



Dynamical Formulation of (Anti-)Bunching

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Identical Particles in Quantum Mechanics



$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \pm \psi(\mathbf{r}_2, \mathbf{r}_1)$$

“+” : Bose-Einstein statistics ——— “bosons” (photons, ...)

“−” : Fermi-Dirac statistics ——— “fermions” (electrons, neutrons, protons, ...)

Two fermions (in the same spin states)

at the same position $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$

$$\psi(\mathbf{r}, \mathbf{r}) = -\psi(\mathbf{r}, \mathbf{r}) = 0 \quad \text{——— antibunching}$$

Fermions avoid each other.

The spin degree of freedom is omitted in this page.

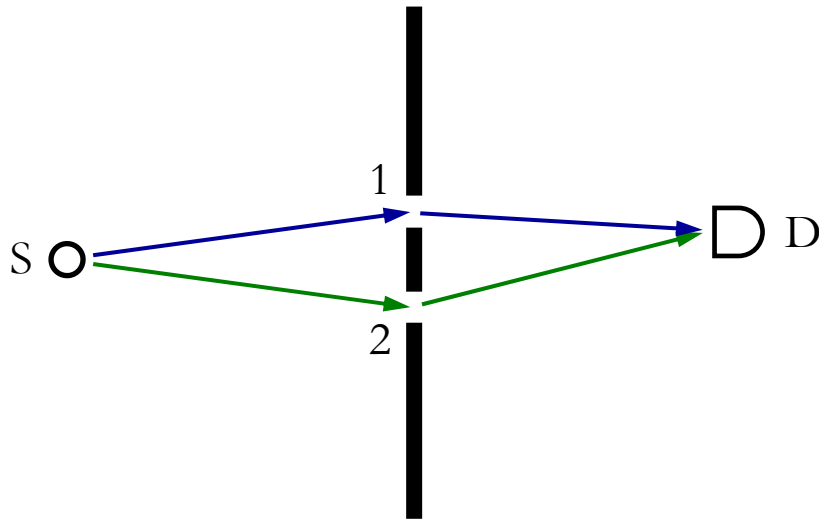
Motivations

- (Anti-)Bunching
 - HBT effects in Astronomy, Heavy-Ion Collisions
 - Generation of Entangled Photons
- Coherence Properties of Multi-Particle Systems
 - Coherence Lengths (Longitudinal/Lateral)
 - Entanglement, ...
- Dynamical Model for Coincidence Exp.
 - Particle Source as a Dynamical System
 - Characteristics of the Source
 - Temperature, Size, ...

First-/Second-Order Interference

Double-Slit Experiment

single detector

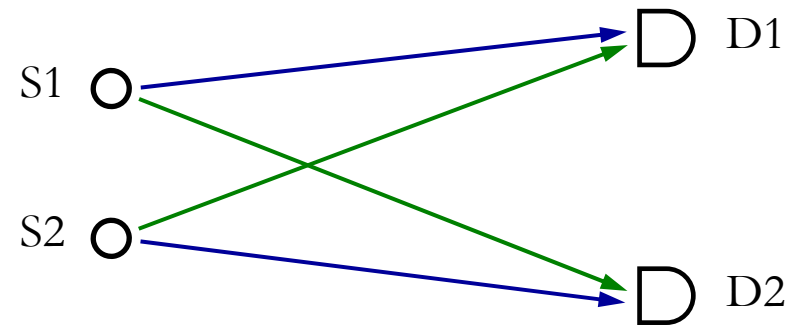


$$|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1^* \psi_2| \cos(\phi_1 - \phi_2)$$

phase difference

Coincidence Experiment

two detectors



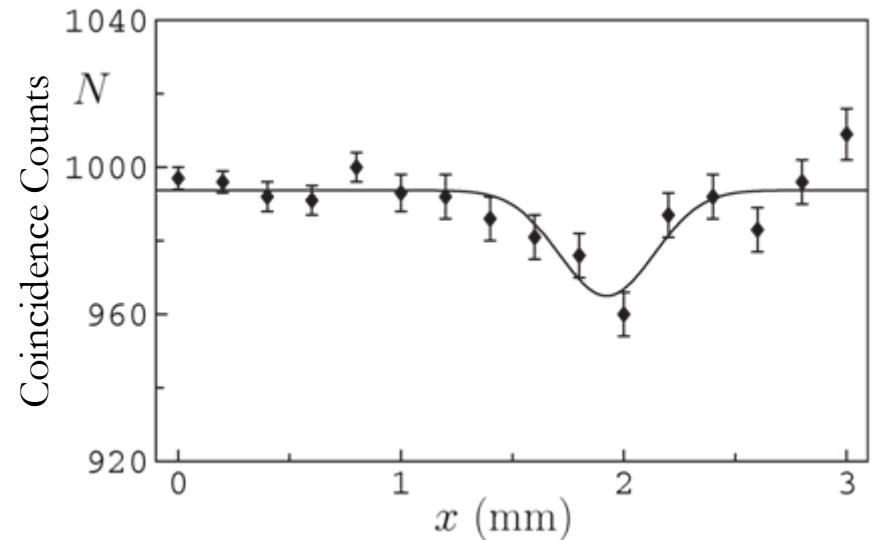
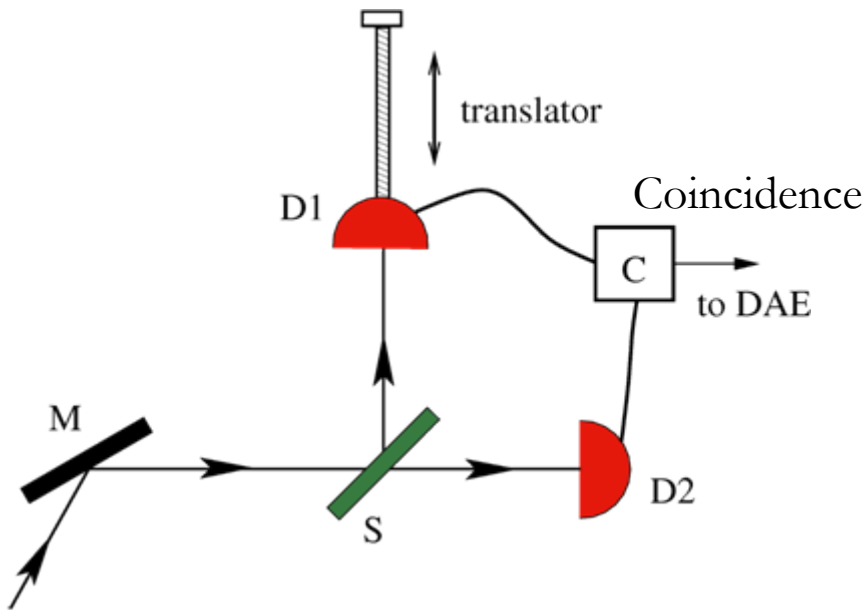
fermion/boson

