Basic observables are given by three spin-1 operators.

• M.A. Can, A.A. Klyachko, and A.S. Shumovsky, J. Opt. B: Quant. Semiclass. Opt. **7**, L1 (2005).

* Single-particle entanglement


NON-VIOLENCE

State

$$|\psi\rangle = \psi_+|+1\rangle + \psi_0|0\rangle + \psi_-|-1\rangle$$

of a single spin-qutrit with basic observables

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

can manifest entanglement. For example, the states $|0\rangle$ and

$$\frac{1}{\sqrt{2}}(|+1\rangle + |-1\rangle)$$

obey the condition of complete entanglement

$$\langle S_\alpha\rangle = 0 \quad \alpha = x, y, z.$$

What does it mean?

Clebsch-Gordon decomposition

$$\mathcal{H}_2 \otimes \mathcal{H}_2 = \mathcal{H}_3 \oplus \mathcal{H}_A.$$

If $|\uparrow\rangle$ and $|\downarrow\rangle$ are basis states of $\mathcal{H}_2$ $(\dim \mathcal{H}_2 = 2)$, then the symmetric states

$$|s\rangle = \begin{cases} |\uparrow\uparrow\rangle, & \text{projection of total spin } s = 1 \\ \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), & \text{projection of total spin } s = 0 \\ |\downarrow\downarrow\rangle, & \text{projection of total spin } s = -1 \end{cases}$$

form the basis of $\mathcal{H}_3$, while the antisymmetric singlet

$$|A\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle).$$

is associated with $\mathcal{H}_A$.

Examples

✠ Isotriplet $I = 1$ of π-mesons

$$|s\rangle = \begin{cases} \pm 1 & \text{charged } \pi^{\pm} \text{ mesons} \\ 0 & \text{neutral } \pi^{0} \text{ meson} \end{cases}$$

Quark structure of π mesons:

$$\begin{cases} |+1\rangle & \sim \ \pi^{+} = u\bar{d} \\ |0\rangle & \sim \ \pi^{0} = (u\bar{u} + d\bar{d})/\sqrt{2} \\ |-1\rangle & \sim \ \pi^{-} = \bar{u}d \end{cases}$$

✠ Biphoton

✠ Deuteron

✠ Cooper pairs in superfluid ^{3}He.

∗ Mini-max principle for robust entanglement:

Maximum of quantum fluctuations of basic observables at minimum of energy of the system.

Can MA, Klyachko AA and Shumovsky AS 2002 *Appl. Phys. Lett.* **81** 5072

• Can MA, Çakır Ö, Klyachko AA and Shumovsky AS 2003 *Phys. Rev. A* **68** 022395

• Çakır Ö, Klyachko AA and Shumovsky AS 2005 *Phys. Rev. A* **71** 034303

Thank you for attention!