DIVERGENCE OF THE ENTANGLEMENT RANGE

IN LOW DIMENSIONAL QUANTUM SYSTEMS

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SUMMARY

- **MODELS** : ANTIFERROMAGNETIC EXCHANGE
  vs UNIFORM MAGNETIC FIELD

- **PHENOMENOLOGY** : QUANTUM CRITICALITY, SATURATION,
  FACTORIZATION

- **TOOLS** : CONCURRENCE
  PARALLEL & ANTIPARALLEL ENTANGLEMENT
  RANGE OF THE CONCURRENCE R

- **RESULTS** : ENTANGLEMENT "PHASE DIAGRAM"
  DIVERGENCE OF R

- **MORE MODELS**

- CONCLUSIONS AND BIBLIOGRAPHY

MODELS

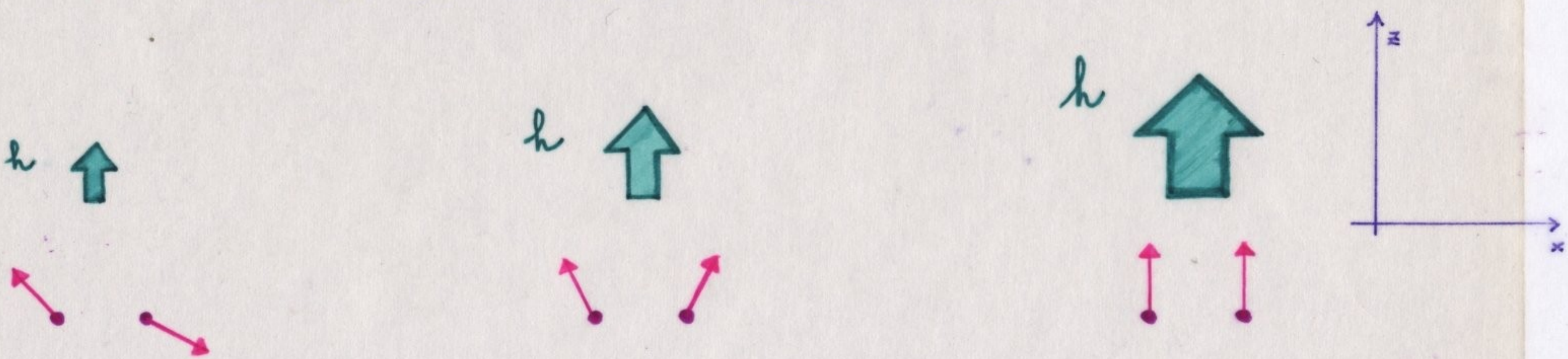
$$\mathcal{H} = J \sum_i S_i^x S_{i+1}^x + \Delta_y S_i^y S_{i+1}^y + \Delta_z S_i^z S_{i+1}^z - h S_i^z$$

$$J > 0 \; ; \; 0 \leq \Delta_{y,z} \leq 1 \; ; \; S = \frac{1}{2}$$

AN ANTIFERROMAGNETIC ZOOLOGY

Ising	Kosterlitz-Thouless	isotropic
XY(Z)	XX(Z)	XXX

WITH A COMMON FEATURE



ANTIFERROMAGNETIC EXCHANGE VERSUS UNIFORM MAGNETIC FIELD

$\parallel x$ $\parallel z$

PHENOMENOLOGY $(T=0)$

(**TOOLS :** $g_r^{\alpha\beta} \equiv \langle S_i^\alpha S_{i+r}^\beta \rangle$ & $M^\alpha \equiv \langle S_i^\alpha \rangle$; $\alpha, \beta = x, y, z$)

$M^z \neq 0$ whenever $h_z \neq 0$

- **QUANTUM CRITICALITY** [XY(Z) & XX(Z)]

$\exists\, h_c > 0$ such that

$$h < h_c \qquad g_r^{xx} \underset{r\to\infty}{\sim} \frac{1}{r^\eta} \quad \begin{cases} \eta = 0 & \Rightarrow \ M^x > 0 \quad \text{ordered phase (Ising)} \\ \eta > 0 & \Rightarrow \ M^x = 0 \quad \text{quasi-ordered phase (KT)} \end{cases}$$

$$h > h_c \qquad g_r^{xx} \underset{r\to\infty}{\sim} e^{-r/\xi^{xx}} \qquad \text{disordered phase}$$