$$\frac{1}{\sqrt{2}} \left(\psi_{S1}(1)\psi_{S2}(2) \mp \psi_{S1}(2)\psi_{S2}(1) \right)$$

$$\xrightarrow{D1=D2} \begin{cases} 0 & \dots \text{antibunching} \\ \sqrt{2} \psi_{S1}(D)\psi_{S2}(D) & \dots \text{bunching} \end{cases}$$

Neutron Antibunching Experiment

M. Iannuzzi, A. Orecchini, F. Sacchetti, P. Facchi, and S. Pascazio, PRL **96**, 080402 (2006).



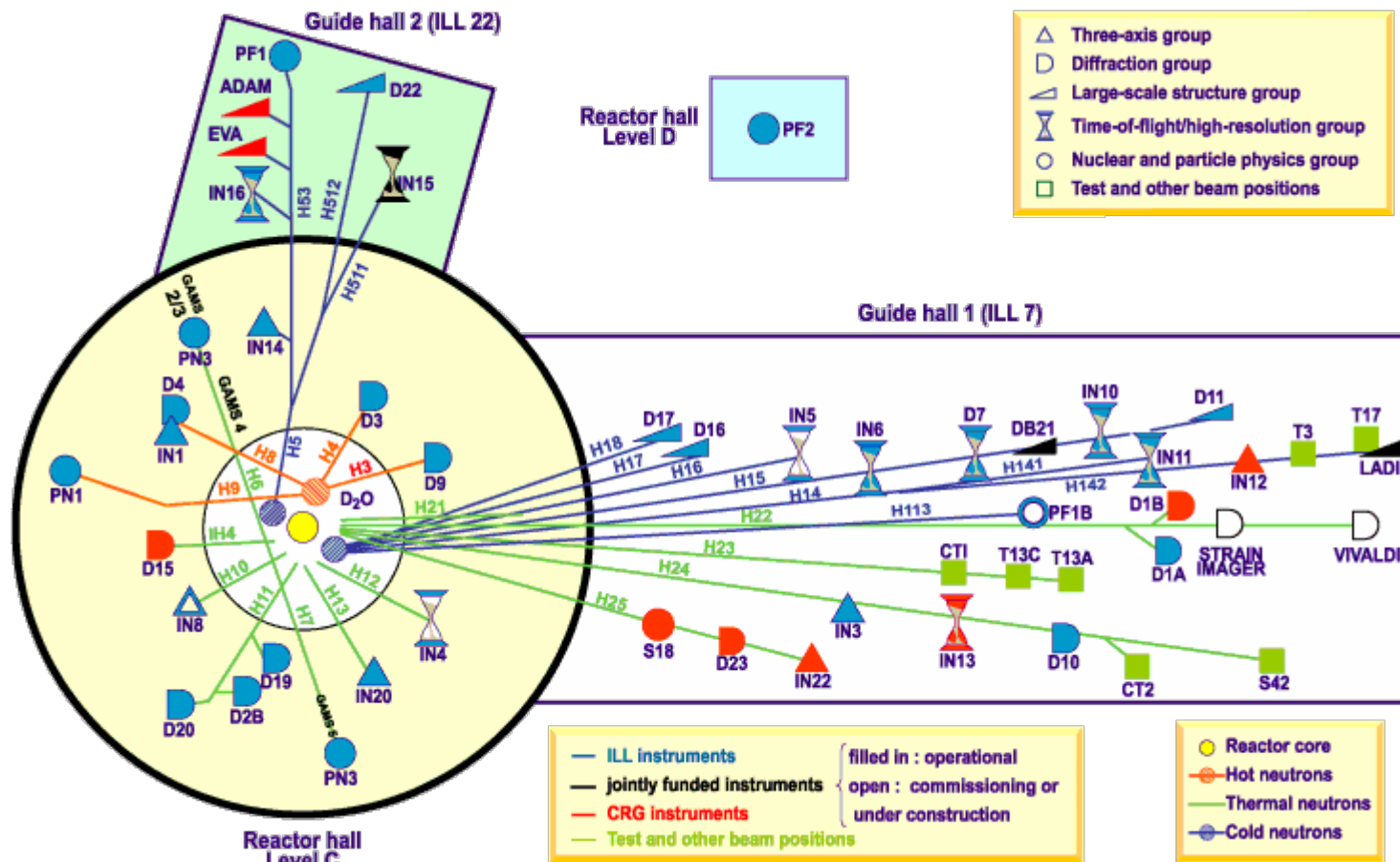
Several experiments with electrons:

Liu *et al.* (1998), Henny *et al.* (1999), Oliver *et al.* (1999), Kiesel *et al.* (2002), Ji *et al.* (2003).

