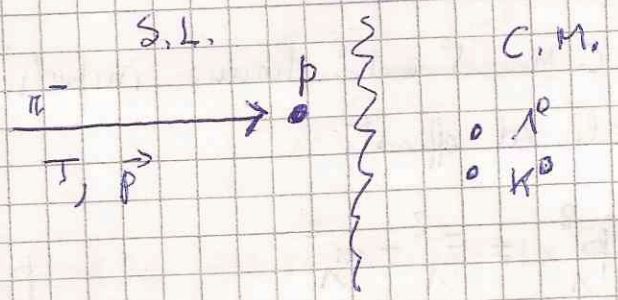
18/2/2013

DIPARTIMENTO DI FISICA
UNIVERSITA' DEGLI STUDI
Via Irnerio, 46
40126 BOLOGNA

④

$$\pi^- + p \rightarrow \Lambda^0 + K^0$$



①

$$P_{SL} \equiv (T + m_\pi + m_p, \vec{p})$$

$$P_{CM} \equiv (m_\Lambda + m_K, 0)$$ alla soglia Λ^0 e K^0 sono ferme nel C.M.

$$P_{SL}^2 = P_{CM}^2$$

$$(T + m_\pi + m_p)^2 - p^2 = (m_\Lambda + m_K)^2$$

esprimo p in termini di en. cinetica T $\quad p = \sqrt{T^2 + 2Tm_\pi}$

$$T^2 + 2T(m_\pi + m_p) + (m_\pi + m_p)^2 - T^2 - 2Tm_\pi = (m_\Lambda + m_K)^2$$

$$2Tm_p = (m_K + m_\Lambda)^2 - (m_\pi + m_p)^2$$

$$T = \frac{(m_\Lambda + m_K)^2 - (m_\pi + m_p)^2}{2m_p} =$$ Energia di soglia del π^-

$$T = \frac{(1115.683 + 492.622)^2 - (139.570 + 938.272)^2}{2 \times 938.272} = \boxed{762.992 \text{ MeV}}$$

② En. Totale disponibile nel C.M. per π^- con $T = 1$ GeV

$$P_{SL} \equiv (T + m_\pi + m_p, \vec{p}) \qquad P_{CM} \equiv (E_{CM}, 0)$$

$$P_{SL}^2 = P_{CM}^2 \qquad E_{CM}^2 = (T + m_\pi + m_p)^2 - p^2 =$$

$$= T^2 + 2T(m_\pi + m_p) + (m_\pi + m_p)^2 - T^2 - 2Tm_\pi$$

$$E_{CM} = \{2Tm_p + (m_\pi + m_p)^2\}^{1/2} = \{2 \times 10^3 \times 938.272 + (139.57 + 938.272)^2\}^{1/2} =$$

$$E_{CM} = \boxed{1243.02 \text{ MeV}}$$

③ Energia totale e impulsi delle Λ^0 e K^0 nel C.M.

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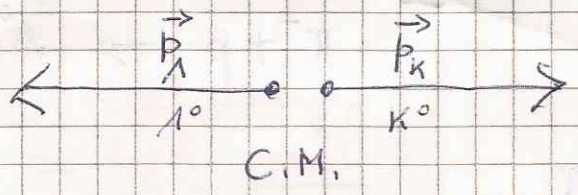
nel C.M. Λ^0 e K^0 hanno impulsi uguali ed opposti:



$$p_\Lambda^2 = E_\Lambda^2 - m_\Lambda^2 \qquad p_K^2 = E_K^2 - m_K^2$$

$$p_\Lambda = p_K \Rightarrow E_\Lambda^2 - m_\Lambda^2 = E_K^2 - m_K^2$$

ma $E_\Lambda + E_K = E_{CM}$ $\qquad E_K = E_{CM} - E_\Lambda$

$$\rightarrow E_\Lambda^2 - m_\Lambda^2 = (E_{CM} - E_\Lambda)^2 - m_K^2$$

$$E_\Lambda^2 - m_\Lambda^2 = E_{CM}^2 - 2E_{CM}E_\Lambda + E_\Lambda^2 - m_K^2$$