- **SATURATION** [XXX & XX(Z)]

$\exists\, h_s < +\infty$ such that

$$h \geq h_s \qquad g_r^{zz} = (M^z)^2 = \frac{1}{4} \quad \forall r \qquad \text{paramagnetic disordered phase}$$

FACTORIZATION [XY(z) & XX(z)]

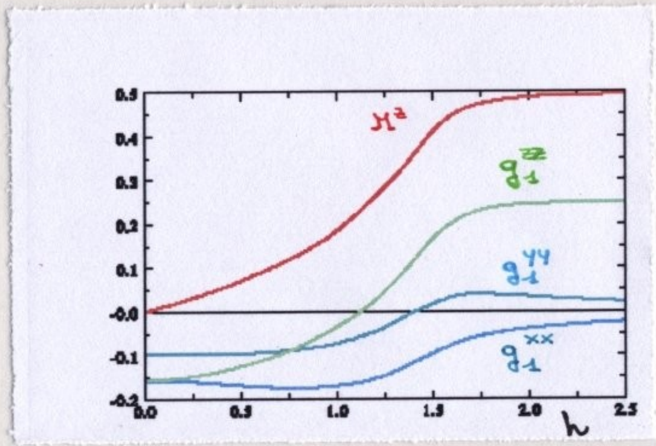
Kurmann, Thomas, and Müller : Physica 112A, 235 (1982)

$$\exists\, h_f = \sqrt{(1+\Delta_z)(\Delta_y+\Delta_z)} \quad \text{such that}$$

$$|GS\rangle = \prod_i |\varphi_i\rangle \quad \text{"classical-like GS"} \quad @\, h_f$$

$$|\varphi_{2i}\rangle = |\alpha\rangle \quad \& \quad |\varphi_{2i+1}\rangle = |\beta\rangle$$

$$\varepsilon_0 = -\frac{1+\Delta_x+\Delta_y}{4}$$



no peculiarities @ h_f

!

"new" TOOLS

ENTANGLEMENT BETWEEN A COUPLE OF SPINS

S_i & S_{i+r}

↓

CONCURRENCE C_r

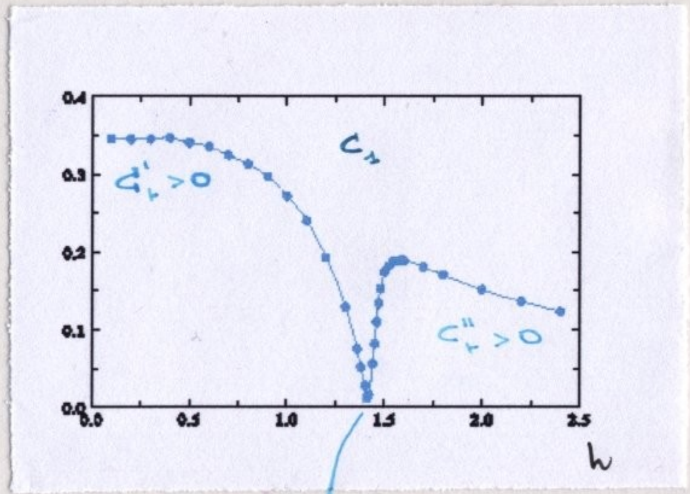
$$C_r \equiv 2\max\{0, C'_r, C''_r\}$$

$$C'_r = |g^{xx}_r + g^{yy}_r| - \sqrt{\left(\tfrac{1}{4} + g^{zz}_r\right)^2 - M_z^2} \quad ; \quad C''_r = |g^{xx}_r - g^{yy}_r| + g^{zz}_r - \frac{1}{4}$$

$h < h_f$

$C'_r > 0$

$C''_r < 0$



$h > h_f$

$C'_r < 0$

$C''_r > 0$

@h_f → $C'_r = C''_r = 0$

PARALLEL & ANTIPARALLEL ENTANGLEMENT (bipartite)

$|e_1\rangle \equiv \dfrac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}$; $|e_2\rangle \equiv \dfrac{|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle}{\sqrt{2}}$ parallel Bell states