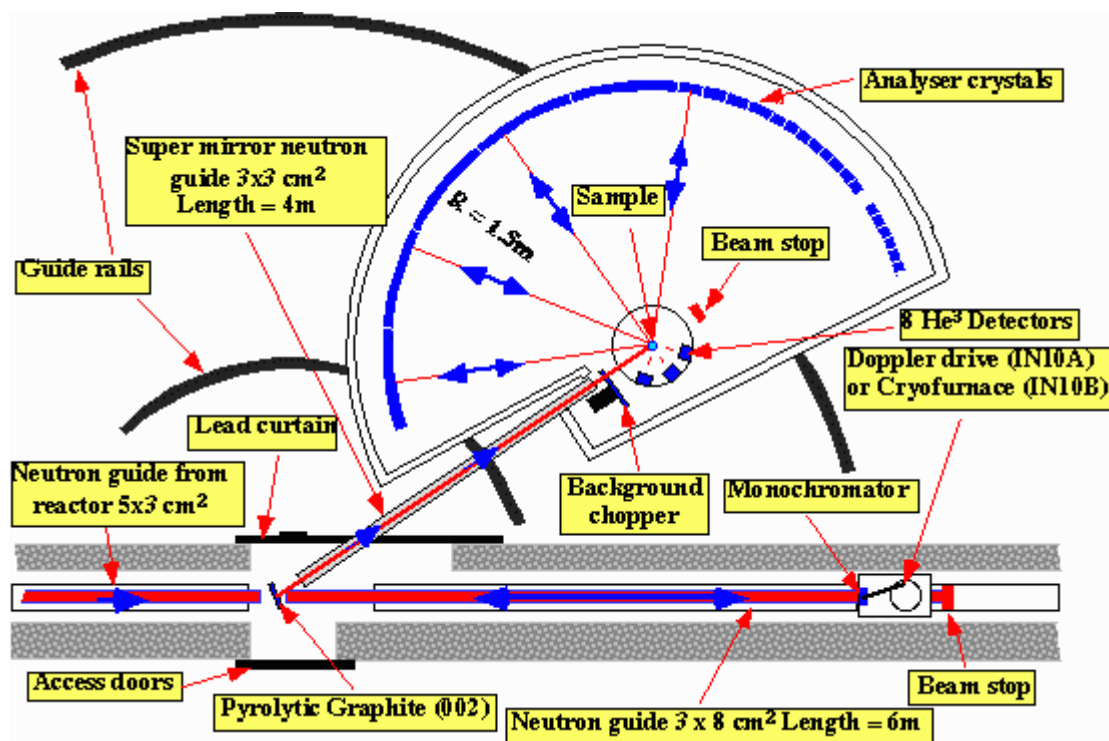
Institut Laue-Langevin (ILL), Grenoble



Instruments Map

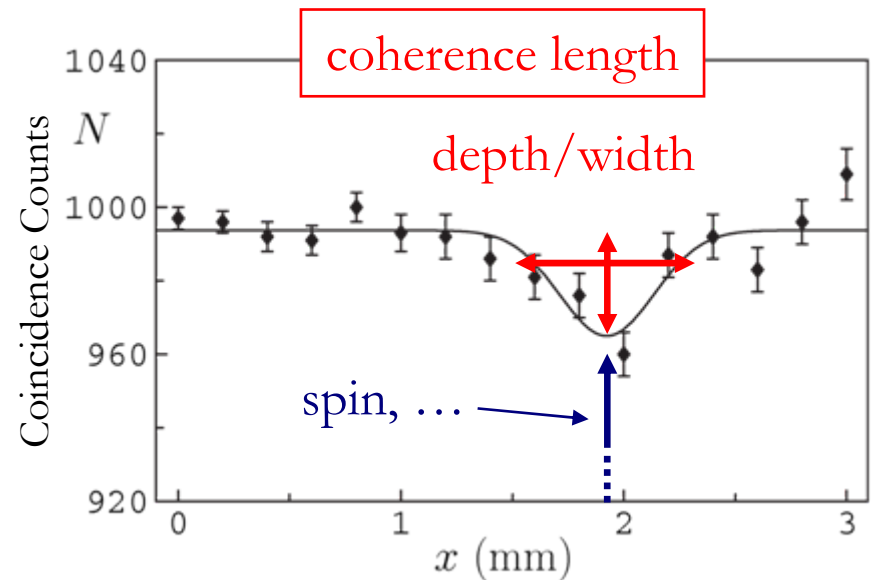
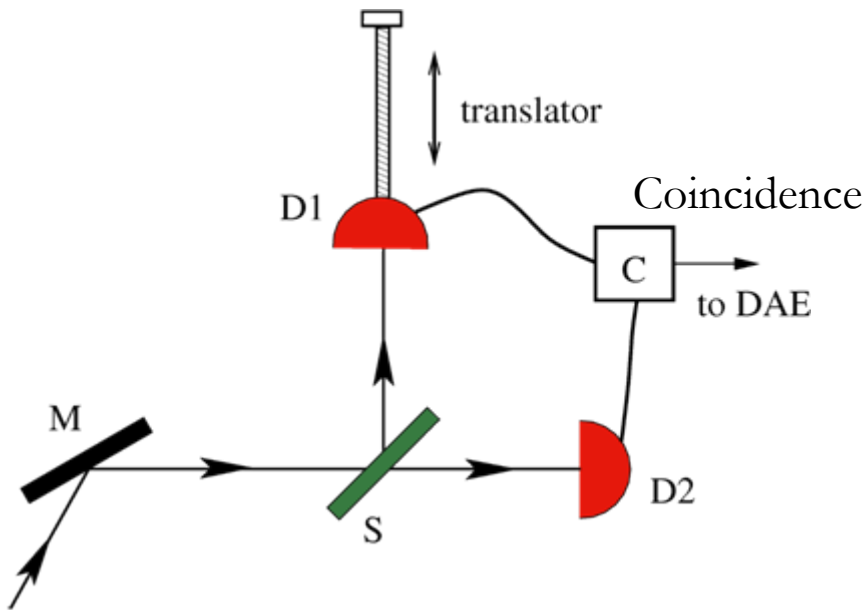