$$E_\Lambda = \frac{E_{CM}^2 + m_\Lambda^2 - m_K^2}{2E_{CM}} = \frac{1243.0^2 + 1115.683^2 - 493.677^2}{2 \times 1243.0} = \boxed{1152.545\ \text{MeV}}$$

$$p_\Lambda = \sqrt{E_\Lambda^2 - m_\Lambda^2} = \{1152.545^2 - 1115.683^2\}^{1/2} = \boxed{308.483\ \text{MeV/c}}$$

$$E_K = E_{CM} - E_\Lambda = 1243.0 - 1152.545 = \boxed{585.525\ \text{MeV}}$$

$$p_K = \sqrt{E_K^2 - m_K^2} = \{585.525^2 - 493.677^2\}^{1/2} = \boxed{308.483\ \text{MeV/c}}$$

p_K

④ β_{CM} e γ_{CM} del moto del CM rispetto al S.L.

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impulso del π^- con $T = 1\,\text{GeV}$

$$p = \sqrt{T^2 + 2Tm_\pi} = \{(10^3)^2 + 2 \times 10^3 \times 139.57\}^{1/2} = 1130.991\ \text{MeV/c}$$

$$\beta_{CM} = \frac{pc}{(T + m_\pi) + m_p} = \frac{1130.991}{10^3 + 139.57 + 938.272} = \boxed{0.54431}$$

$$\gamma_{CM} = \frac{1}{\sqrt{1 - \beta_{CM}^2}} = \frac{1}{\sqrt{1 - 0.54431^2}} = \boxed{1.19206}$$

(5) Energia e impulso del K^0 nel S.L. nell'ipotesi che sia emesso nella direzione del π^-

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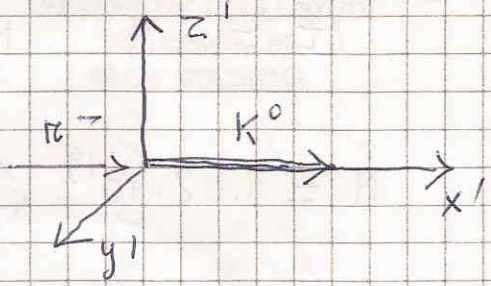

CM $\begin{cases} p'_x = 308.483 \text{ MeV/c} \\ p'_y = p'_z = 0 \end{cases}$

S.L. $\begin{cases} p_x = \gamma_{CM}\left(p'_x + \beta_{CM} E_K^{CM}\right) = 1.19206\left(308.483 + 0.54431 \times 585.525\right) \\ p_y = p_z = 0 \end{cases}$

$= \boxed{747.648} \text{ MeV/c}$

$$E_K^{SL} = \sqrt{p_x^2 + m_K^2} = \left\{747.648^2 + 497.672^2\right\}^{1/2} = \boxed{898.139} \text{ MeV}$$

Verifica: $E_K^{SL} = \gamma_{CM}\left(E_K^{CM} + \beta_{CM} p'_x\right) =$ ← O.K.

$= 1.19206\left(585.525 + 0.54431 \cdot 308.483\right) = 898.140 \text{ MeV}$

(8) 1) Attività del ^{14}C. All'equilibrio
Tanti ^{14}C si formano $\equiv \phi$, Tanti decadono $\equiv dt$

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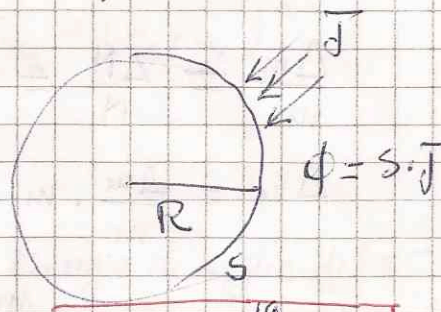