$|e_3\rangle \equiv \dfrac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$; $|e_4\rangle \equiv \dfrac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$ antiparallel Bell states

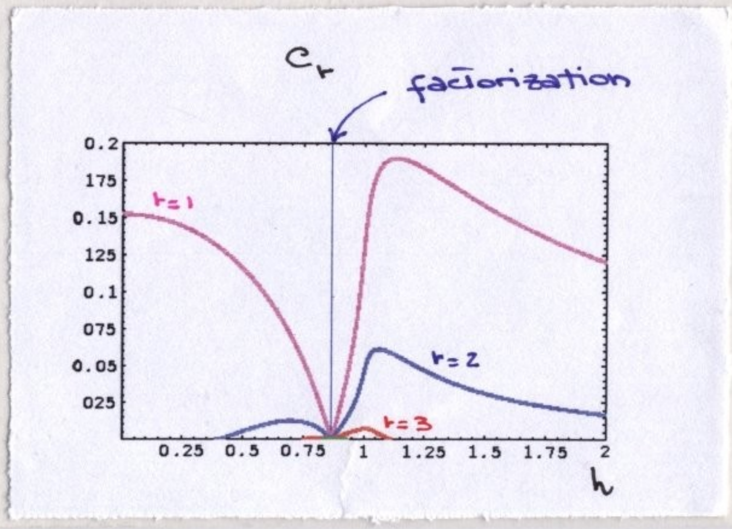
$p_\alpha \Rightarrow$ probability that the pair S_i & S_{i+r} be in the state $|e_\alpha\rangle$

$+ \; p_{\uparrow\uparrow} , p_{\downarrow\downarrow}$

$\Downarrow$

$2C'_r = |p_3 - p_4| - 2\sqrt{p_{\uparrow\uparrow} p_{\downarrow\downarrow}}$ $\qquad$ $2C''_r = 2\max\{p_1, p_2\} - 1$

antiparallel entanglement



parallel entanglement

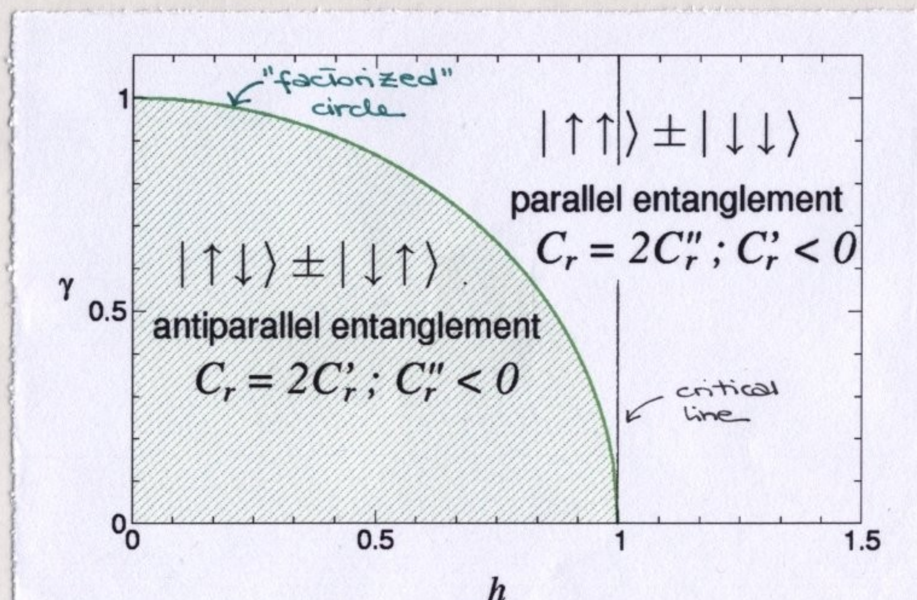
ENTANGLEMENT "PHASE DIAGRAM"

$$\left.\begin{matrix} C'_r \geq 0 \\ C''_r \leq 0 \end{matrix}\right\} \quad h < h_f \qquad \forall r \ !$$

$$C'_r = 0 = C''_r \qquad @ h_f \qquad \forall r \ !$$

$$\left.\begin{matrix} C'_r \leq 0 \\ C''_r \geq 0 \end{matrix}\right\} \quad h > h_f \qquad \forall r \ !$$