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Neutron Antibunching Experiment

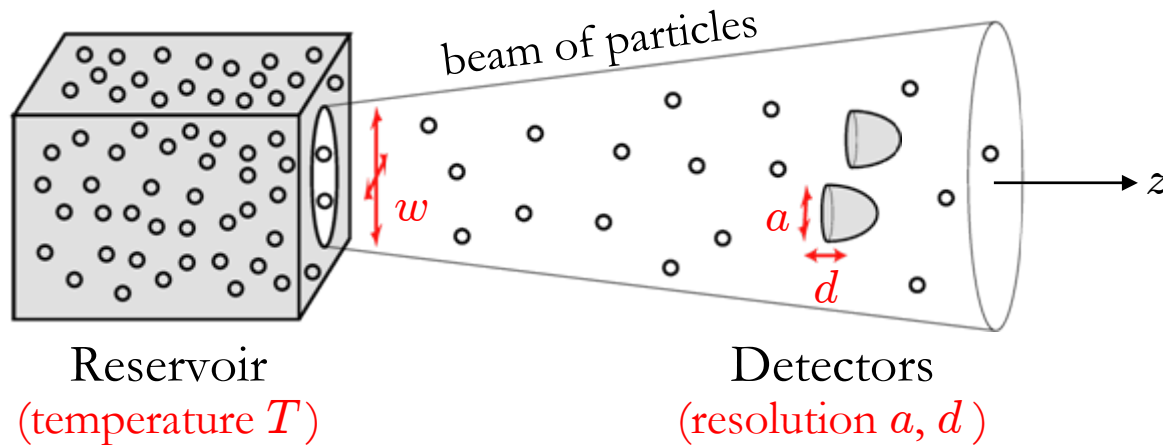
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Dynamical Formulation of Coincidence Exp.



- Hamiltonian $H = H_{\text{beam}} + H_{\text{reservoir}} + \lambda V$ in 3D

$$\begin{cases} H_{\text{beam}} = \sum_{\sigma} \int d^3 \mathbf{p} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^{\dagger} c_{\mathbf{p}\sigma}, & H_{\text{reservoir}} = \sum_{\sigma} \int d^3 \mathbf{k} \omega_{\mathbf{k}} a_{\mathbf{k}\sigma}^{\dagger} a_{\mathbf{k}\sigma} \\ V = \sum_{\sigma} \int d^3 \mathbf{p} \int d^3 \mathbf{k} (T_{\mathbf{p}\mathbf{k}} c_{\mathbf{p}\sigma}^{\dagger} a_{\mathbf{k}\sigma} + \text{h.c.}) & \{c_{\mathbf{p}\sigma}, c_{\mathbf{p}'\sigma'}^{\dagger}\} = \delta_{\sigma\sigma'} \delta^3(\mathbf{p} - \mathbf{p}') \\ & \{a_{\mathbf{k}\sigma}, a_{\mathbf{k}'\sigma'}^{\dagger}\} = \delta_{\sigma\sigma'} \delta^3(\mathbf{k} - \mathbf{k}') \end{cases}$$

- Initial State (Temp. $T \otimes$ Vacuum) $\xrightarrow[\text{Hamiltonian}]{t \rightarrow \infty}$ Stationary Beam

Correlation Functions

- First-Order Correlation (in interaction picture)

$$\langle \psi_{\sigma}^{\dagger}(\mathbf{r}, t) \psi_{\sigma'}(\mathbf{r}', t) \rangle = \delta_{\sigma\sigma'} \rho_t^{(1)}(\mathbf{r}|\mathbf{r}'), \quad \psi_{\sigma}(\mathbf{r}, t) = \int \frac{d^3\mathbf{p}}{\sqrt{(2\pi)^3}} c_{\mathbf{p}\sigma} e^{i(\mathbf{p}\cdot\mathbf{r} - \varepsilon_{\mathbf{p}}t)}$$

- Spin-Summed Two-Particle Distribution

$$\begin{aligned} \rho_t^{(2)}(\mathbf{r}_1, \mathbf{r}_2) &= \sum_{\sigma_1, \sigma_2} \langle \psi_{\sigma_1}^{\dagger}(\mathbf{r}_1, t) \psi_{\sigma_2}^{\dagger}(\mathbf{r}_2, t) \psi_{\sigma_2}(\mathbf{r}_2, t) \psi_{\sigma_1}(\mathbf{r}_1, t) \rangle \\ &= 4\rho_t^{(1)}(\mathbf{r}_1|\mathbf{r}_1)\rho_t^{(1)}(\mathbf{r}_2|\mathbf{r}_2) - 2\rho_t^{(1)}(\mathbf{r}_1|\mathbf{r}_2)\rho_t^{(1)}(\mathbf{r}_2|\mathbf{r}_1) \end{aligned}$$

Wick's Theorem

- Detector Resolution

$$\bar{\rho}_t^{(2)}(\bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2) = \int d^3\mathbf{r}_1 R_{\bar{\mathbf{r}}_1, \mathbf{d}}(\mathbf{r}_1) \int d^3\mathbf{r}_2 R_{\bar{\mathbf{r}}_2, \mathbf{d}}(\mathbf{r}_2) \rho_t^{(2)}(\mathbf{r}_1, \mathbf{r}_2)$$

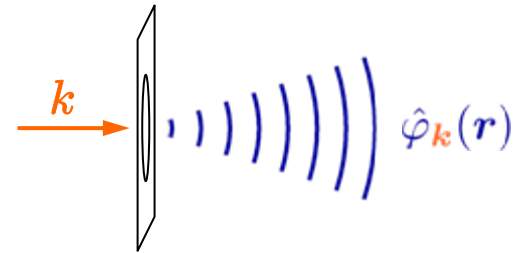
In the following, we consider the Gaussian resolution $R_{\bar{\mathbf{r}}, \mathbf{d}}(\mathbf{r})$ centered at $\bar{\mathbf{r}}$ with the width $\mathbf{d} = (a, a, d)$