$$dt(^{14}C) = \Phi = S \cdot J = 4\pi R^2 \cdot J$$

$$dt(^{14}C) = 4\pi\left(6321 \cdot 10^5 \text{ cm}\right)^2 \cdot 120 \frac{\text{n}}{\text{cm}^2 \cdot 60 \text{ s}} = \boxed{1.020 \cdot 10^{19} \text{ Bq}}$$

$$\frac{\Delta dt}{dt} = 2\frac{\Delta R}{R} + \frac{\Delta J}{J} \sim \frac{\Delta J}{J} = \frac{30}{120} = 0.25 \quad 25\%$$

$$\Delta dt = \frac{\Delta dt}{dt} \cdot dt = 0.25 \times 1.02 \cdot 10^{19} = \boxed{2.55 \cdot 10^{18}} \text{ Bq}$$

$$dt(^{14}C) = \left(1.02 \pm 0.25\right) \cdot 10^{19} \text{ Bq}$$

2) N° di atomi di ^{14}C presenti $\qquad \lambda = \frac{\ln 2}{T_{1/2}}$

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$$dt = \lambda N \qquad N = \frac{dt}{\lambda} = \frac{dt \cdot T_{1/2}}{\ln 2}$$

$$N = 1.02 \cdot 10^{19} \; \frac{5730 \times 3.156 \cdot 10^7 \text{ s}}{\ln 2} = \boxed{2.661 \cdot 10^{30}}$$

$$\frac{\Delta N}{N} = \frac{\Delta dt}{dt} + \frac{\Delta T_{1/2}}{T_{1/2}} = 0.25\% + \frac{40}{5730} = 0.25\%$$

$$\Delta N = \frac{\Delta N}{N} \cdot N = 0.25\% \times 2.661 \cdot 10^{30} = \boxed{6.8 \cdot 10^{29}}$$

$$N = (2.66 \pm 0.68) \cdot 10^{30} \text{ atomi}$$

che corrispondono ad una massa di

$$m = N \cdot \frac{A}{N_A} = 2.661 \cdot 10^{30} \; \frac{14.0032}{6.022 \cdot 10^{23}} = 6.19 \cdot 10^3 \text{ g}$$

$$= \boxed{6.19 \cdot 10^4 \text{ kg}}$$

$$\frac{\Delta m}{m} = \frac{\Delta N}{N} = 0.25\%$$

$$\Delta m = \frac{\Delta m}{m} \cdot m = 0.25\% \times 6.19 \cdot 10^4 = 1.59 \cdot 10^4$$

$$m = (62 \pm 16) \cdot 10^3 \text{ kg}$$

3) Attività specifica $\qquad \frac{dt(^{14}C)}{m} = \frac{1.02 \cdot 10^{19}}{6.19 \cdot 10^7} = \boxed{1.65 \cdot 10^{11} \; \frac{\text{Bq}}{\text{g}}}$

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4) produzione annua di ^{14}C $\quad 1y = 3.156 \cdot 10^7 s$

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$$\Delta m = \frac{dN}{dt} \cdot \frac{A}{N_A} \cdot t = J \cdot S \cdot \frac{A}{N_A} \cdot t = ct \cdot \frac{A}{N_A} \cdot t$$

$$\Delta m = 1.02 \cdot 10^{19} \cdot \frac{14.0032}{6.022 \cdot 10^{23}} \cdot 3.156 \cdot 10^7 = 2.5 \cdot 10^3 g = 2.5 \text{ Kg}$$

5) Massa di ^{12}C in equilibrio col ^{14}C

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$$N(^{12}C) = \frac{10^{12}}{1.5}$$

$$\frac{m(^{12}C)}{A(^{12}C)} \cdot N_A = \frac{10^{12}}{1.5} \cdot \frac{m(^{14}C)}{A(^{14}C)} \cdot N_A$$

$$m(^{12}C) = \frac{10^{12}}{1.5} \cdot \frac{A(^{12}C)}{A(^{14}C)} \quad m(^{14}C) = \frac{10^{12}}{1.5} \frac{12.0004}{14.0032} \cdot 6.19 \cdot 10^4$$

$$m(^{12}C) = 3.5 \cdot 10^{16} \text{ Kg}$$