XY model in a transverse field

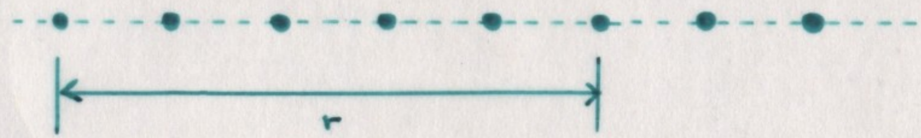


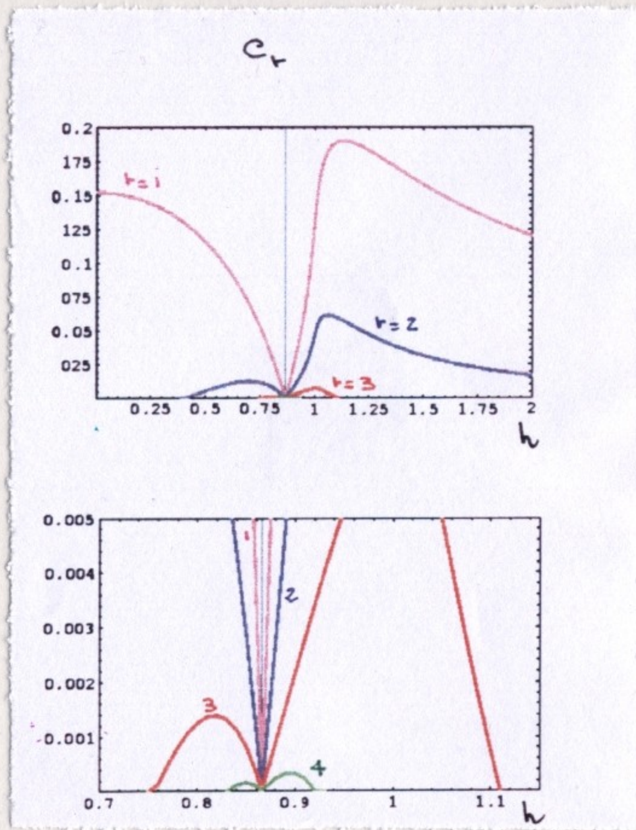
$h_f = \sqrt{1-\gamma^2}$

$h_c = 1$

?

along the chain





as $h \to h_f$

all e_r get positive

$\Downarrow$

long-ranged bipartite entanglement

but

$$\sum_r e_r^2 \leq \tau \leq 1$$

$\Downarrow$

$$C_r \to 0$$
$$h \to h_f$$
$$\forall r$$

ENTANGLEMENT RANGE

it is observed that

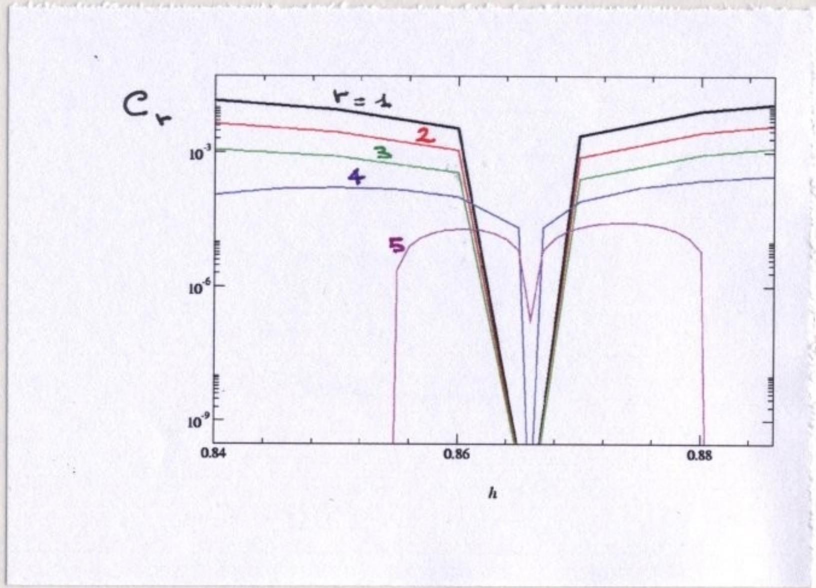
$$\text{if} \quad C_r > 0 \Rightarrow C_m > 0 \ \forall m \leq r$$