Structure of the Correlation Functions

Mixture over the Reservoir Modes

$$\rho_t^{(1)}(\mathbf{r}_1|\mathbf{r}_2) \xrightarrow{t \rightarrow \infty} \lambda^2 \int d^3\mathbf{k} N(\omega_{\mathbf{k}}) \underbrace{\hat{\varphi}_{\mathbf{k}}^*(\mathbf{r}_1) \hat{\varphi}_{\mathbf{k}}(\mathbf{r}_2)}_{\text{“pure” wave from mode } \mathbf{k}}$$

mixture over the Fermi distribution



“pure” wave from mode \mathbf{k}

$$\hat{\varphi}_{\mathbf{k}}(\mathbf{r}) = \int \frac{d^3\mathbf{p}}{\sqrt{(2\pi)^3}} \frac{T_{\mathbf{p}\mathbf{k}} e^{i\mathbf{p}\cdot\mathbf{r}}}{i \epsilon_{\mathbf{p}} - \omega_{\mathbf{k}} - i0^+}$$

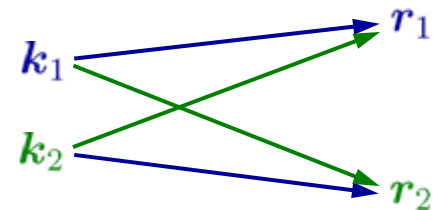
Triplet/Singlet Contributions

$$\rho_t^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = 4\rho_t^{(1)}(\mathbf{r}_1|\mathbf{r}_1)\rho_t^{(1)}(\mathbf{r}_2|\mathbf{r}_2) - 2\rho_t^{(1)}(\mathbf{r}_1|\mathbf{r}_2)\rho_t^{(1)}(\mathbf{r}_2|\mathbf{r}_1)$$

$$\xrightarrow{t \rightarrow \infty} \lambda^4 \int d^3\mathbf{k}_1 \int d^3\mathbf{k}_2 N(\omega_{\mathbf{k}_1})N(\omega_{\mathbf{k}_2}) \left(3 |\Psi_{\mathbf{k}_1\mathbf{k}_2}^{(-)}(\mathbf{r}_1, \mathbf{r}_2)|^2 + |\Psi_{\mathbf{k}_1\mathbf{k}_2}^{(+)}(\mathbf{r}_1, \mathbf{r}_2)|^2 \right)$$

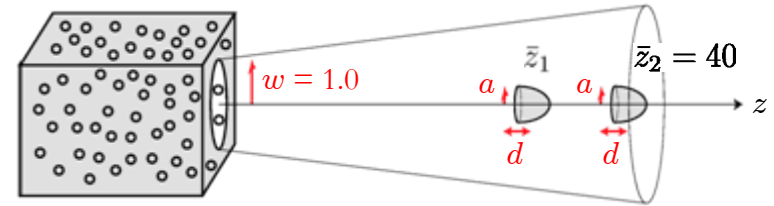
$$\Psi_{\mathbf{k}_1\mathbf{k}_2}^{(\pm)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} \left(\hat{\varphi}_{\mathbf{k}_1}(\mathbf{r}_1)\hat{\varphi}_{\mathbf{k}_2}(\mathbf{r}_2) \pm \hat{\varphi}_{\mathbf{k}_1}(\mathbf{r}_2)\hat{\varphi}_{\mathbf{k}_2}(\mathbf{r}_1) \right)$$

$\left\{ \begin{array}{l} \text{Anti-Symmetric Wave Func.} \longleftrightarrow \text{Triplet Spin} \rightarrow 3 \\ \text{Symmetric Wave Func.} \longleftrightarrow \text{Singlet Spin} \rightarrow 1 \end{array} \right.$

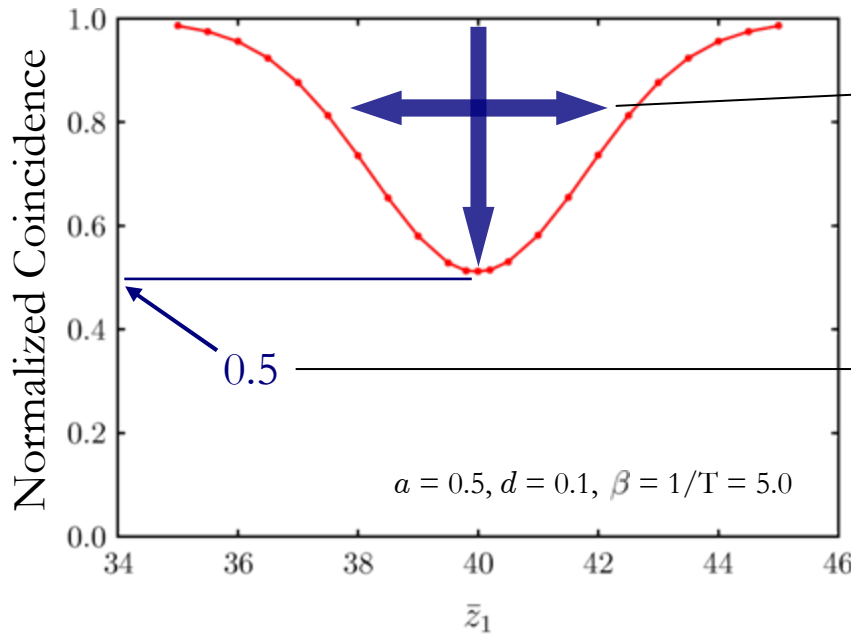


Depth/Width

(with fine resolution)



monochromator $k_0 = 5.0, \delta k_z = 0.5$
 Fermi level $E_F = 15$ ($\hbar = m = 1$)