the range of the concurrence may be hence defined as

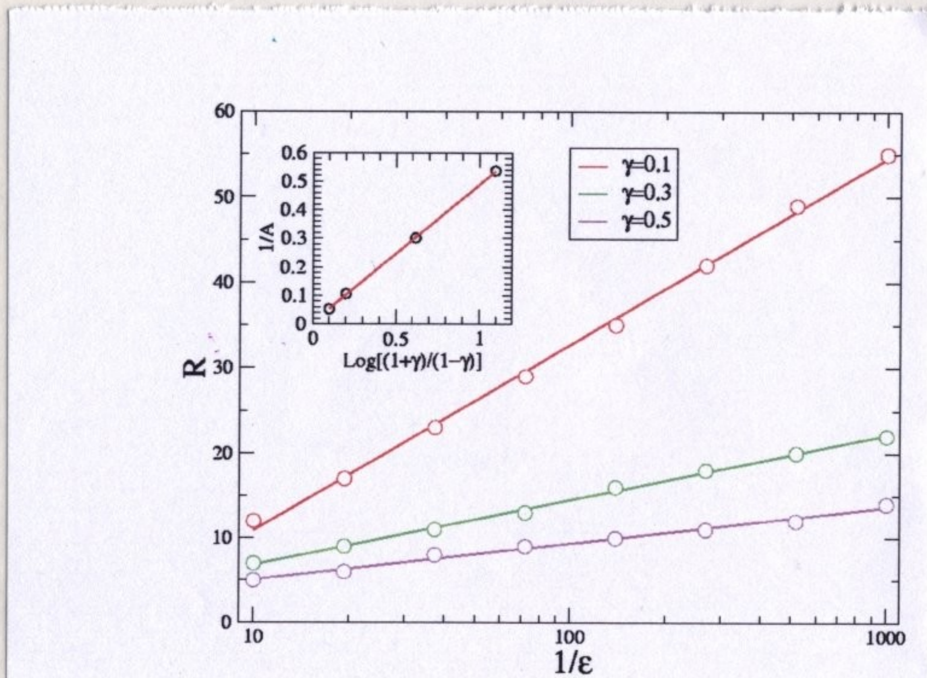
$$R \quad \text{such that} \quad \begin{array}{ll} C_r > 0 & \text{if } r \leq R \\ C_r < 0 & \text{if } r > R \end{array}$$

for a given model R depends on h

DIVERGENCE of R @ h_f



$$\lim_{h \to h_f} R = \infty$$!



the way **R** diverges @ h_f depends on the model :

$$R^{XY} \sim \frac{1}{\ln\left(\frac{1-\gamma}{1+\gamma}\right)} \frac{1}{\varepsilon} = \frac{1}{\ln(\Delta_Y)} \frac{1}{\varepsilon}$$

$$\varepsilon \equiv |h - h_f| \quad ; \quad h_f = \sqrt{1-\gamma^2} = \sqrt{\Delta_Y(1+\Delta_Y)}$$

$$(A^{-1} \equiv \varepsilon R^{XY} \sim \ln(\Delta_Y))$$

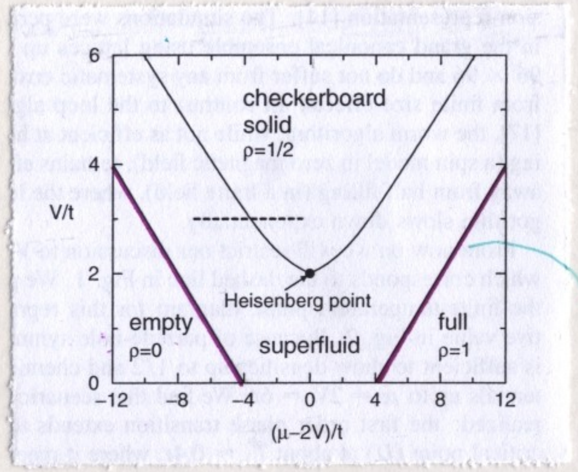
MORE MODELS

- factorization occurs in any dimension provided the lattice is bipartite
  → (ladders, square lattices ...)