width of the spectrum

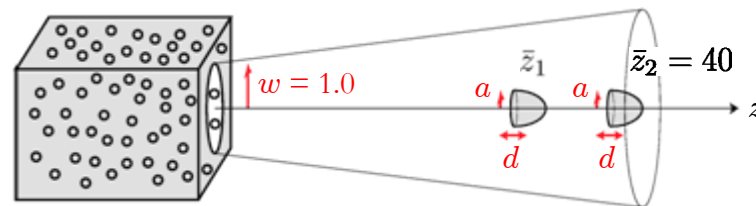
$$\rho_t^{(1)}(\mathbf{r}_1|\mathbf{r}_2) \sim \int d^3\mathbf{k} N(\omega_{\mathbf{k}}) f^2(k_z) \dots e^{-ik_z(\bar{z}_1 - \bar{z}_2)}$$

$$= 1 - \frac{3}{4} + \frac{1}{4}$$

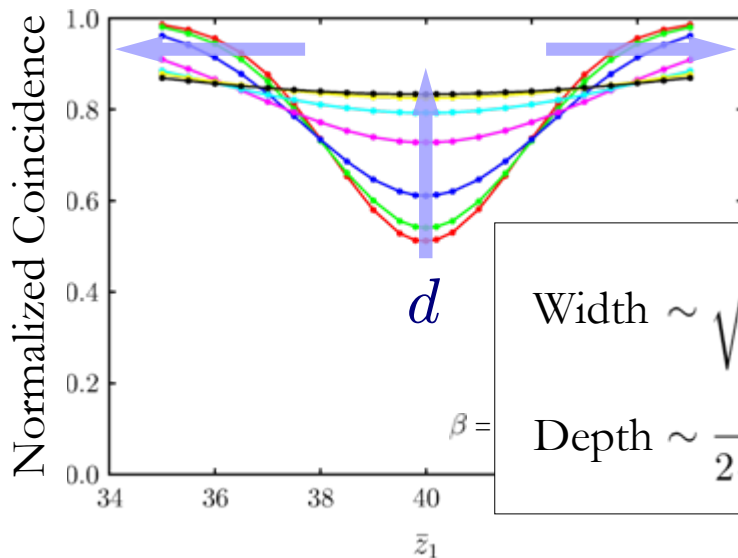
singlet ... bunching

triplet ... antibunching

Detector Resolution

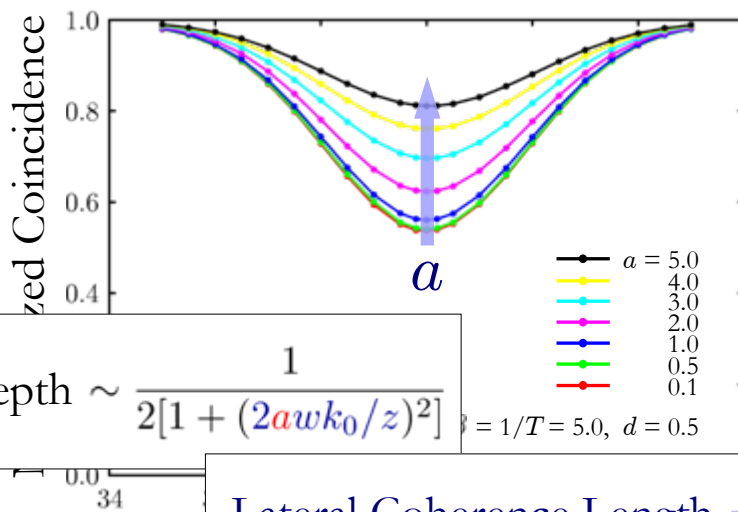


monochromator $k_0 = 5.0, \delta k_z = 0.5$
 Fermi level $E_F = 15$ ($\hbar = m = 1$)



$$\text{Width} \sim \sqrt{\frac{1}{2(\delta k_z)^2} + 2d^2}$$

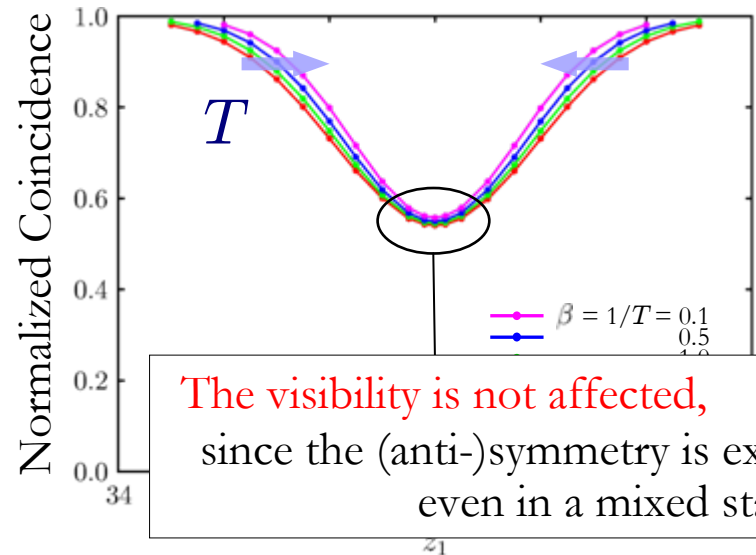
$$\text{Depth} \sim \frac{1}{2\sqrt{1 + 4(\delta k_z)^2 d^2}}$$