- factorization is observed in magnetic models with long-ranged interactions
  (Vidal et al. PRL 90, 227902 (2003))

- XXZ models in a field are exactly mapped into the
  Hard-core boson Hubbard models

Schmid et al. PRL 88, 167208 (2002)



H-c boson Hubbard model on the square lattice

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left(a_i^+ a_j + a_j^+ a_i \right) + V \sum_{\langle ij \rangle} n_i n_j - \mu \sum_i n_i$$

→ XXZ via $t \to \frac{1}{2}$; $V \to j_z$; $\mu - 2V \to h$

~ $h = 2(1 + j_z)$ saturation in 2d

...

T > 0 ?

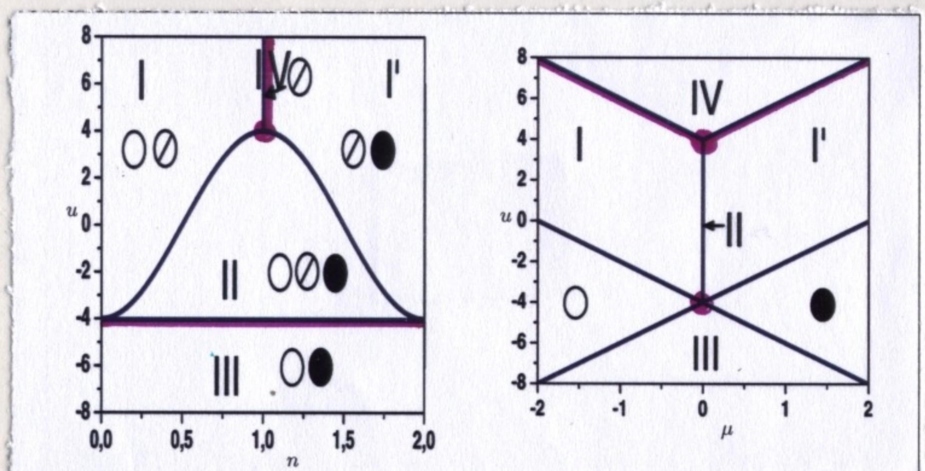
EVEN MORE MODELS

the bond-charge extended Hubbard model

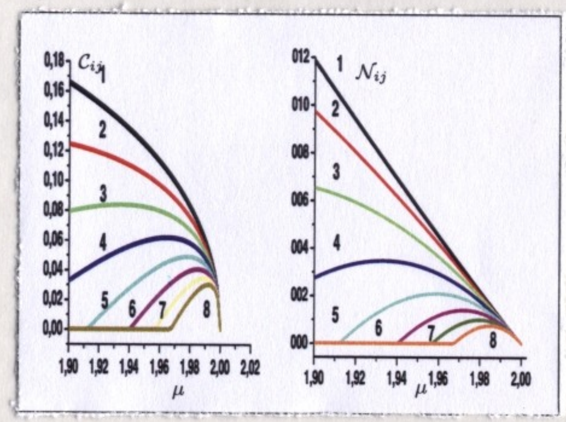
$$\mathcal{H}_{BC} = -\sum_{i,\sigma} \left[1 - x\left(n_{i\bar{\sigma}} + n_{i+1,\bar{\sigma}}\right)\right] c^{+}_{i\sigma} c_{i+1,\sigma} - \mu \sum_{i\sigma} n_{i\sigma} + u \sum_{i} \left(n_{i\uparrow} - \tfrac{1}{2}\right)\left(n_{i\downarrow} - \tfrac{1}{2}\right)$$

$c^{+}_{i\sigma}, c_{i\sigma}$: fermionic operators ; $\sigma = \uparrow, \downarrow$; $\bar{\sigma}$ opposite of σ ; $n_{i\sigma} = c^{+}_{i\sigma} c_{i\sigma}$

Anfossi et al. cond-mat/0611094



$\longrightarrow$



I' → IV

@ —— long-ranged bipartite entanglement ⟶ divergent R

?

? FACTORIZATION $\overset{vs}{\simeq}$ QUANTUM CRITICALITY ?

studied models (always with a uniform field // $\hat{z}$)

1d : XY

XYX with $\Delta_y = 0, 0.25, 0.5, 0.75$

XXX

ladder : XXX

2d : XYX with $\Delta_y = 0, 0.25, 0.5, 0.75$

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THANKS !