$$\text{Depth} \sim \frac{1}{2[1 + (2awk_0/z)^2]}$$

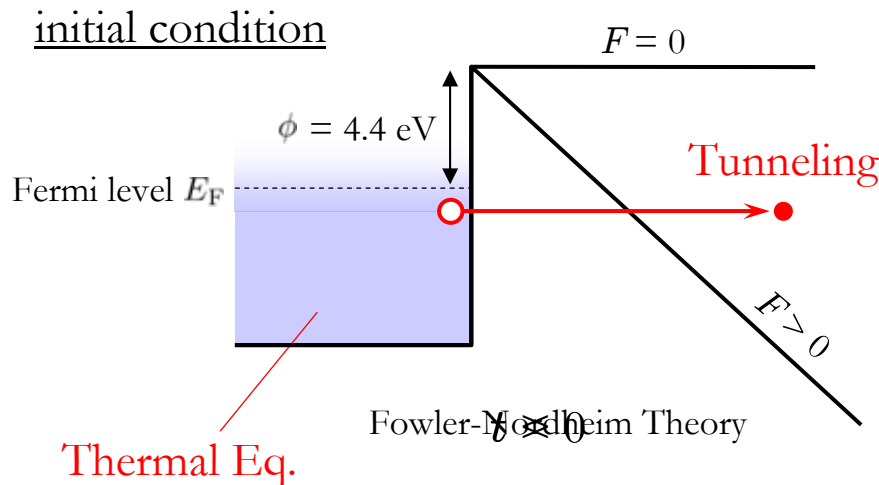
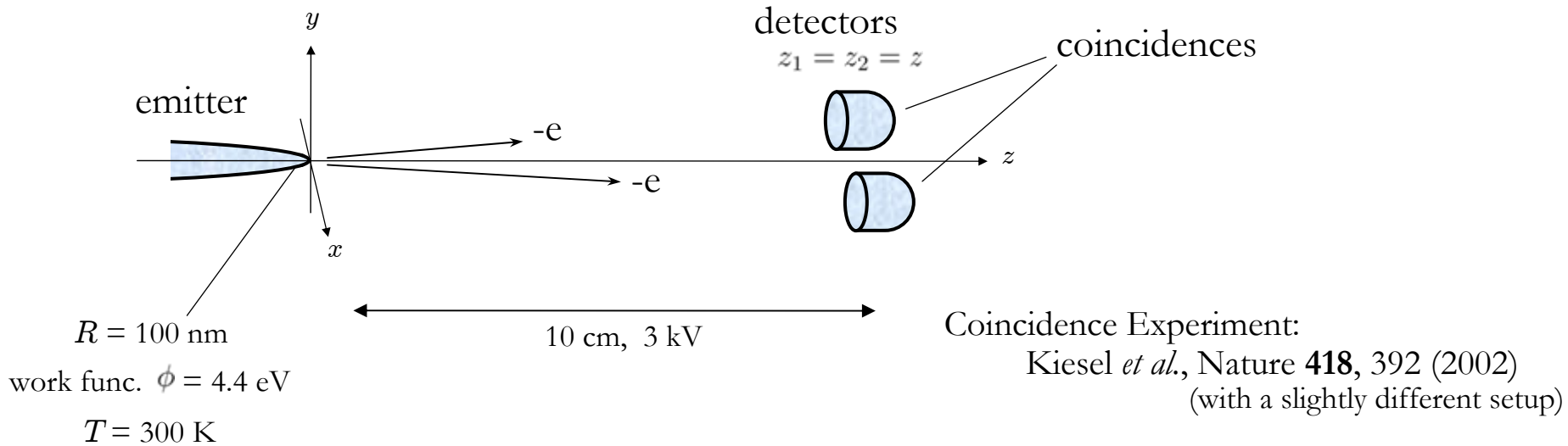
$$\text{Lateral Coherence Length} \frac{z}{2wk_0}$$

Temperature



The visibility is not affected, since the (anti-)symmetry is exact even in a mixed state.

Field Emission



- tunneling Hamiltonian*
- initial-value problem
- 3D
- acceleration
- (Coulomb force neglected)

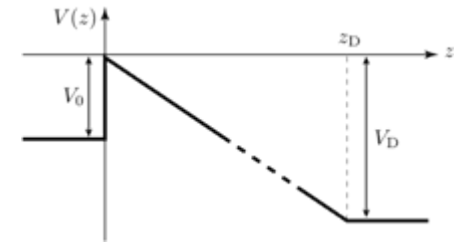
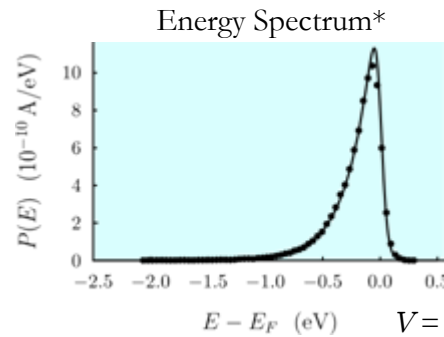
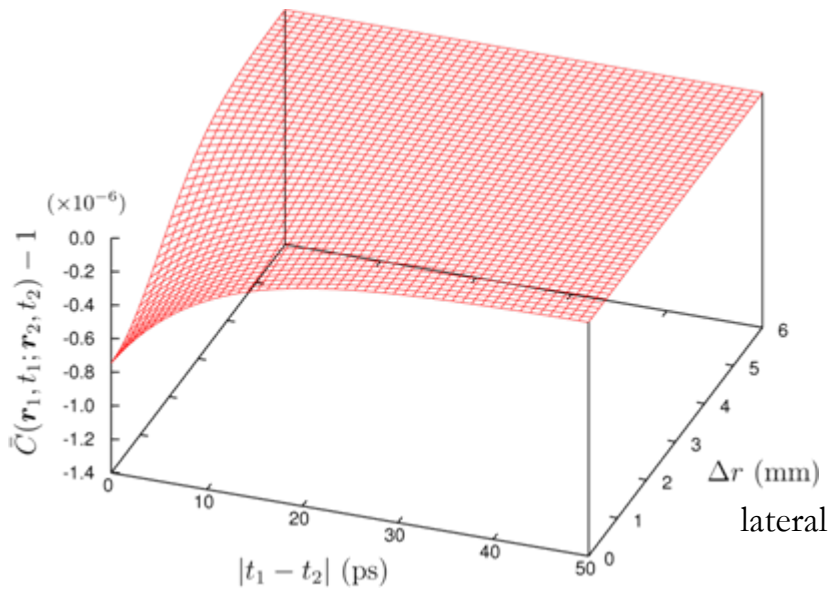
*J. Bardeen, PRL **6**, 57 (1961); etc.

$$\bar{\rho}^{(2)}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) = \bar{\rho}^{(1)}(\mathbf{r}_1)\bar{\rho}^{(1)}(\mathbf{r}_2) \left[1 - \frac{\pi t_c}{4\tau} e^{-|t_1-t_2|/\tau} \frac{1}{1 + a^2(w^2/\ell_c^2)/[a^2 + \Delta^2(z)]} \times \exp\left(-\frac{(x_1-x_2)^2 + (y_1-y_2)^2}{4[1 + a^2/\Delta^2(z)]\{a^2 + (\ell_c^2/w^2)[a^2 + \Delta^2(z)]\}}\right) \right]$$

$z_1 = z_2 = z$
 $T = 0$

response time of the detectors

$$\begin{cases} \ell_c^2 = 3S_F/2\kappa_F^2, & \Delta^2(z) = w^2 + (1 + \ell_c^2/w^2)\frac{(z+z_D)^2}{4\kappa_F z_D}, \\ t_c = 3\hbar S_F/\phi, & \text{beam expansion} \end{cases} \quad \begin{cases} S_F = \frac{2\sqrt{2m\phi^3}}{3\hbar eF} \\ \kappa_F = \sqrt{\frac{2m\phi}{\hbar^2}} \end{cases} \quad \text{tunnel action}$$



$V = 3.8 \text{ kV}$
 $S_F \sim 6.81$

$$\begin{cases} S_F = 7 \\ w = 10 \text{ nm}, T = 0 \\ z = z_1 = z_2 = 10 \text{ cm} \\ z_D = 0.1 \mu\text{m} \end{cases}$$

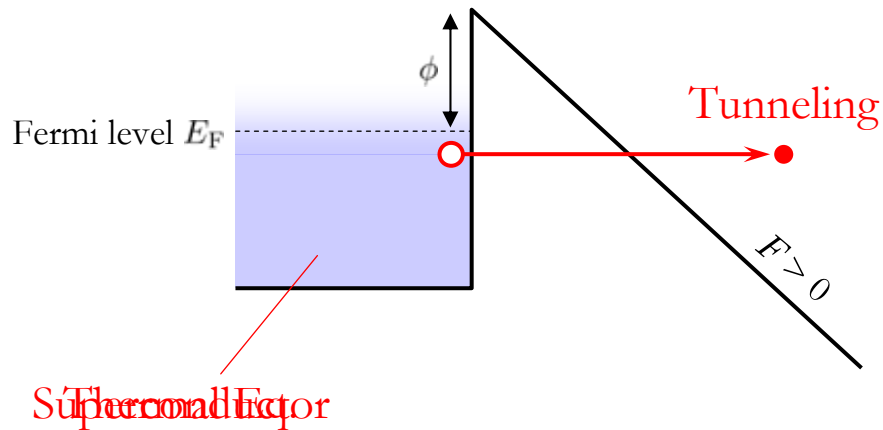
$$\begin{cases} t_c = 3.1 \text{ fs} \\ \ell_c = 0.30 \text{ nm} \\ \Delta(z) = 1.5 \text{ mm} \end{cases}$$

$$\begin{cases} a = 1 \text{ mm} \\ \tau = 10 \text{ ps} \end{cases}$$

- Magnification of the beam would be necessary.
- Coulomb force ?

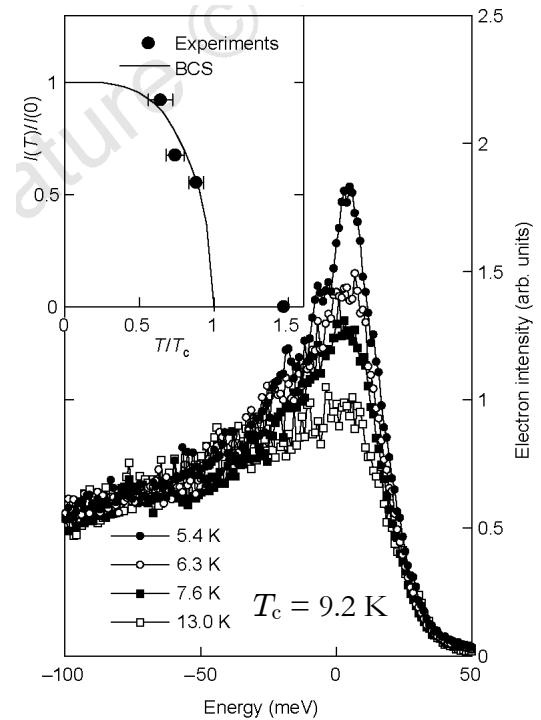
$$\bar{C}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) = \frac{\bar{\rho}^{(2)}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2)}{\bar{\rho}^{(1)}(\mathbf{r}_1, t_1)\bar{\rho}^{(1)}(\mathbf{r}_2, t_2)}, \quad \Delta r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Field Emission from a Superconductor



- sharp energy spectrum
 - ➡ long coherence time/length expected
- Cooper pairs
 - ➡ resource of entanglement ?

To be explored ...



K. Nagaoka *et al.* (Waseda, Tokyo)
Nature **396**, 557 (1998).

Summary

- Dynamical Formulation of Coincidence Exp.
 - The reservoir is treated as a quantized dynamical system.
 - ⇒ The temperature is naturally introduced.
 - The emission process is described dynamically.
 - ⇒ The stationary beam is prepared naturally.
 - Detector Resolution / Lateral Effect / Temperature Effect
- An Interesting Future Subject
 - Thermal Reservoir \Leftrightarrow Superconductor*
Cooper-Pair Correlation \longrightarrow Entanglement ??

*Nagaoka *et al.* (Waseda, Tokyo), Nature **396**, 557 (1998).